Doctoral School of Economics, Business and Informatics

THESIS COLLECTION

Szádoczkiné Varga Veronika

Ph.D. dissertation titled

Modelling of the insurance market

Thesis supervisor:

Ágoston Kolos Csaba
associate professor

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Department of Operations Research and Actuarial Sciences

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I. Research background and relevance of the chosen field

The main topic of the dissertation is modelling the insurance market. Several economic studies deal with empirical and analytical problems in this area. On the one hand, the actuarial science focuses on the question of estimating the life time, mortality, number of damage and the level of claim, it provides solutions to the problem of pricing, reserve calculation and risk dispersion with statistical and probability theoretical methods. On the other hand, the insurance market is suitable for examining many interesting economic phenomena, such as decision under uncertainty, information asymmetry, anti-selection and moral hazard. In the aging societies of the developed world, these results are useful not only for private health insurers, pension and health funds but also for the public health and pension problems.

Although, the insurance market has an important role in developed countries, COVID 19 caused its temporary slowdown in 2020. The restrictions due to the pandemic influenced the sales processes (agents) and the development of claim payments. In 2021, insurance companies' premium income picked up, especially in the life sector. According to an OECD report, the Hungarian insurance market growth was over the average of OECD countries in 2021 (OECD, 2023).

In Hungary, the premium income of insurance companies has been increasing year by year recently. The trend continued in 2022, the premium income reported by insurers was HUF 1.469 billion, which is 6.9% higher than the premium income of the previous year. According to expectations, this growing trend will continue in 2023. Compared to the previous year, the premium income of life insurances increased by 1.7%, while the income of non-life insurances increased by 11.3%, which is why in 2022, within the total premium income, the share of life insurances was 43.6% compared to the non-life insurance branch (MABISZ, 2023).

The macroeconomic challenges of the sector and its regulation, the evolution of its market structure and its analytical modelling are the topics of many international and domestic actuarial conferences (MAT, 2024). At the Corvinus University of Budapest an academic research group is studying this area, which served as the subject of many articles (Ágoston, 2004; Balog, 2023; Banyár and Regős, 2012; Gyetvai, 2022; Kovács, 2023; Szepesváry, 2022; Vaskövi, 2024; Vékás, 2017).

We present the results of three different studies, which complement each other well and seek answers to equally important and relevant questions. The literature on decisions based on uncertainty is very extensive (Szentpéteri, 1980; Varian, 2014; Winston, 2004). Most of the interesting economic and mathematical phenomena in the insurance market can be traced back to this problem. We examine the impact of risk aversion and the capital requirement under uncertainty with the help of analytical models, using the tools of game theory, microeconomics and market structures. In addition, we analyze the structure of the Hungarian insurance sector with the help of an empirical model and indicators, which is of great importance in terms of the assumptions used in the analytical models.

The first study deal with the effect of the risk aversion in a Bertrand duopoly under uncertainty. We model risk aversion under uncertainty, so players maximize the expected value of their concave utility functions. We introduce a new type of grouping of utility functions, the substance preference. We show that this classification fundamentally affects the evolution of the market equilibrium and does not correspond to any previously used grouping. The research classifies the frequently used utility functions, illustrated by numerical examples or proved analytically.

After the financial crisis of 2008 the regulation of the Systematically Important Financial Institutions were transformed. The bank sector is regulated by the Basel (BIS, 2010) and the insurance sector works under the Solvency II directive since 2016 (Directive 2009/138/EC, 2009). Thus, this also provides an opportunity for new analyses. In the second study we examine the effect of the Solvency Capital Requirement in a Bertrand doupoly on the equilibrium prices and expected profits. In this model the insurance companies are risk neutral, they maximize their expected profit. Using comparative statistics, we examine how the level of security, the number of players, and the capital of insurers influence the equilibrium prices.

In the analytical models, the insurance sector was modeled as an oligopoly. This assumption fundamentally influenced the results. Therefore, the question arises that what form of market structure is closest to the functioning of the sector. In the third study we examined the Hungarian insurance sector using structural (concentration ratios, Herfindahl-Hirschman Index) and non-structural (Panzar and Rosse, 1987) methods. The Panzar and Rosse model gives a testable hypothesis about the factor price elasticity of the sector, from which we can deduce the market structure. We estimate the factor price elasticity with a static and a dynamic panel model between 2010 and 2019. In addition to further analytical models, the market structure analysis may contain interesting and important results for consumer protection, competition supervision as well as insurance supervision.

The presented results open up further research directions and are not restricted to the field of insurance. The three main questions of the thesis are closely related, since knowledge of the market structure is extremely important when using analytical models. Modelling risk aversion and the capital requirement similar results can be observed, higher equilibrium prices may also develop, insurers may fare better (higher expected utility and positive profit), but it may also happen that there is only one insurer operating in the market. A natural continuation of the research could be the joint modelling of the two phenomena, during which risk-averse insurers are faced with the capital requirement constraint.

II. Methodology

II..1 Analytical modelling approach

The analysis of the insurance market (Dionne, 2000) is often based on traditional microeconomic models (Varian, 2014). These models can be defined as non-cooperative games (Osborne and Rubinstein, 1996), and they often use the Nash equilibrium concept. The analytical models of the thesis (Study I and II) use similar models. We present the similarities and the differences of them here.

Firstly, both models are *Bertrand models* (Bertrand, 1883), where two or more companies make simultaneous decision about the price level, while maximizing their own profit. The goods are homogenous, so the consumers buy the cheapest one. This is a perfect information game. In the unique Nash equilibrium of Bertrand price competition the companies set their prices at the level of the marginal costs, so the profits of the companies are zero, if there are no fixed cost in the model. The result is the same as in the case of perfect competition even if there are only two companies on the market.

Secondly, we determine the *Nash equilibrium* in the models (Nash, 1951). This solution concept is a strategy profile in which each player sets the best response to the strategy of the other(s). This is not a unique equilibrium concept. We are looking for the equilibrium on the set of pure strategies (we do not examine the mixed expansion of the game), so the solution does not necessarily exist. In order to determine the price level from which no insurer should unilaterally deviate, we used a visualization in the price and contract number plane.

Thirdly, assumptions about the consumers are also similar. The consumers have the same risk, which can be described with independent random variables, they have the same level of claim with the same level of probability. There is a decreasing demand function in both models. The insurances are homogeneous, so all consumers buy from the insurer that determines the lowest price. If more companies charge the same price, the market is divided equally. The companies cannot refuse a customer. The insurance company offers full insurance, so in the case of damage, it pays the entire damage amount to the injured party. Thus, the insurer's expenses are uncertain.

The main difference between the two models (Study I and Study II) is the risk attitude of the insurance companies. In Study I we assume risk averse insurance companies with concave utility functions, they maximize their expected utility, while in Study II the insurers are risk neutral, they maximize their expected profit. In addition, as the investigated phenomenon, the capital constraint, appears in the second model, there is a cost of holding capital, thus, a fixed cost in this model.

We study the effect of different parameters, we compare equilibria with *Comparative statics*.

II..2 Market structure analysis

There are several structural and non-structural models to determine the market structure of a sector. As structural indicators we use the Herfindahl Hirschman Index (HHI) and the concentration ratios of the biggest three, five and ten companies. The calculating formula of these indexes are the following:

$$C_3 = \sum_{i=1}^{3} r_i$$

$$C_5 = \sum_{i=1}^{5} r_i$$

$$C_{10} = \sum_{i=1}^{10} r_i$$

$$HHI = \sum_{i=1}^{n} r_i^2$$

where r_i is the market share of the ith largest firm and n is the number of the firms. The value of the often used HHI index can take a number between $\frac{1}{n}$ and 1. In the case of a monopoly, the value is 1, the market is completely concentrated, while the closer the market is to perfect competition, the closer the value of the HHI index is to 0, since in this case all actors have an equal share (the market is not concentrated), and the number of actors tends to infinity, due to which $\frac{1}{n}$ tends to 0. Higher concentration may also highlight market collusion. It is also common to give the r_i values as a percentage, in which case the HHI value is given as 10000 times the value listed here.

Matsumoto et al. (2009) pointed out an unfavorable feature of the measure, namely if all the companies in the industry collude, the value of the HHI is the same as the index value without collusion. In addition, the value of the HHI stays similar if the market shares are of a similar size, even if the order of the actors changes from year to year. A change in order can also mean strong competition (Kovács, 2011), but the HHI does not reflect this.

The analysis of the insurance market is based on the Panzar-Rosse model (Panzar and Rosse, 1987), which gives testable implications of profit maximizing companies, so we can deduce the market structure of the sector. The reduced form revenue equation is the following:

$$\pi = R\left(y, z\right) - C\left(y, w, t\right),$$

where $R(y, z)$ is the reduced form revenue function, y is the decision variable and z are further exogenous variables which influence the revenue function. The vector of y contains the decision variables that influence the company's revenue and, directly or indirectly, its costs. In addition to the output level, this can include prices or even advertising expenditures or quality levels. $C(y, w, t)$ is the cost function, where w is the vector of exogenous factor prices and t is the vector of additional exogenous variables that influence cost. This simple model assumes profit maximizing companies. The testable expression is the sum of the factor price elasticities of the reduced form revenue equation:

$$H = \sum_i \frac{\partial R^*}{\partial w_i} \frac{w_i}{R^*},$$

where * indicates the profit maximizing values.

Panzar and Rosse (1987) offers different theorems about the value of the sum of elasticities of gross revenue with respect to input prices. In the case of a neoclassical monopolist or collusive oligopolist, the elasticity is nonpositive, it is equal to unity in the case of a competitive price-taking market in long-run equilibrium. Between these two extreme situations, the factor price elasticity is between 0 and 1 and the market is monopolistic competition. An assumption is that in the case of perfect competition and monopolistic competition the companies are in long-run equilibrium and entry and exit in the market are free, thus, this also should be tested. In long run equilibrium the return rates are not correlated with input prices. To test the long run equilibrium empirically, return on assets (ROA) can be estimated with the same independent variables used in the estimation of the factor price elasticity. In long-run competitive equilibrium, both of the factor price elasticities are zero.

The model has received some criticism in recent years, but it is one of the most common forms of analysis in the banking and insurance sectors. The different size of the firms can cause some problems (Bikker et al., 2012), according to Shaffer and Spierdijk (2015) it may happen that the H statistic is negative or positive at any level of the competition and it would be better to present the statistic as a pass-through rate not a market power measure (Sanchez-Cartas, 2020). Goddard and Wilson (2009) showed that the dynamic rather than a static formulation of the revenue equation should be used to identify the Panzar–Rosse H-statistic, because the fixed effect estimation can be biased towards zero. According to Bikker et al. (2012) a further improvement of the model is that only an unscaled revenue equation gives unbiased estimation, the dependent variable of income should not be scaled, and the model should not contain the total asset as a control variable. These suggestions related to the empirical application were taken into account when performing the analysis.

Both the static and the dynamic approach were used to analyse the *Panel model* (Baltagi, 2021). The panel data have time and cross-sectional dimensions, we can distinguish the database based on which one dominates, or whether the sample is balanced. *The static* panel equation is the following:

$$y = X\beta + u,$$

where y is the dependent variable vector, X is the matrix of the independent variables and u is the residual vector.

The dynamic panel estimation uses the autoregressive specification of the dependent variable as an explanatory variable. The Arellano–Bond estimator (Arellano and Bond, 1991) is a standard estimation tool for dynamic panel models. They apply Generalized Method of Moments (GMM) estimation in which they use first differences to eliminate the individual effects. They solve the endogeneity problem by using all the lagged values of the dependent variables as instruments. The method is also called one-step GMM panel estimation. Two model diagnostic tools are offered, first the over-identification test to check the specification; and second to test the second-order autocorrelation of the error term of the differentiated model to check the dynamics.

III. The main results of the thesis

The main results of the article based dissertation are published in academic journals.

Ágoston, K. Cs. and **Varga, V.** (2020). Bertrand-árverseny állománypreferenciák mellett a biztosítási piacokon (Bertrand price competition with substance preferences in insurance markets). *Szigma, 51*(2), 149-167. https://journals.lib.pte.hu/index.php/szigma/article/view/3261/3066

Varga, V. and Madari, Z. (2023). The Hungarian insurance market structure: an empirical analysis. *Central European Journal of Operations Research 31*(3), 927-940. https://doi.org/10.1007/s10100-023-00842-8

Varga, V. and Ágoston, K. Cs. (2024). Modelling insurance market under solvency capital requirement. *Szigma, 55*(2), 239-255.

The relation of research questions, methods, and results of the studies included in the Ph.D. thesis can be seen in Figure 1.

The central topic of the dissertation is the analysis of the insurance market. The special nature of the sector lies in the uncertainty. Customers are exposed to some kind of risk, which they want to cover by purchasing insurance. In return for the insurance premium, the insurer pays compensation. However, it is not possible to foresee how many and to what extent damage events will occur during the insurance period, or whether a payment obligation will arise at all. In the framework of actuarial analysis, risk is considered to be something that can be measured, so it can be estimated statistically or mathematically. The risk can thus be described by a random variable. Uncertain payments in exchange for insurance premiums appear in all three studies of the dissertation and fundamentally determine the results obtained.

The studies of the effect of the capital requirement and risk aversion were carried out with the help of analytical models. In these models, we have to assume a form of market structure, so for more precise and realistic results, the question arises that which form of market structure approximates the real market structure the most closely. To this end, we examined the Hungarian insurance market with the help of concentration indicators and input price elasticity, and deduced the market structure based on the hypotheses given by Panzar and Rosse (1987).

We examined the effect of risk aversion and the capital requirement using two basically very similar analytical models. In accordance with the market structure results, oligopolistic (duopoly) markets were assumed. The assumptions about customers are also similar. During the analysis of risk aversion, we assumed risk-averse insurers, while companies are risk-neutral if the capital requirement is examined. The results are also very similar. Depending on the parameters, in both cases there may be a continuum of many symmetric Nash equilibria in the market or only one company sells while the other does not. We have also shown that in both cases we can see examples of insurers achieving some kind of surplus compared to their initial wealth, this is an expected utility greater than the utility of the initial assets or a positive expected profit. If this phenomenon can be observed in practice, it greatly affects the income of companies, including the balance sheet data used for empirical analysis. Thus, the phenomena analyzed in the research mutually influence each other.

The research leads to interesting and similar results, which open up new research directions. During the market structure analysis, we cannot reject the hypothesis of perfect competition, so it may be worthwhile to examine the effects of risk aversion and capital constraint in such a framework. Examining the two phenomena together can also be a new direction; risk-averse decision makers face the capital constraint. In addition, the development of lower, clear equilibrium prices can be an important issue from a supervisory point of view, so it is worth investigating another equilibrium concept or the new dynamics of the course of the game.

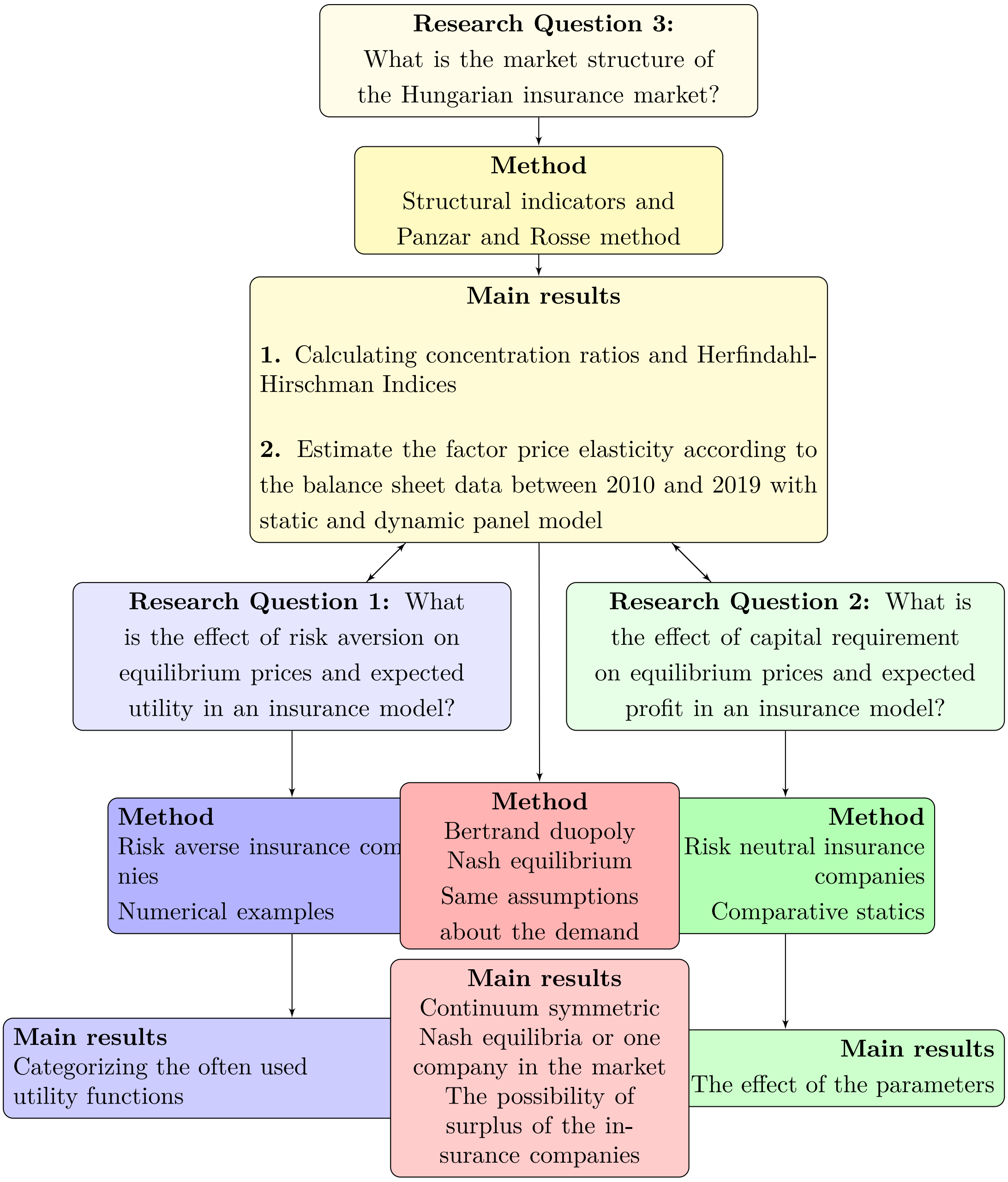

Figure 1: The connection of research questions, methods, and main results

In this section we summarize the research questions and the main results of the included studies, list the individual and joint (inseparable) findings.

III..1 Study I. – The oligopolistic model of the insurance market with substance preference

Study I. deals with the risk aversion of the insurance companies in a Bertrand duopoly. One can think that the insurance companies try to sell as many contracts as they can, but if we assume this risk averse behavior, it is not necessarily true. The risk aversion of the firms occur in the literature, because the companies decide under uncertainty so their risk attitude could be important. We define the substance preference, and show the equilibrium in different cases through examples, and analyze the connections of substance preference and risk preference in more details.

The early literature defined the measure of risk aversion with the absolute risk aversion measurement (Pratt, 1964). According to this, we can talk about Constant, Decreasing and Increasing Absolute Risk Aversion utility functions (CARA, DARA and IARA). Later the fact that risk aversion is not accurate enough in the case when we examine more than one risks at the same time, got bigger attention. Pratt and Zeckhauser (1987) defined a strict property in connection with the utility function that we call proper risk aversion.

We assume risk averse insurance companies with concave utility functions. Based on the utility function, the attitude of insurers to the size of the substance can be different, which fundamentally affects the equilibrium on the market. In order to understand this, let's define the concept of substance preference. For a given number of contracts (n) the *indifferent price* is the price at which level the insurance company is indifferent to sell n contracts or to do not sell any of them. The utility function is *substance averse* if selling one more insurance on this indifferent price level is not preferred, it is *substance seeking* if it is preferred, and *substance neutral* if the company is indifferent.

To see the market equilibrium in the different cases we show examples for each of them. We describe a Bertrand duopoly market, where the companies maximize their expected utility. The behavior of the customers is described with a linear demand function. There is a price competition on the homogeneous insurance market, so they buy at the cheapest company. If the prices are equal the companies share the market equally. At the quoted price the company should serve all the customers, who choose their firm.

We examine the market equilibrium in three groups (substance neutral, averse and seeking). First of all, we prove that the exponential utility function is substance neutral. It is a Constant Absolute Risk Averse function. In this case the equilibrium price is the indifferent price and the insurers' expected utility is equal to the utility of the initial wealth. It is similar to the case of traditional Bertrand game, when the marginal cost is the equilibrium price and the profits are zero.

In the case of substance averse insurers there are continuum symmetric Nash equilibria and extra utility can be achieved. We can also show some cases when an asymmetric market share can be an equilibrium too. We proved that the mixed exponential (DARA) and the quadratic utility functions (IARA) are substance averse, and illustrated with numerical examples that the often used square root and logarithm functions can be listed here.

Last but not least, the case of substance seeking can be illustrated with a modified example from Pratt and Zeckhauser (1987). They propose to avoid such a function describing risk averse behavior. This is an improper function, which does not satisfy the proper risk aversion property. In equilibrium there is only one firm in the market, but it sells on a lower price, than the so-called monopoly price.

Table 1 summarizes the connections between the substance and absolute risk aversion measures. Blue color shows the proper risk aversion property. Substance aversion has a really strong connection with the property of proper risk aversion. Typically the proper risk aversion means substance aversion, however, substance aversion is a more general property, the not proper risk aversion case can be substance averse as well, for instance in the case of the quadratic utility function.

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="3">Absolute Risk Aversion</td></tr>
<tr><td>Constant</td><td>Decreasing</td><td>Increasing</td></tr>
<tr><td rowspan="3">Substance</td><td>Neutral</td><td>Exponential</td><td></td><td></td></tr>
<tr><td>Averse</td><td></td><td>Mixed exponential,
Logarithm,
Square root</td><td>Quadratic</td></tr>
<tr><td>Seeking</td><td></td><td>Modified example</td><td></td></tr>
</table>

Table 1: The connections between the substance and absolute risk aversion measures.

All of the results are equally joint (inseparable) **with our co-author** (*Kolos Ágoston*)**:**

- Extensive literature review connected to risk aversion and insurance models;
- Running simulations to find numerical examples;
- Proof, check the calculations;
- Illustrative figures;
- Editing and writing the article.

III..2 Study II. – Modelling the insurance market under solvency capital requirement

In Study II we analyze the effect of the capital constraint on equilibrium prices and profits. Since 2016 the operation of insurance companies in the European Union is regulated by the Solvency II directive. According to the regulation, the solvency capital requirement of insurance companies should ensure that bankruptcy occurs not more often than once in every 200 cases (Directive 2009/138/EC, 2009). This capital requirement can be calculated as a 99.5% Value at Risk (VaR).

We assume a Bertrand model with profit maximizing companies. There are I insurance companies, and they decide on the price level (P_i) simultaneously. The companies have the same level of capital. Holding the capital has some costs, so there is a fixed cost in the model.

The customers are homogeneous with respect to risk. They face independent risks with the same distribution. Customers have different reservation prices, as shown by a decreasing demand function. If someone buys an insurance and financial loss occurs, then the insurance company will cover it completely (full coverage). Insurance contracts are homogeneous products, thus customers are indifferent between buying it from any of the insurers. Thus, they choose the cheapest insurance company.

In the search for equilibrium, there are four notable prices. The first is the net price, which is the expected level of the damage of one consumer. Since the insurer companies maximize their expected profit, it is not worth selling a contract at a lower price. The second, the monopoly price, is the price at which a single market player would maximize its expected income, which is twice the net price. The third, the intersection of the demand curve and the capital constraint (denoted by P_U), is the lowest price at which a single insurer can serve the market alone while meeting the capital requirement. The fourth, the Ith part of the demand and the intersection of the capital constraint (denoted by P_L) is the lowest price at which the insurers can jointly cover the market while complying with the capital constraint.

The evolution of equilibrium prices is determined by whether the above intersections are located in the increasing or decreasing part of the capital constraint curve. If the increasing section is relevant, then $P_L \leq P_U$. In this case, there are a continuum of many symmetric equilibria on the interval $[P_L, P_U]$, if these prices are higher than the net price. If the decreasing part is relevant ($P_U < P_L$), then in equilibrium there is one insurer on the market who sets a price of P_U, the others quote a higher price. Provided that this price is higher than the monopoly price.

The size of the expected profit depends on the parameters. At net premium the expected profit is the fixed cost, if the premium is higher, then the profit could become positive. If the interest rate is lower, positive profit occurs more often.

We examined how the evolution of individual parameters affect the endpoints of the equilibrium price interval. The increasing of the confidence level (more safety) both of the endpoints are increasing. Only higher prices can ensure the higher confidence level. Higher levels of capital can lead to lower equilibrium premiums. The increasing of the number of companies causes decreasing in the lower endpoint of the equilibrium interval (P_L), while the higher endpoint is unchanged. If the total capital in the market is fixed, the increasing of the number of companies leads to a higher lower endpoint of the equilibrium interval. If the total capital level is fixed in the market, the higher number of companies leads to a lower level of individual capital leading to higher possible equilibrium premiums in the sets. Because this means that more companies share the same level of capital, so the capital of each firms decreases. Table 2 summarizes the results of the comparative statics.

	P_L	P_U
Increasing level of confidence	+	+
Increasing capital level	-	-
Increasing number of companies	-	*no effect*
Increasing number of companies with fixed level of market capital	+	+

Table 2: The results of the comparative statics modelling the insurance market under solvency capital.

All of the results are equally joint (inseparable) **with our co-author** (*Kolos Ágoston*)**:**

- Literature review;
- Running simulations to find equilibrium;
- Calculations to prove the theorems;
- Illustrative figures;
- Editing and writing the article.

III..3 Study III. – The empirical analysis of the market structure of the Hungarian insurance market

In insurance models, monopoly, oligopoly and perfect competitive markets occur. However, these assumptions greatly influence the results, so it is worth examining which form of market structure fits best the real operation of the sector. Many quantitative and qualitative studies deal with a similar question, examining different countries and periods. There are several methodological options for market structure analysis. We focus on the Hungarian market between 2011 and 2019, analyzing the operation of the sector with structural and non-structural methods.

We list the number of insurance corporations in the examined period and calculate the market share of the largest three, five and ten insurance companies. The latter is shown in Figure 2.

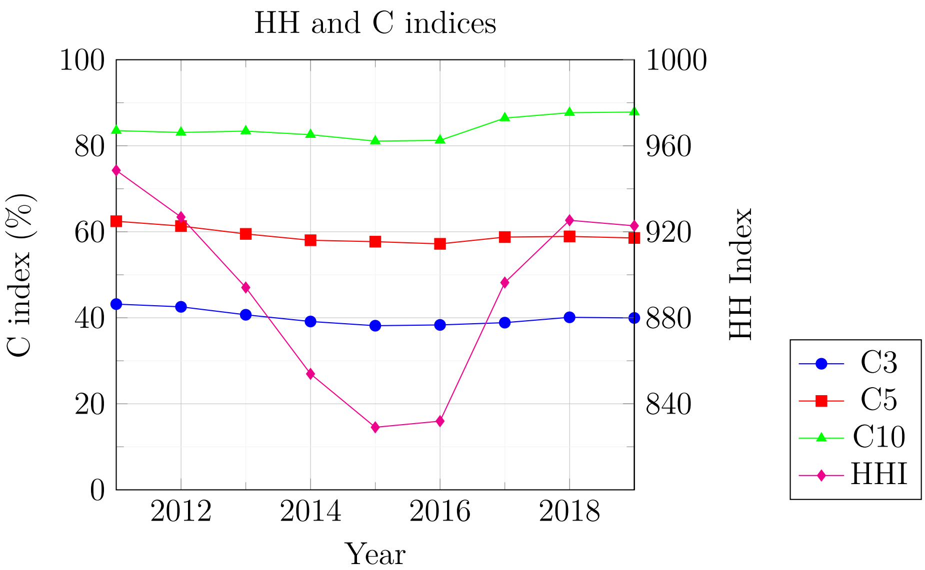

Figure 2: The evolution of the Herfindahl-Hirschman and C indices of the Hungarian insurance market.

The position of the market leaders is stable, although the 43% share of the three largest companies decreased to 40% by the end of the period. The largest 5 insurers already cover more than half of the market. The market share of the ten largest companies was 88% in 2019, and we will consider these companies for the subsequent analysis.

Among non-structural models, the Panzar and Rosse model is most often used to analyze the sector, which infers the market structure based on the input price elasticity (H statistic). Using data from the annual financial reports of ten companies covering a significant part of the market, we performed a static and dynamic panel estimation. In the case of the static model, we used a fixed effect estimation, and in the case of the dynamic panel model, we used the generalized method of moments estimation with 1 or 2 autoregressive variables. The following three factors are usually considered as input prices: the unit price of labor, business services and the financial capital. In order to check the robustness of the results, we also performed the analysis with two dependent variables. In the first case, the goal variable is the insurance and technical income together, in the second case only the income from the insurance was taken into account.

Table 3 summarizes the conclusions of the four models. For the first outcome variable (insurance and technical income) the estimated H statistic was 0.081 according to the static and -0.006 according to the dynamic model. In both cases the hypothesis of the monopoly market (or colluding oligopolies) cannot be rejected, so the market is monopoly or monopolistic competition. If the dependent variable contains only the insurance incomes we get more significant variables. The static estimate is 0.49 and the dynamic estimate is 0.758 for the input price elasticity. Based on the results of the static model, we reject the hypothesis of perfect competition and monopoly, the market is monopolistic competition. While in the case of the dynamic model, the market can be either monopolistic competition or perfect competition. The conclusion regarding these two forms of market structure is correct only if the assumption regarding the long-term equilibrium of the market is fulfilled. We checked that the Hungarian insurance sector was in long-term equilibrium between 2011 and 2019 using a model about the return on equity.

		Static panel model	Dynamic panel model
Dependent variable	Insurance and technical income	*Monopoly or Monopolistic competition*	*Monopoly or Monopolistic competition*
	Insurance income	*Monopolistic competition*	*Monopolistic competition or Perfect competition*

Table 3: The conclusions of the four examined models to determine the market structure of the Hungarian insurance market.

In our study, we summarize the results of 9 additional Panzar and Rosse methods for the insurance market of other countries and periods. Most studies support monopolistic competition, but there are also examples of the other two forms of market structure.

Individual contributions:

- Literature review;
- Data collection and calculating variables;
- Editing and writing the text.

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V. List of publications

1. Publications in English

Madari, Z. and Varga, V. (2021) Empirical Analysis of the Hungarian Insurance Market. *In: Drobne, Samo; Stirn, Lidija Zadnik; Kljajić, Borštnar Mirjana; Povh, Janez; Žerovnik, Janez (eds.) Proceedings of the 16th International Symposium on Operational Research in Slovenia : SOR'21 in Slovenia*

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2. Publications in Hungarian

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