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Theses of the PhD Dissertation

Systemic Risk and Contagion in Interbank Networks

From Ex-Post Measurement to
Forward-Looking Valuation

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Budapest, 2026

Contents

I Research background and justification of the topic

The global financial crisis of 2007–2009 exposed critical vulnerabilities in the interconnected structure of modern financial systems. The collapse of Lehman Brothers in September 2008 triggered a cascade of failures that propagated through the global financial network, demonstrating that the failure of sufficiently interconnected institutions could threaten the stability of the entire system (Haldane and May, 2011; Reinhart and Rogoff, 2009). Since then, systemic risk—the risk that the failure of one or more financial institutions triggers widespread failures—has become a central concern for regulators, policymakers, and academics. The Financial Stability Board explicitly identified the measurement and management of systemic risk as a key priority (Financial Stability Board, 2009).

The challenge is fundamentally rooted in the network structure of financial obligations. Financial institutions are connected through interbank loans, derivatives contracts, and payment obligations. This interconnectedness facilitates risk sharing and liquidity provision, but also creates channels through which shocks propagate (Acemoglu et al., 2015; Allen and Gale, 2000). Two primary channels of contagion have been identified: direct contagion through contractual obligations and indirect contagion through common exposures or fire sales (Glasserman and Young, 2016). Understanding and quantifying systemic risk therefore requires mathematical models that capture both the network structure and the mechanisms of default propagation.

The foundational framework for analyzing clearing in networks of financial obligations was established by Eisenberg and Noe (2001), who determine equilibrium payments when multiple institutions are simultaneously insolvent. Subsequent extensions have incorporated bankruptcy costs (Rogers and Veraart, 2013), multiple maturities (Kusnetsov and Veraart, 2019), and decentralized clearing (Csóka and Herings, 2018). Beyond modeling default cascades, a crucial question concerns the allocation of systemic risk to individual participants, with approaches based on cooperative game theory (Denault, 2001), marginal contribution measures such as CoVaR (Adrian and Brunnermeier, 2016), and network-based indicators such as DebtRank (Battiston et al., 2012).

While the static clearing framework captures defaults at the moment of settlement, it does not account for the buildup of systemic stress before actual default. A second generation of models introduces ex-ante valuation adjustments (Barucca et al., 2020; Bardoscia et al., 2019), and Feinstein and Sojmark (2022) develop a fully endogenous framework where mark-to-market valuations and default probabilities are jointly determined in equilibrium. However, the computational cost of their tree-based method grows exponentially with the number of banks.

Meanwhile, the interaction of asset correlation and network contagion has received limited analytical treatment. Many network models assume either independent or perfectly correlated shocks, but intermediate correlation regimes—which are empirically most relevant (Banerjee and Feinstein, 2022)—can produce qualitatively different behavior. This gap motivates a systematic analysis of how the correlation structure interacts with network topology.

This dissertation addresses these gaps by developing three stylized, mechanism-focused models of solvency contagion in interbank liability networks. All chapters share a common balance-sheet backbone built on the Eisenberg–Noe clearing framework, but each isolates a different mechanism. The formal object throughout is a network of fixed nominal bilateral obligations among banks, combined with exogenous external assets and chapter-specific default or valuation rules. The framework deliberately does not represent collateralization, close-out netting, margining, CCP novation, fire sales, or liquidity dynamics, so results should be read as mechanism-focused analytical contributions rather than complete supervisory stress tests.

The three research questions are:

1. How can systemic importance be measured and allocated to individual institutions using minimally required capital injections within a network clearing framework?

2. How does the correlation structure of external asset shocks interact with network topology to determine default probabilities, and what is the role of contagion?

3. How can forward-looking, mark-to-market valuation of interbank claims be made self-consistent, and what new contagion mechanisms emerge from this valuation channel?

II Methods used

The three chapters employ progressively richer methodological frameworks while maintaining a common foundation in network clearing theory.

Chapter 2 uses cooperative game theory and coherent risk measures within a static clearing framework. The financial system is modeled as a network $(\mathcal{N}, z, L)$ of agents with endowments z and a liability matrix L, where equilibrium payments are determined by the clearing model and proportional bankruptcy rule in Eisenberg and Noe (2001). For each shock scenario and coalition of banks, the minimal capital injection needed to restore solvency is computed. These scenario-wise injections are aggregated across scenarios using Expected Shortfall, yielding a transferable-utility cooperative game whose characteristic function captures systemic risk. The Shapley value is then applied to obtain a systemic-importance indicator for each institution. A simulation with 200,000 Monte Carlo scenarios and bootstrap confidence intervals illustrates the framework.

Chapter 3 introduces uncertainty via a one-factor Gaussian copula model for external asset shocks: $\mathscr{A}_i = \sigma(\sqrt{\rho}\,Z + \sqrt{1-\rho}\,\varepsilon_i) + \mu$, where Z is a common factor, the ε_i are idiosyncratic factors, and $\rho \in [0,1]$ controls pairwise correlation. Two complementary analytical approaches are developed:

- A *Bayesian network* formulation that exploits the symmetries of fully connected and ring topologies to derive exact conditional default probabilities without Monte Carlo simulation.
- A *mean-field approximation* that replaces the many-body interaction with a representative-agent fixed-point equation, characterizing behavior as the number of agents grows to infinity.

Both approaches use the zero-recovery version of the clearing mechanism of Rogers and Veraart (2013) and select the greatest equilibrium survival vector.

Chapter 4 extends the model to multiple discrete time periods $\mathscr{T} = \{0, 1, \ldots, T\}$ by introducing interim mark-to-market revaluation of interbank claims. Haircuts are defined as conditional to-maturity default probabilities, and the key requirement is *strong self-consistency*: the probabilities used to discount claims must coincide with the actual default probabilities implied by the system dynamics those very discounts generate. This amounts to a forward–backward fixed point.

Computing this fixed point is achieved via a Machine Learning method resembling the *temporal-difference (TD) learning* scheme (Sutton and Barto, 2018). Neural networks are trained backward in time: at each date t, the post-clearing survival indicator at t and the predicted default probability at $t+1$ are combined to form a one-step bootstrap target for the predictor at t. Clearing is enforced at each step to guarantee weak self-consistency during training. A permutation-equivariant DeepSets architecture exploits the symmetry of homogeneous systems, enabling experiments with larger networks.

Three benchmarks are developed: (i) state-independent haircuts that condition only on survival up to time t; (ii) a comonotonic benchmark where all external assets move in lockstep, collapsing the problem to a scalar recursion; and (iii) a static-barrier-crossing proxy based on the reflection principle for arithmetic Brownian motion.

III Scientific results of the dissertation

Thesis 1. *(Chapter 2)*

I proposed a framework for measuring systemic importance in financial networks with circular obligations using minimally required capital injections.

For any subset (coalition) of banks C in the system and any realized shock scenario s, the model computes the minimum capital injection k_C^s needed to keep all members of C solvent under proportional clearing. These scenario-wise injections are collected into a realization vector $X(C)_s = -k_C^s$, aggregated across scenarios by a coherent risk measure ρ (Expected Shortfall), and the resulting cooperative game with characteristic function $v(C) = -\rho(X(C))$ is solved via the Shapley value to obtain a systemic-importance indicator φ_i for each institution. This indicator attributes the system's total risk to individual participants, capturing coalitional effects that marginal-contribution measures cannot.

Thesis 2. *(Chapter 2)*

I proved the equivalence of restricted and unrestricted minimal capital injections.

Under the assumptions of the model, the minimal capital injection required to rescue a coalition C is the same regardless of whether only members of C or any agent in the system may receive the injection ($\kappa_C^s = k_C^s$). The proof shows that any amount of cash is utilized most effectively when given directly to coalition members, because the relative liabilities matrix channels payments through the network with a telescoping-series bound that sums to at most one. This result implies that computing the optimal injection reduces to a single modified clearing step followed by a direct balance-sheet computation, making the framework computationally tractable even with circular obligations.

Thesis 3. *(Chapter 2)*

I showed that the systemic risk capital allocation game based on capital injections is superadditive, and demonstrated that different risk-measurement objects produce materially different systemic-importance rankings.

Superadditivity ($v(C \cup T) \geq v(C) + v(T)$ for disjoint C, T) guarantees a diversification effect: the systemic risk of a union of coalitions never exceeds the sum of their separate risks. This follows from the monotonicity and subadditivity of coherent risk measures.

In a simulation example with a central-agent–lender–borrower topology, capital-injection-based and loss-based indicators produce substantially different systemic relevance. As interbank connectivity increases, the capital-injection indicator assigns dramatically rising importance to the central (hub) agent, while the loss-based indicator remains relatively flat. This demonstrates that the choice of risk-measurement object—nonbank creditor losses versus solvency-restoration costs—materially affects which institutions are deemed systemically important.

Thesis 4. *(Chapter 3)*

I showed that default probabilities in interbank networks are non-monotonic in the pairwise correlation of external asset shocks, exhibiting a characteristic hump shape.

Using both the Bayesian network approach (with fully connected and ring topologies) and the mean-field approximation (for the large-system limit), I demonstrated that default probabilities increase with correlation ρ, peak at intermediate values (typically $\rho \approx 0.8$–0.9), and then decrease rapidly as $\rho \to 1$. This holds for both individual and systemic default probabilities.

Key comparative statics: larger interbank exposure $k\lambda$ increases the peak default probability and shifts the peak toward smaller ρ; smaller distance to default DD makes the hump more pronounced; the ring topology produces substantially higher contagion at all correlations than the fully connected topology. The peak correlation range coincides with empirically observed bank asset correlations (Banerjee and Feinstein, 2022), suggesting that assuming perfect correlation may seriously underestimate default probabilities.

Thesis 5. *(Chapter 3)*

I proved that, in a homogeneous symmetric interbank network, comonotonic endowments minimize each agent's default probability, and identified the underlying mechanism as a symmetry property.

Under the homogeneity assumptions and the one-factor Gaussian copula, $\rho = 1$ implies $\mathscr{A}_i = \sigma Z + \mu$ for all i, so all agents cross the fundamental-default boundary at the same realization of the common factor. No state exists in which some agents are fundamentally solvent while others are insolvent, hence contagion defaults disappear and $P(i) = P_f(i)$.

I termed this mechanism *heterogeneity in asset values at clearing* (HAVC): the hump shape arises from the tension between weakening diversification (pushing default probabilities up as ρ rises) and disappearing contagion states (pulling them down near $\rho = 1$). Crucially, this is a symmetry result: with ex-ante heterogeneity (e.g., different distances to default across agents), contagion can reappear even under $\rho = 1$, and the optimality of perfect correlation can break down.

Thesis 6. *(Chapter 4)*

I developed a temporal-difference (TD) learning approach for computing strongly self-consistent, state-dependent, forward-looking valuations of interbank claims.

The method trains time-indexed neural networks backward from maturity: at each date t, the post-clearing survival indicator and the next-step predictor's output form a one-step TD target. Clearing is enforced at each step, so weak self-consistency holds by construction even before convergence. The approach sidesteps the curse of dimensionality that makes the tree-based method of Feinstein and Sojmark (2022) infeasible for networks with more than a handful of banks.

Robustness checks demonstrate that the learned self-consistent default probabilities are stable to substantial perturbations: training on different buffer-generation procedures (static-barrier proxy vs. external-default overlay) and different warm-starts (optimistic vs. pessimistic initial predictors at roughly 0%, 50%, and 100% default probability) all converge to essentially the same equilibrium.

Thesis 7. *(Chapter 4)*

I identified a new contagion mechanism—the synchronous valuation spiral (SVS)—that arises exclusively under multi-period, state-dependent, mark-to-market valuation, and showed that it can shift the minimal default probability away from perfect correlation.

Unlike the HAVC mechanism of Chapters 2–3, SVS operates through anticipatory markdowns: when interbank claims are repriced at interim dates using state-dependent haircuts, a bad common shock lowers all counterparties' asset values simultaneously, triggering synchronous markdowns that can cause defaults *before maturity* and *before any payment obligation is missed.*

Because SVS is strongest at high correlation (synchronous markdowns are most severe when assets move in lockstep), it pushes in the opposite direction from HAVC, which favors $\rho = 1$. When SVS dominates near $\rho = 1$, the safest correlation shifts to an interior value or zero. In the one-period model or with state-independent haircuts, SVS is inactive and the $\rho = 1$ minimum is preserved—providing a clean identification of the valuation-spiral channel. The vertical gap between state-dependent and state-independent default probability curves at a given ρ directly quantifies the additional default risk attributable to SVS.

IV Main references

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V Own and co-authored publications related to the topic

Peer-reviewed publication

1. Csóka, P. and Sass, Z. (2023). Rendszerkockázati jelentőség mérése minimálisan szükséges tőkeinjekció segítségével . SZIGMA Matematikai-közgazdasági folyóirat, 54(1), 1–20 [ZS: 50%]

   Basis for Chapter 2.

Working papers forming the basis of the dissertation

2. Sass, Z. (2026). Correlated external shocks in a stylized financial network: Bayesian network and mean-field approaches. *Working paper.* [ZS: 100%]

   Basis for Chapter 3.

3. Sass, Z. (2026). Forward-looking network valuation using machine learning. *Working paper.* [ZS: 100%]

   Basis for Chapter 4.