Szádoczkiné Varga Veronika

Modelling of the insurance market

Department of Operations Research and Actuarial Sciences

Supervisor:

Ágoston Kolos Csaba Ph.D.

Corvinus University of Budapest

Doctoral School of Economics, Business and Informatics

Modelling of the insurance market

Ph.D. Thesis

Szádoczkiné Varga Veronika

Budapest, 2024

Contents

List of Figures

List of Tables

Acknowledgements

I would like to express my gratitude to my supervisor, *Kolos Ágoston*, who always provided me his time and professional support to finish my doctoral dissertation and ensured the quality of my publications.

Among my former professors *Imre Csekő, Erzsébet Kovács, Dénes Pálvölgyi* and *Imre Szabó* had important roles in my decision to choose the doctoral studies.

I would like to thank my co-authors, *Kolos Ágoston* and *Zoltán Madari*, who largely contributed to the studies included in this thesis.

I got useful feedback from *Mánuel Mágó*, *Róbert Somogyi*, *Anastasios Dosis*, *Giorgos Stamatopoulos* and *Attila Tasnádi* on different conferences. I am grateful to *Tamás Solymosi*, who provided me the opportunity to present my research at the Corvinus Game Theory Seminar.

I would like to acknowledge the Department of Operations Research and Actuarial Sciences, Institute of Operations Research and Decision Sciences, Corvinus University of Budapest to ensure the conditions of my research.

I am grateful to my mother and father, and my brother for their support and encouragement through the years; to my most enthusiastic readers, my grandparents and my son, *Márton.* I would like to thank my husband, *Zsombor*, for the discussions, ideas and for his consistent help.

1 Dissertation summary

The main topic of the dissertation is modelling the insurance market. Several economic studies deal with empirical and analytical problems in this area. On the one hand, the actuarial science focuses on the question of estimating the life time, mortality, number of damage and the level of claim, it provides solutions to the problem of pricing, reserve calculation and risk dispersion with statistical and probability theoretical methods. On the other hand, the insurance market is suitable for examining many interesting economic phenomena, such as decision under uncertainty, information asymmetry, anti-selection and moral hazard. In the aging societies of the developed world, these results are useful not only for private health insurers, pension and health funds but also for the public health and pension problems.

Although, the insurance market has an important role in developed countries, COVID-19 caused its temporary slowdown in 2020. The restrictions due to the pandemic influenced the sales processes (agents) and the development of claim payments. In 2021, insurance companies' premium income picked up, especially in the life sector. According to an OECD report, the Hungarian insurance market growth was over the average of OECD countries in 2021 (OECD, 2023).

In Hungary, the premium income of insurance companies has been increasing year by year recently. The trend continued in 2022, the premium income reported by insurers was HUF 1.469 billion, which is 6.9% higher than the premium income of the previous year. According to expectations, this growing trend will continue in 2023. Compared to the previous year, the premium income of life insurances increased by 1.7%, while the income of non-life insurances increased by 11.3%, which is why in 2022, within the total premium income, the share of life insurances was 43.6% compared to the non-life insurance branch (MABISZ, 2023).

The macroeconomic challenges of the sector and its regulation, the evolution of its market structure and its analytical modelling are the topics of many international and domestic actuarial conferences (MAT, 2024). At the Corvinus University of Budapest an academic research group is studying this area, which served as the subject of many articles (Ágoston, 2004; Balog, 2023; Banyár and Regős, 2012;

Gyetvai, 2022; Kovács, 2023; Szepesváry, 2022; Vaskövi, 2024; Vékás, 2017).

We present the results of three different studies, which complement each other well and seek answers to equally important and relevant questions. The literature on decisions based on uncertainty is very extensive (Szentpéteri, 1980; Varian, 2014; Winston, 2004). Most of the interesting economic and mathematical phenomena in the insurance market can be traced back to this problem. We examine the impact of risk aversion and the capital requirement under uncertainty with the help of analytical models, using the tools of game theory, microeconomics and market structures. In addition, we analyze the structure of the Hungarian insurance sector with the help of an empirical model and indicators, which is of great importance in terms of the assumptions used in the analytical models.

The first study deals with the effect of risk aversion in a Bertrand duopoly under uncertainty. We model risk aversion under uncertainty, so players maximize the expected value of their concave utility functions. We introduce a new type of grouping of utility functions, the substance preference. We show that this classification fundamentally affects the evolution of the market equilibrium and does not correspond to any previously used grouping. The research classifies the frequently used utility functions, illustrated by numerical examples or proved analytically.

After the financial crisis of 2008 the regulation of the Systematically Important Financial Institutions were transformed. The bank sector is regulated by the Basel (BIS, 2010) and the insurance sector works under the Solvency II directive since 2016 (Directive 2009/138/EC, 2009). Thus, this also provides an opportunity for new analyses. In the second study we examine the effect of the Solvency Capital Requirement in a Bertrand duopoly on the equilibrium prices and expected profits. In this model the insurance companies are risk neutral, they maximize their expected profit. Using comparative statistics, we examine how the level of security, the number of players, and the capital of insurers influence the equilibrium prices.

In the analytical models, the insurance sector was modeled as an oligopoly. This assumption fundamentally influenced the results. Therefore, the question arises that what form of market structure is closest to the functioning of the sector. In the third study we examined the Hungarian insurance sector using structural (concentration ratios, Herfindahl-Hirschman Index) and non-structural (Panzar and Rosse, 1987) methods. The Panzar and Rosse model gives a testable hypothesis about the factor price elasticity of the sector, from which we can deduce the market structure. We estimate the factor price elasticity with a static and a dynamic panel model between 2010 and 2019. In addition to further analytical models, the market structure analysis may contain interesting and important results for consumer protection, competition supervision as well as insurance supervision.

The presented results open up further research directions and are not restricted to the field of insurance. The three main questions of the thesis are closely related since knowledge of the market structure is extremely important when using analytical models. Modelling risk aversion and the capital requirement similar results can be observed, higher equilibrium prices may also develop, insurers may fare better (higher expected utility and positive profit), but it may also happen that there is only one insurer operating in the market. A natural continuation of the research could be the joint modelling of the two phenomena, during which risk-averse insurers are faced with the capital requirement constraint.

The structure of the article-based dissertation is the following: Chapter 1 is the dissertation summary that contains a literature review in Section 1.1; the methodology of the studies in Section 1.2; the list of presented publications in Section 1.3; the connection of the studies in Section 1.4; the summary of the three publications, the main results, the individual and joint contributions, and the list of the attended conferences in Section 1.5; the future research questions in Section 1.6 the publications and working papers are also listed at the end. The rest of the dissertation contains the three studies, as they are published. Chapter 2 is the first study, the oligopolistic model of the insurance market with substance preference; Chapter 3 contains the second study, the analysis of insurance market under solvency capital requirement. Chapter 4 is the third study about the Hungarian insurance market structure, which is an empirical analysis. Finally, Chapter 5 contains a discussion about the suggestions raised during the defense of the dissertation draft.

1.1 Literature review

In this section, we present the results of the early insurance models, which are still central to the literature (Arrow, 1963; Borch, 1962; Mossin, 1968). We also list the results of later research, which are connected to the three studies of the dissertation. In connection with the first study, we detail the modelling of risk aversion, particularly the problem of managing multiple risks together. In the second study, insurers face a capital constraint, which can also be interpreted as a generalization of the capacity constraint, so we briefly present the development of the capacity constraint literature. Finally, we emphasize that the market structures of the models used in the literature are very diverse, we can find examples of perfect competition, monopoly market and oligopoly framework as well, therefore the empirical investigation of the question is also interesting and important.

The insurance market serves as a good example for many interesting economic phenomena, so its analysis soon began. Due to the nature of the service, risk and uncertainty also appear in the models, which were previously dealt with in finance (Dusek, 2022) and decision theory. This led to the development of the expected utility theory (Neumann and Morgenstern, 1947). This market is also a good example for modelling situations burdened with information asymmetry, such as the moral risk (Pauly, 1968) or the anti-selection (Akerlof, 1970).

Arrow's 1963 article on health insurance draws attention to the problem of information asymmetry, which hinders market mechanisms (Arrow, 1963). In addition, transaction costs and insurer risk aversion explain the lack of risk transfer. Arrow stated for the first time that with a positive loading, incomplete insurance is optimal for risk-averse customers who maximize their expected utility. In such cases, the insurance company imposes a higher premium than the net premium by a fixed percentage of the expected loss. Arrow also showed that when both the customer and the insurer are risk-averse the Pareto-optimal contract includes coinsurance of the deductible and the risk above the deductible.

Karl Borch (1962) applied the model of general equilibrium under uncertainty (Arrow and Debreu, 1954) to determine the optimal level of reinsurance. Based on the results, risk-averse decision-makers create and diversify individual risk into a risk pool. Non-diversifiable social risks are distributed among individuals. The degree of Pareto optimal risk exchange depends on the risk tolerance of individuals (this is the reciprocal of absolute risk aversion) (Wilson, 1968).

According to Mossin (1968) at net premium the full coverage insurance is optimal, and with positive loading (higher premium level) a risk averse customer choose partial insurance. In terms of income effect, the insurance is an inferior good, if the customer has decreasing absolute risk aversion (DARA). These statements are true under the implicit condition that the client faces only one risk and the level of risk aversion does not depend on the wealth.

In the case of managing several risks together, the tools modelling habitual risk aversion have not proven to be sufficient. Thus, the detailed examination of this area has also begun, and several risk aversion concepts have been born, the relationship of which may be interesting to the concept of substance preference introduced in our first study. The risk aversion of the decision maker corresponds to the concavity of the utility function u. Arrow and Pratt are associated with the definition of absolute risk aversion (ARA) (Pratt, 1964).

$$ARA(x) = \frac{-u''(x)}{u'(x)}$$

where $u(.)$ is a concave utility function and x is the level of wealth. This measure shows how the level of risk aversion changes with the growth of wealth. In economic models, the degree of risk aversion is either constant (CARA, Constant Absolute Risk Aversion) or decreasing (DARA, Decreasing Absolute Risk Aversion). The quadratic utililty function—often used in finance—is Increasing Absolute Risk Averse. Around the turn of the millennium, it became the focus of investigation that if the decision-maker is faced with several risks, then the previous classification is not sufficient. Gollier and Pratt (1996) describe a phenomenon they call a paradox, where in the case of a risk-averse (even DARA) utility function, the decision-maker reacts to an increase in risk by preferring a previously unpreferred risk. They conclude that stricter requirements must be met by the utility function so that such a situation does not occur. Several such attempts are known, such as risk vulnerability (Gollier and Pratt, 1996), standard risk aversion (Kimball, 1993) and proper risk aversion (Pratt and Zeckhauser, 1987). *Risk vulnerability* means that as a result of every unfair background risk, the consumer becomes more risk-averse. In the case of *Proper risk aversion* a non-preferred risk cannot become preferred due to the effect of a non-preferred independent background risk. And if every risk that has a negative interaction with a small decrease in wealth also has a negative interaction with any undesirable, independent risk, that means *Standard risk aversion*. Decreasing absolute risk aversion and decreasing absolute prudence is necessary and sufficient for standard risk aversion, the prudence of the utility function is the following:

$$\frac{-u'''(x)}{u''(x)}$$

We illustrate with examples that both risk-neutral and risk-averse insurers, which we also assume in our studies, occur in the models. In the early models assuming risk-averse, expected utility-maximizing insurers are common (Polborn, 1998; Wambach, 1999; Raviv, 1979; Borch, 1962), sometimes they even deal with a decreasing degree of risk aversion (Hardelin and Lemoyne de Forges, 2012); but risk-neutral, expected profit-maximizing insurers also appear (Schlesinger and Graf von der Schulenburg, 1991; Sonnenholzner and Wambach, 2004; Stiglitz, 1977). We see examples of both cases in the literature. However, most models agree that insurers decide on prices, not volumes.

In the case of Bertrand model, the players of the oligopoly engage in price competition (not quantitative), it is a reasonable assumption that insurance companies also decide on prices and not on produced quantities. In the traditional Bertrand model, companies produce at their marginal cost, which will be the equilibrium price, similar to the case of perfect competition. The profits of the companies are zero, even if there are only two of them in the market. This phenomenon is also known as the Bertrand paradox. However, there may be cases when price competition results in extra, positive profit even in an oligopoly market. Sonnenholzner and Wambach (2004) list several model modifications as to why positive profits can be achieved in an oligopoly market, such as uncertainty, asymmetric or non-linear cost functions, the sharing rule (Dastidar, 1997), capacity constraints, search costs, entry barriers, product differentiation, and the assumption of risk-averse insurers (Polborn, 1998; Wambach, 1999).

Polborn (1998) models insurance companies with an oligopoly assuming risk-averse insurers who are in price competition with each other. The probability of the damage occurring is uncertain, and the payouts are therefore also uncertain, which is a specific factor of insurance. This cannot be reduced by selling more insurance, so price competition is reduced. The insurer decides on the premiums in a Bertrand price competition. Insurers have an exponential, constant risk aversion rate, the so-called CARA-type utility function. All customers go to the insurer with the lowest price, who is obliged to serve these customers. If they determine the same price, the insurers will distribute the customers evenly among themselves. The latter is a frequently used assumption in the case of price competition. It can be shown that in this way insurance companies can earn a positive profit in equilibrium, in contrast to the zero profit experienced in the case of the traditional product market.

Wambach (1999) also models the market using Bertrand price competition, however, the uncertainty resulting from damage events appears in his model in the assumption of uncertain costs. It builds a model similar to Polborn's, in the equilibrium situation a positive profit can also be achieved in this model. Risk-averse insurers do not necessarily do well by acquiring the entire market, as this represents a kind of risk for them due to the uncertain costs. If the market grows, the price also grows, which is also a phenomenon contrary to our expectations, which can be traced back to the emergence of uncertain costs.

A different equilibrium can be obtained from the Bertrand model if the costs are convex. In the market of homogeneous products with price competition with strictly convex costs, Dastidar (1995) showed that there always exists pure strategy Nash equilibrium if the output is demand determined. This equilibrium is not unique.

The reason for achieving a positive profit may be the capacity constraint in addition to risk aversion. The Bertrand paradox might not hold in the case of capacity constraint. According to Edgeworth (1897), Bertrand's assumption that the company offering the lowest price is obliged to serve the entire demand is not realistic, since the company is not always able to do so in practice. Therefore, he extended it with capacity constraints and realized that the Bertrand solution in the capacity-constrained model is not an equilibrium. After all, if the company cannot serve the entire market, it is not worth undercutting the other's price. Edgeworth's criticism led to the development of Bertrand-Edgeworth oligopolies (Tasnádi, 2001). In Nash equilibrium, firms set a price above the marginal costs. In this model, the capacity constraint is exogenous. However, later Kreps and Scheinkman (1983) describe a two stage model (simultaneous capacity choice in the first stage and simultaneous price setting in the second stage), where capacities are determined endogenously. In equilibrium, the firms can get the Cournot outcome with positive profits (Cournot, 1838).

Somogyi (2024) focuses on the dual capacity constraints of optimal firm behavior, where one is the number of consumers and the other is the quantity of products sold. The analysis distinguishes two types of consumers and describes the optimal behavior of the monopoly market, which in equilibrium differs from the model of firms bound by a single capacity. In equilibrium, the exogenous capacity reduction in the number of consumers is a monopoly that lowers prices for high-type consumers and raises prices for low-type consumers. On the other hand, the optimal response to reducing the production level is to increase the price of high-type consumers and decrease the price of low-type consumers. Moreover, total welfare increases at both capacity levels. In particular, relaxing the regulatory restrictions on one limit could harm the average consumer. If the capacity choice is endogenous and the cost of building capacity is strictly positive, then the monopoly will choose its prices and capacity levels so that both constraints are bound.

Although the theoretical results lead to different conclusions, the empirical analyses show that companies with a larger capacity constraint set lower prices than smaller ones. Private information about this level of capacity constraint may be behind this. Somogyi et al. (2023) introduces capacity uncertainty and shows that these capacity constraints under incomplete information lead to lower prices in equilibrium in the case of larger firms.

The Value at Risk capital requirement can be considered a kind of generalization of capacity constraints if the amount of contracts the insurance company can sell depends on the set price. Thus, the modelling of the introduction of the capital constraint is related to the literature of capacity-constraint models, but the models are not fully comparable. Due to the uncertainty in the insurance models, it is more difficult to compare the relationship with other results. Furthermore, production in general has physical limits, although we may also encounter physical capacity during the sale of insurance, it is much more common that other types of limitations arise during the service, for instance, the capital requirement.

Cabon-Dhersin and Drouhin (2014) and Cabon-Dhersin and Drouhin (2020) combine models with capacity constraints and convex costs in their studies. They construct a two-factor sequential game, in which a payout-dominated equilibrium is sought in order to determine the unique equilibrium. Similar to the capital requirement model we examined, they also found that the equilibrium price can increase as the number of companies increases.

In Bertrand-Edgeworth models, Dixon (1990) solved the problem of the non-existence of pure strategic equilibria by introducing a cost that companies pay when they have to send a customer away. If the market is large enough and the cost is low, the competition price is the equilibrium in this model.

In the classic Bertrand model with constant marginal costs, the cost uncertainty can intensify price competition with asymmetric information; see Lagerlöf (2016). Thus, with the appearance of imperfect information, a result opposite to the findings of Polborn (1998) and Wambach (1999) is obtained.

The literature on Solvency II regulations covers Doff (2016); Kouwenberg (2018); Escobar et al. (2019); Bi and Cai (2019); Zhang et al. (2016). However, the effect of the Value at Risk constraint on equilibrium prices has only been investigated by few in a traditional model framework. Dutang et al. (2013) show a non-cooperative game to see the market premium, solvency level, market share and underwriting results of non-life insurance companies. The study examines the insurance market with similar tools, but it differs from our research in important conditions regarding the demand. In their article, the number of insurance buyers does not depend on the market price, and consumers do not necessarily buy the cheapest insurance, contrary to the terms of our model. These assumptions ensure a unique equilibrium. Mouminoux et al. (2022) developed a similar repeated game and determine long run market shares, leadership and ruin probabilities and the effect of deviation from the regulated market.

The models dealing with insurance are also diverse in terms of market structure. Imperfect information markets are modeled with perfect competition by Rothschild and Stiglitz (1976) and Azevendo and Gottlieb (2017). The role of information asymmetry shows a significant difference in the case of a monopoly market. Stiglitz (1977)'s insurance model shows that a monopolist can discriminate against customers through non-linear pricing, thereby making the equilibrium different from the competitive situation. In the imperfect information model there are low risk and high risk individuals. In equilibrium the utility of the low risk individuals is the same when they do not buy insurance. The two groups never purchase the same insurance, and the optimal contract for the high-risk individuals is complete insurance.

Ágoston (2004) examines the relationship between risk aversion and stock size in a monopolistic market. He compares the pricing of a risk-averse monopolist in the case of product markets and insurance markets, taking into account the number of customers and the effect of uncertainty. In the traditional product market, the number of buyers is uncertain, and the monopolist does not face other risks, unlike insurers, who are exposed to risk even after sales. The increase in the size of the customer base can have opposite effects in the two sectors. In the traditional product market, the increase in the number of buyers increases the expected utility, while in the insurance market, the insurance price may have decreased if more people buy the insurance.

Among the presented models, several oligopoly (duopoly) markets were assumed (Sonnenholzner and Wambach, 2004; Polborn, 1998; Wambach, 1999).

So the models are also diverse in terms of risk attitude and market structure. Therefore, the question arises as to which approach and which market structure is realistic in the sector.

1.1.1 Market structure

Joskow (1973) was one of the first to investigate the problem of choosing the realistic market structure in the insurance market in a paper, which dealt with the question of market concentration, barriers to entry, returns to scale, and discusses insurance distribution systems and rate regulation. According to the empirical results, the property and liability insurance industry in the United States was associated with competitive markets. However, the price setting mechanism worked through cartel-like rating bureaus and has been subjected to pervasive state rate regulation. This results in supply shortages and over capitalization. The free entry and a movement away from cartel pricing and state regulation to open competition could tend the prices towards the cost of capital.

Due to the nature of the insurance service, however, it may happen that several companies can operate more safely by cooperating and sharing information. Espeli (2020) shows an example of this in the Norwegian market, where a multi-member cartel system supported by the state existed in the insurance market for many years. Fog (1956) also analyzes this question from an analytical point of view and examines an alternative cartel pricing technique that maximizes profits in a non-monopol pattern. The aim of the cartel is often safety, this results in good profit for the insurers, but not the maximum profit.

A good example of state intervention and the centralization of decision-making is that the insurance market of several Eastern European countries operated as a monopoly during the years of socialism. Tipuric et al. (2008) examined the development and concentration of the insurance market in Central and Eastern Europe between 1998 and 2006 for Croatia, Slovenia, Slovakia, the Czech Republic, Poland and Hungary. In each country, the history of insurance started from a monopoly situation before the system change, more than 90% of the market was in one hand, mostly owned by the state. The product range was not very diverse, the non-life branch of insurance generated the main income, which is also a characteristic of underdeveloped markets. In the 1980s, regulations were loosened, the state control decreased, and the process of transformation began. After that, many entrants appeared in the market because the sector seemed to be extremely profitable. This reduced concentration and increased competition, oligopolistic competition began to develop in the market.

The advantage of the concentration ratio is that only the data of the largest and most influential companies and the aggregate indicators of the market are needed, we do not need to know the individual results of all companies to calculate it. Its disadvantage stems from the same phenomenon, as it does not describe a more precise distribution of market share.

Several additional indicators can be used to measure this, the Herfindahl-Hirschman Index (HHI), the Hall-Tideman Index, the Rosenbluth Index, the Comprehensive Industrial Concentration Index, the Entropy measure, the Hannah and Kay Index (HKI), the U Index (U), and the Hause Indeces (Bikker and Haaf, 2002). The difficulty of calculating these indicators is that we have to determine which companies are considered to belong to a sector, since the analysis will be accurate if the market share of each company is taken into account (Uhrin, 2010). The rank of the highest companies can be also important information about the market structure, but these indicators do not reflect to it. The Markov chain model may be another suitable tool for analyzing structural dynamics (Kovács, 2011).

In addition to structural indicators, non-structural analysis can also be used, including the Iwata model (Iwata, 1974), Bresnahan model (Bresnahan, 1982) and Panzar and Rosse model (Panzar and Rosse, 1987). The Panzar and Rosse model uses the comparative static properties of the reduced-form revenue approach. The Iwata and Bresnahan model are based on the profit maximizing problem of the oligopolies.

The Panzar and Rosse analysis is most often used to study the banking and insurance sectors. The biggest advantage of it is the small data requirement, only revenues and factor prices of the companies are required. The results are easy to interpret, however, several criticisms of the method have been formulated over the years (Bikker et al., 2012; Shaffer and Spierdijk, 2015; Sanchez-Cartas, 2020; Goddard and Wilson, 2009). Nevertheless, it has also been used in recent years to perform similar analyses, see for instance, Prayoonrattana et al. (2020), Guidi (2021), Zhang and Matthews (2019).

Based on the suggestion of Goddard and Wilson (2009), for the sake of unbiased results, it is worth using dynamic panel estimation to estimate the input price elasticity of Panzar and Rosse. The Arellano–Bond estimator is a standard estimation tool for dynamic panel models. One of the weaknesses of this analysis procedure is that for integrated variables close to the random walk, differentiation greatly increases the noise, so the estimate is very uncertain, i.e., it is difficult to obtain a significant coefficient estimate. Therefore, Blundell and Bond proposed an alternative system estimate (Baltagi, 2021).

1.2 Methodology

1.2.1 Analytical modelling approach

The analysis of the insurance market (Dionne, 2000) is often based on traditional microeconomic models (Varian, 2014). These models can be defined as non-cooperative games (Osborne and Rubinstein, 1996), and they often use the Nash equilibrium concept. The analytical models of the thesis (Study I and II) use similar models. We present the similarities and the differences of them here.

Firstly, both models are *Bertrand models* (Bertrand, 1883), where two or more companies make simultaneous decision about the price level while maximizing their own profit. The goods are homogenous, so the consumers buy the cheapest one. This is a perfect information game. In the unique Nash equilibrium of Bertrand price competition the companies set their prices at the level of the marginal costs, so the profits of the companies are zero, if there are no fixed cost in the model. The result is the same as in the case of perfect competition even if there are only two companies on the market.

Secondly, we determine the *Nash equilibrium* in the models (Nash, 1951). This solution concept is a strategy profile in which each player sets the best response to the strategy of the other(s). This is not a unique equilibrium concept. We are looking for the equilibrium on the set of pure strategies (we do not examine the mixed expansion of the game), so the solution does not necessarily exist. In order to determine the price level from which no insurer should unilaterally deviate, we used a visualization in the price and contract number plane.

Thirdly, assumptions about the consumers are also similar. The consumers have the same risk, which can be described with independent random variables, they have the same level of claim with the same level of probability. There is a decreasing demand function in both models. The insurances are homogeneous, so all consumers buy from the insurer that determines the lowest price. If more companies charge the same price, the market is divided equally. The companies cannot refuse a customer. The insurance company offers full insurance, so in the case of damage, it pays the entire damage amount to the injured party. Thus, the insurer's expenses are uncertain.

The main difference between the two models (Study I and Study II) is the risk attitude of the insurance companies. In Study I we assume risk averse insurance companies with concave utility functions, they maximize their expected utility, while in Study II the insurers are risk neutral, they maximize their expected profit. In addition, as the investigated phenomenon, the capital constraint, appears in the second model, there is a cost of holding capital, thus, a fixed cost in this model.

We study the effect of different parameters, we compare equilibria with *Comparative statics*.

1.2.2 Market structure analysis

There are several structural and non-structural models to determine the market structure of a sector. As structural indicators we use the Herfindahl Hirschman Index (HHI) and the concentration ratios of the biggest three, five and ten companies. The calculating formula of these indexes are the following:

$$C_3 = \sum_{i=1}^{3} r_i$$

$$C_5 = \sum_{i=1}^{5} r_i$$

$$C_{10} = \sum_{i=1}^{10} r_i$$

$$HHI = \sum_{i=1}^{n} r_i^2$$

where r_i is the market share of the ith largest firm and n is the number of the firms. The value of the often used HHI index can take a number between $\frac{1}{n}$ and 1. In the case of a monopoly, the value is 1, the market is completely concentrated, while the closer the market is to perfect competition, the closer the value of the HHI index is to 0, since in this case all actors have an equal share (the market is not concentrated), and the number of actors tends to infinity, due to which $\frac{1}{n}$ tends to 0. Higher concentration may also highlight market collusion. It is also common to give the r_i values as a percentage, in which case the HHI value is given as 10000 times the value listed here.

Matsumoto et al. (2009) pointed out an unfavorable feature of the measure, namely if all the companies in the industry collude, the value of the HHI is the same as the index value without collusion. In addition, the value of the HHI stays similar if the market shares are of a similar size, even if the order of the actors changes from year to year. A change in order can also mean strong competition (Kovács, 2011), but the HHI does not reflect this.

The analysis of the insurance market is based on the Panzar-Rosse model (Panzar and Rosse, 1987), which gives testable implications of profit maximizing companies, so we can deduce the market structure of the sector. The reduced form revenue equation is the following:

$$\pi = R\left(y, z\right) - C\left(y, w, t\right),$$

where $R(y, z)$ is the reduced form revenue function, y is the decision variable and z are further exogenous variables which influence the revenue function. The vector of y contains the decision variables that influence the company's revenue and, directly or indirectly, its costs. In addition to the output level, this can include prices or even advertising expenditures or quality levels. $C(y, w, t)$ is the cost function, where w is the vector of exogenous factor prices and t is the vector of additional exogenous variables that influence cost. This simple model assumes profit maximizing companies. The testable expression is the sum of the factor price elasticities of the reduced form revenue equation:

$$H = \sum_i \frac{\partial R^*}{\partial w_i} \frac{w_i}{R^*},$$

where * indicates the profit maximizing values.

Panzar and Rosse (1987) offers different theorems about the value of the sum of elasticities of gross revenue with respect to input prices. In the case of a neoclassical monopolist or collusive oligopolist, the elasticity is nonpositive, it is equal to unity in the case of a competitive price-taking market in long-run equilibrium. Between these two extreme situations, the factor price elasticity is between 0 and 1 and the market is monopolistic competition. An assumption is that in the case of perfect competition and monopolistic competition the companies are in long-run equilibrium and entry and exit in the market are free, thus, this also should be tested. In long run equilibrium, the return rates are not correlated with input prices. To test the long run equilibrium empirically, return on assets (ROA) can be estimated with the same independent variables used in the estimation of the factor price elasticity. In long-run competitive equilibrium, both of the factor price elasticities are zero.

The model has received some criticism in recent years, but it is one of the most common forms of analysis in the banking and insurance sectors. The different size of the firms can cause some problems (Bikker et al., 2012), according to Shaffer and Spierdijk (2015) it may happen that the H statistic is negative or positive at any level of the competition and it would be better to present the statistic as a pass-through rate not a market power measure (Sanchez-Cartas, 2020). Goddard and Wilson (2009) showed that the dynamic rather than a static formulation of the revenue equation should be used to identify the Panzar–Rosse H-statistic, because the fixed effect estimation can be biased towards zero. According to Bikker et al. (2012) a further improvement of the model is that only an unscaled revenue equation gives unbiased estimation, the dependent variable of income should not be scaled, and the model should not contain the total asset as a control variable. These suggestions related to the empirical application were taken into account when performing the analysis.

Both the static and the dynamic approaches were used to analyze the *Panel model* (Baltagi, 2021). The panel data have time and cross-sectional dimensions, we can distinguish the database based on which one dominates, or whether the sample is balanced. *The static* panel equation is the following:

$$y = X\beta + u,$$

where y is the dependent variable vector, X is the matrix of the independent variables and u is the residual vector.

The dynamic panel estimation uses the autoregressive specification of the dependent variable as an explanatory variable. The Arellano–Bond estimator (Arellano and Bond, 1991) is a standard estimation tool for dynamic panel models. They apply Generalized Method of Moments (GMM) estimation in which they use first differences to eliminate the individual effects. They solve the endogeneity problem by using all the lagged values of the dependent variables as instruments. The method is also called one-step GMM panel estimation. Two model diagnostic tools are offered, first the over-identification test to check the specification; and second to test the second-order autocorrelation of the error term of the differentiated model to check the dynamics.

1.3 List of publications included in the Ph.D. thesis

All of the research we present here have been developed as academic journal articles. The summary and the main results of the following studies are detailed in Section 1.5. We compiled the studies in the dissertation without re-editing, and present them in the form as they were—or are planned to be–published.

I. Ágoston, K. Cs. and **Varga, V.** (2020). Bertrand-árverseny állománypreferenciák mellett a biztosítási piacokon (Bertrand price competition with substance preferences in insurance markets). *Szigma 51*(2):149-167. https://journals.lib.pte.hu/index.php/szigma/article/view/3261/3066

II. **Varga, V.** and Ágoston, K. Cs. (2024). A biztosítási piac modellezése tőkekövetelmény korlát mellett (Modelling insurance market under solvency capital requirement). *Szigma 55*(2-3):239-255.

III. **Varga, V.** and Madari, Z. (2023). The Hungarian insurance market structure: an empirical analysis. *Central European Journal of Operations Research 31*(3):927-940. https://doi.org/10.1007/s10100-023-00842-8

1.4 Connection of the studies

The relation of research questions, methods, and results of the studies included in the Ph.D. thesis can be seen in Figure 1.

The central topic of the dissertation is the analysis of the insurance market. The special nature of the sector lies in the uncertainty. Customers are exposed to some kind of risk, which they want to cover by purchasing insurance. In return for the insurance premium, the insurer pays compensation. However, it is not possible to foresee how many and to what extent damage events will occur during the insurance period, or whether a payment obligation will arise at all. In the framework of actuarial analysis, risk is considered to be something that can be measured, so it can be estimated statistically or mathematically. The risk can thus be described by a random variable. Uncertain payments in exchange for insurance premiums appear in all three studies of the dissertation and fundamentally determine the results obtained.

The studies of the effect of the capital requirement and risk aversion were carried out with the help of analytical models. In these models, we have to assume a form of market structure, so for more precise and realistic results, the question arises that which form of market structure approximates the real market structure the most closely. To this end, we examined the Hungarian insurance market with the help of concentration indicators and input price elasticity, and deduced the market structure based on the hypotheses given by Panzar and Rosse (1987).

We examined the effect of risk aversion and the capital requirement using two basically very similar analytical models. In accordance with the market structure results, oligopolistic (duopoly) markets were assumed. The assumptions about customers are also similar. During the analysis of risk aversion, we assumed risk-averse insurers, while companies are risk-neutral if the capital requirement is examined. The results are also very similar. Depending on the parameters, in both cases there may be a continuum of many symmetric Nash equilibria in the market or only one company sells while the other does not. We have also shown that in both cases we can see examples of insurers achieving some kind of surplus compared to their initial wealth, this is an expected utility greater than the utility of the initial assets or a positive expected profit. If this phenomenon can be observed in practice, it greatly affects the income of companies, including the balance sheet data used for empirical analysis. Thus, the phenomena analyzed in the research mutually influence each other.

The research leads to interesting and similar results, which open up new research directions. During the market structure analysis, we cannot reject the hypothesis of perfect competition, so it may be worthwhile to examine the effects of risk aversion and capital constraint in such a framework. Examining the two phenomena together can also be a new direction; risk-averse decision makers face the capital constraint. In addition, the development of lower, clear equilibrium prices can be an important issue from a supervisory point of view, so it is worth investigating another equilibrium concept or the new dynamics of the course of the game.

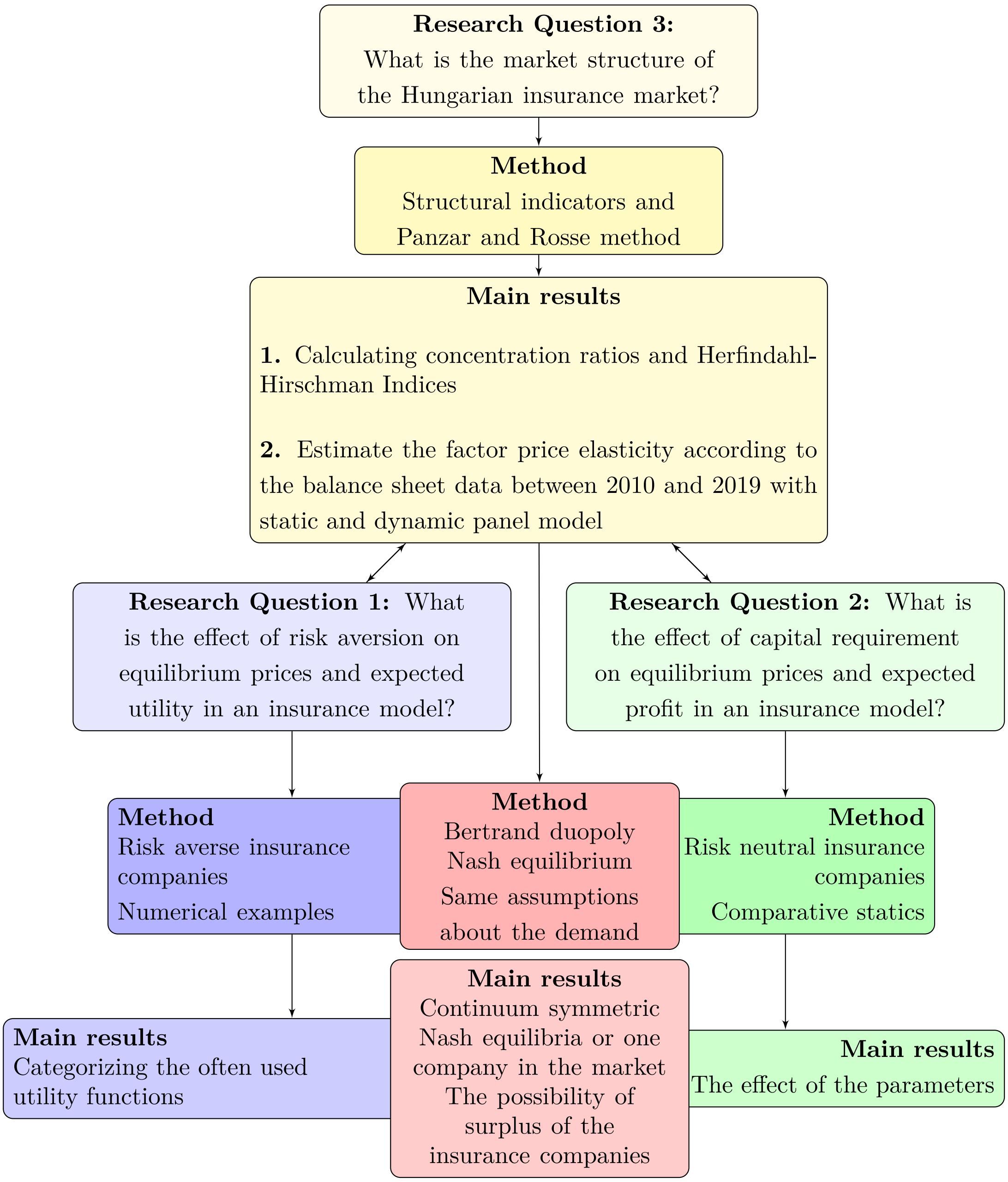


Figure 1: The connection of research questions, methods, and main results

1.5 Summary, results and contributions

In this section we summarize the research questions and the main results of the included studies, list the individual and joint (inseparable) findings and the conferences, where the results were presented.

Study I. – The oligopolistic model of the insurance market with substance preference

Study I. deals with the risk aversion of the insurance companies in a Bertrand duopoly. One can think that the insurance companies try to sell as many contracts as they can, but if we assume this risk averse behavior, it is not necessarily true. The risk aversion of the firms occur in the literature, because the companies decide under uncertainty so their risk attitude could be important. We define the substance preference, and show the equilibrium in different cases through examples, and analyze the connections of substance preference and risk preference in more details.

The early literature defined the measure of risk aversion with the absolute risk aversion measurement (Pratt, 1964). According to this, we can talk about Constant, Decreasing and Increasing Absolute Risk Aversion utility functions (CARA, DARA and IARA). Later the fact that risk aversion is not accurate enough in the case when we examine more than one risks at the same time, got bigger attention. Pratt and Zeckhauser (1987) defined a strict property in connection with the utility function that we call proper risk aversion.

We assume risk averse insurance companies with concave utility functions. Based on the utility function, the attitude of insurers to the size of the substance can be different, which fundamentally affects the equilibrium on the market. In order to understand this, let's define the concept of substance preference. For a given number of contracts (n) the *indifferent price* is the price at which level the insurance company is indifferent to sell n contracts or to do not sell any of them. The utility function is *substance averse* if selling one more insurance on this indifferent price level is not preferred, it is *substance seeking* if it is preferred, and *substance neutral* if the company is indifferent.

To see the market equilibrium in the different cases we show examples for each of them. We describe a Bertrand duopoly market, where the companies maximize their expected utility. The behavior of the customers is described with a linear demand function. There is a price competition on the homogeneous insurance market, so they buy at the cheapest company. If the prices are equal the companies share the market equally. At the quoted price the company should serve all the customers, who choose their firm.

We examine the market equilibrium in three groups (substance neutral, averse and seeking). First of all, we prove that the exponential utility function is substance neutral. It is a Constant Absolute Risk Averse function. In this case the equilibrium price is the indifferent price and the insurers' expected utility is equal to the utility of the initial wealth. It is similar to the case of traditional Bertrand game, when the marginal cost is the equilibrium price and the profits are zero.

In the case of substance averse insurers there are continuum symmetric Nash equilibria and extra utility can be achieved. We can also show some cases when an asymmetric market share can be an equilibrium too. We proved that the mixed exponential (DARA) and the quadratic utility functions (IARA) are substance averse, and illustrated with numerical examples that the often used square root and logarithm functions can be listed here.

Last but not least, the case of substance seeking can be illustrated with a modified example from Pratt and Zeckhauser (1987). They propose to avoid such a function describing risk averse behavior. This is an improper function, which does not satisfy the proper risk aversion property. In equilibrium, there is only one firm in the market, but it sells on a lower price, than the so-called monopoly price.

Table 1 summarizes the connections between the substance and absolute risk aversion measures. The blue color shows the proper risk aversion property. Substance aversion has a really strong connection with the property of proper risk aversion. Typically the proper risk aversion means substance aversion, however, substance aversion is a more general property, the not proper risk aversion case can be substance averse as well, for instance in the case of the quadratic utility function.

		Absolute Risk Aversion		
		Constant	*Decreasing*	*Increasing*
Substance	*Neutral*	Exponential		
	Averse		Mixed exponential, Logarithm, Square root	Quadratic
	Seeking		Modified example	

Table 1: The connections between the substance and absolute risk aversion measures.

All of the results are equally joint (inseparable) **with our co-author** (*Kolos Ágoston*)**:**

- Extensive literature review connected to risk aversion and insurance models;
- Running simulations to find numerical examples;
- Proof, check the calculations;
- Illustrative figures;
- Editing and writing the article.

List of the conferences, where the results were presented:

- Corvinus Game Theory Seminar 2019
- Online International Conference in Actuarial Science, Data Science and Finance 2020
- Oligo Workshop 2020
- VIII. MKE-PTE PhD Conference (Nyári Műhelykonferencia)
- Hungarian Operations Research Conference (Magyar Operációkutatás Konferencia) 2021

Study II. – Modelling the insurance market under solvency capital requirement

In Study II we analyze the effect of the capital constraint on equilibrium prices and profits. Since 2016 the operation of insurance companies in the European Union is regulated by the Solvency II directive. According to the regulation, the solvency capital requirement of insurance companies should ensure that bankruptcy occurs not more often than once in every 200 cases (Directive 2009/138/EC, 2009). This capital requirement can be calculated as a 99.5% Value at Risk (VaR).

We assume a Bertrand model with profit maximizing companies. There are I insurance companies, and they decide on the price level (P_i) simultaneously. The companies have the same level of capital. Holding the capital has some costs, so there is a fixed cost in the model.

The customers are homogeneous with respect to risk. They face independent risks with the same distribution. Customers have different reservation prices, as shown by a decreasing demand function. If someone buys an insurance and financial loss occurs, then the insurance company will cover it completely (full coverage). Insurance contracts are homogeneous products, thus customers are indifferent between buying it from any of the insurers. Thus, they choose the cheapest insurance company.

In the search for equilibrium, there are four notable prices. The first is the net price, which is the expected level of the damage of one consumer. Since the insurer companies maximize their expected profit, it is not worth selling a contract at a lower price. The second, the monopoly price, is the price at which a single market player would maximize its expected income, which is twice the net price. The third, the intersection of the demand curve and the capital constraint (denoted by P_U), is the lowest price at which a single insurer can serve the market alone while meeting the capital requirement. The fourth, the Ith part of the demand and the intersection of the capital constraint (denoted by P_L) is the lowest price at which the insurers can jointly cover the market while complying with the capital constraint.

The evolution of equilibrium prices is determined by whether the above intersections are located in the increasing or decreasing part of the capital constraint curve. If the increasing section is relevant, then $P_L \leq P_U$. In this case, there are a continuum of many symmetric equilibria on the interval $[P_L, P_U]$, if these prices are higher than the net price. If the decreasing part is relevant ($P_U < P_L$), then in equilibrium there is one insurer on the market who sets a price of P_U, the others quote a higher price. Provided that this price is higher than the monopoly price.

The size of the expected profit depends on the parameters. At net premium the expected profit is the fixed cost, if the premium is higher, then the profit could become positive. If the interest rate is lower, positive profit occurs more often.

We examined how the evolution of individual parameters affect the endpoints of the equilibrium price interval. The increasing of the confidence level (more safety) both of the endpoints are increasing. Only higher prices can ensure the higher confidence level. Higher levels of capital can lead to lower equilibrium premiums. The increasing of the number of companies causes decreasing in the lower endpoint of the equilibrium interval (P_L), while the higher endpoint is unchanged. If the total capital in the market is fixed, the increasing of the number of companies leads to a higher lower endpoint of the equilibrium interval. If the total capital level is fixed in the market, the higher number of companies leads to a lower level of individual capital leading to higher possible equilibrium premiums in the sets. Because this means that more companies share the same level of capital, so the capital of each firms decreases. Table 2 summarizes the results of the comparative statics.

	P_L	P_U
Increasing level of confidence	+	+
Increasing capital level	-	-
Increasing number of companies	-	*no effect*
Increasing number of companies with fixed level of market capital	+	+

Table 2: The results of the comparative statics modelling the insurance market under solvency capital.

All of the results are equally joint (inseparable) **with our co-author** (*Kolos Ágoston*)**:**

- Literature review;
- Running simulations to find equilibrium;
- Calculations to prove the theorems;
- Illustrative figures;
- Editing and writing the article.

List of the conferences, where the results were presented:

- XVI-th annual conference of the Doctoral School of Economics, Business and Informatics
- Oligo Workshop 2021
- IX. MKE-PTE PhD Conference (Magyar Közgazdasági Egyesület Doktorandusz Műhely Pécs)
- XVII. Gazdaságmodellezési Szakértői Konferencia 2022

Study III. – The empirical analysis of the market structure of the Hungarian insurance market

In insurance models, monopoly, oligopoly and perfect competitive markets occur. However, these assumptions greatly influence the results, so it is worth examining which form of market structure fits best the real operation of the sector. Many quantitative and qualitative studies deal with a similar question, examining different countries and periods. There are several methodological options for market structure analysis. We focus on the Hungarian market between 2011 and 2019, analyzing the operation of the sector with structural and non-structural methods.

We list the number of insurance corporations in the examined period and calculate the market share of the largest three, five and ten insurance companies. The latter is shown in Figure 2.

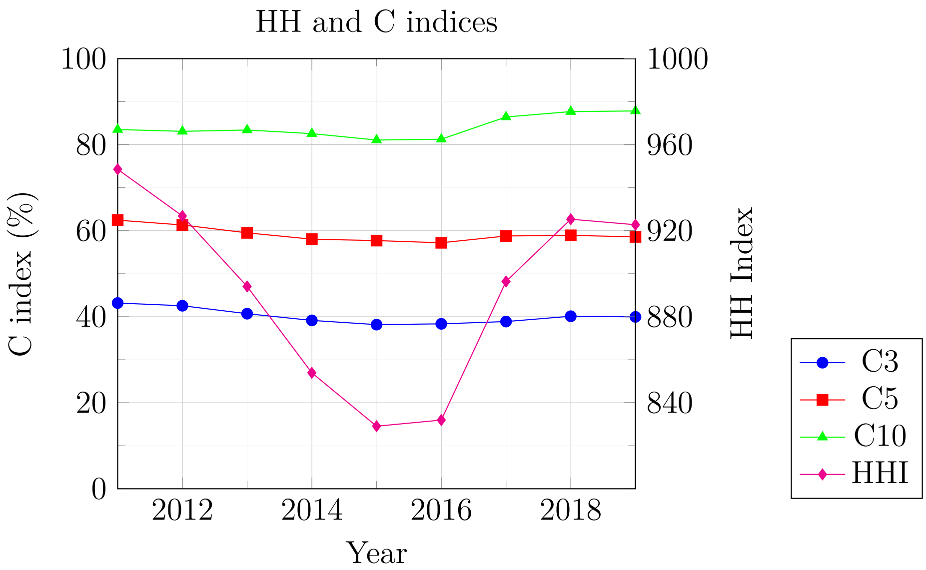


Figure 2: The evolution of the Herfindahl-Hirschman and C indices of the Hungarian insurance market.

The position of the market leaders is stable, although the 43% share of the three largest companies decreased to 40% by the end of the period. The largest 5 insurers already cover more than half of the market. The market share of the ten largest companies was 88% in 2019, and we will consider these companies for the subsequent analysis.

Among non-structural models, the Panzar and Rosse model is most often used to analyze the sector, which infers the market structure based on the input price elasticity (H statistic). Using data from the annual financial reports of ten companies covering a significant part of the market, we performed a static and dynamic panel estimation. In the case of the static model, we used a fixed effect estimation, and in the case of the dynamic panel model, we used the generalized method of moments estimation with 1 or 2 autoregressive variables. The following three factors are usually considered as input prices: the unit price of labor, business services and the financial capital. In order to check the robustness of the results, we also performed the analysis with two dependent variables. In the first case, the goal variable is the insurance and technical income together, in the second case only the income from the insurance was taken into account.

Table 3 summarizes the conclusions of the four models. For the first outcome variable (insurance and technical income) the estimated H statistic was 0.081

according to the static and -0.006 according to the dynamic model. In both cases the hypothesis of the monopoly market (or colluding oligopolies) cannot be rejected, so the market is monopoly or monopolistic competition. If the dependent variable contains only the insurance incomes we get more significant variables. The static estimate is 0.49 and the dynamic estimate is 0.758 for the input price elasticity. Based on the results of the static model, we reject the hypothesis of perfect competition and monopoly, the market is monopolistic competition. While in the case of the dynamic model, the market can be either monopolistic competition or perfect competition. The conclusion regarding these two forms of market structure is correct only if the assumption regarding the long-term equilibrium of the market is fulfilled. We checked that the Hungarian insurance sector was in long-term equilibrium between 2011 and 2019 using a model about the return on equity.

		Static panel model	Dynamic panel model
Dependent variable	Insurance and technical income	*Monopoly or Monopolistic competition*	*Monopoly or Monopolistic competition*
	Insurance income	*Monopolistic competition*	*Monopolistic competition or Perfect competition*

Table 3: The conclusions of the four examined models to determine the market structure of the Hungarian insurance market.

In our study, we summarize the results of 9 additional Panzar and Rosse methods for the insurance market of other countries and periods. Most studies support monopolistic competition, but there are also examples of the other two forms of market structure.

Individual contributions:

- Literature review;
- Data collection and calculating variables;
- Editing and writing the text.

List of the conferences, where the results were presented:

- SOR ’21 - The 16th International Symposium on Operations Research in Slovenia, Online

- XVII-th annual conference of the Doctoral School of Economics, Business and Informatics

1.6 Directions for future research

The results presented in the dissertation shed light on many new research directions. The three studies provide the opportunity for further analysis separately. Since the assumptions and results of the models are strongly related, their connections can also be examined in more detail in the future.

In connection with the empirical analysis of the Hungarian insurance market (Study III), there are several possibilities to extend the data set and make the estimation with a larger sample size (time periods or cross-sectional dimensions in Hungary or even regional or European level), thus, we can get a better estimate. More extensive research would be needed to divide the sector and estimate factor price elasticity for the life and non-life sectors separately. We also work on an analysis about the Hungarian bank sector to understand the market structure of Systematically Important Financial Institutions (SIFIs), where the changes of the H statistic is also a research question.

Let us mention a few extensions of Study I, where we investigate the substance preference. The scope of this research was limited in terms of general exploration of the relationships between substance preference and risk aversion categories, and the formulation of necessary and/or sufficient conditions. Also, the inclusion of insurers with different attitudes (different utility functions or different parameters) in a model can lead to an interesting result. There were continuum many symmetric equilibrium prices in the market, hence an interesting and important additional question could be that how lower or unique equilibrium prices could be reached. For this purpose, the introduction of another market mechanism or equilibrium concept may be considered. For example, in the case of multiple players and sequential games, we can even get a unique equilibrium price.

Study II introduced the capital constraint in a model. Greater effort is needed to simplify the assumptions of the model and derive these results in a more general setting and to show the conditions under which the presented phenomena occur. The greatest disadvantage of the model is that the level of the capital is exogenous. Interesting further research question can be when the companies can decide on prices and capital level. It can be solved two ways: ex-ante or ex-post capital decision. For the latter, we can allow capital adjustment or in the former case, we can study a two stage game with simultaneous capital and price decisions. To make this analysis possible we should calculate the equilibrium price in case of insurances with different capital levels.

All the research focuses on the insurance market. The studies complement each other well, and by connecting the different results, additional interesting research directions are opened up. Based on the empirical research conducted on the form of market structures in Hungary, there are several hypotheses for the market structure that cannot be rejected, so these additional models can be analyzed with other forms of market structure as well. In the case of a capital requirement constraint, we assumed risk-neutral insurers, but at the same time, it can be seen that in the presence of uncertainty, risk aversion can lead to higher prices, so it may be worth looking at these two phenomena together.

The results enable much new research, some of which we have already started to carry out, which are listed in the Working paper chapter.

References

Ágoston, K. Cs. (2004). Hogyan hat a bizonytalanság és a vevőkör nagysága együttesen az árakra. Doktori (PhD) értekezés, How uncertainty and the size of the customer base together affect prices. Doctoral (PhD) thesis, Budapesti Corvinus Egyetem, Közgazdaságtani Doktori Iskola.

Akerlof, J. A. (1970). The market for ‘lemons’: quality uncertainty and the market mechanism, *Quarterly Journal of Economics, 84,* 488-500.

Arellano, M. and Bond, S. (1991). Some tests of specification for panel data:

Monte Carlo evidence and an application to employment equations. *The Review of Economic Studies, 58*(2), 277-297.

Arrow, K. J. and Debreu, G. (1954). Existence of an equilibrium for a competitive economy. *Econometrica 22,* 265-90.

Arrow, K. J. (1963). Uncertainty and the welfare economics of medical care. *American Economic Review, 53,* 941-969.

Azevendo, E. M. and Gottlieb D. (2017). Perfect Competition in Markets With Adverse Selection. *Econometrica, 85*(1), 67-105.

Balog, I. (2023). Biztosítási díj meghatározása összetett kockázati modellben általánosított exponenciális hasznosságfüggvény esetében. Determination of insurance premium in the case of a generalized exponential utility function in a stratified risk model. *Szigma, LIV*(1), 55-79.

Baltagi, B. H. (2021). Econometric Analysis of Panel Data, Springer Nature, 2021.

Banyár J. and Regős G. (2012). Paradoxical price effects on insurance markets, *Economic Modelling, 29*(4), 1399-1407.

Bank for International Settlements (BIS). (2010). Basel III: International Framework for Liquidity Risk Measurement, Standards and Monitoring https://www.bis.org/about/basel_process.htm?m=54), Download: 2023.12.11.

Bertrand, J. (1883). Review of 'Théorie mathématique de la richesse sociale' and 'Recherche sur les principes mathématiques de la théorie des richesses'. *Journal des Savants*, 499-508.

Bi, J. and Cai, J. (2019). Optimal investment–reinsurance strategies with state dependent risk aversion and VaR constraints in correlated markets. *Insurance: Mathematics and Economics 85*(C), 1-14.

Bikker, J. A. and Haaf, A. (2002). Measures of Competition and Concentration in the Banking Industry: a Review of the Literature. *Economic & Financial Modelling* Summer 2002.

Bikker, J. A. , Shaffer, S. and Spierdijk, L. (2012). Assessing competition with the Panzar-Rosse model: The role of scale, costs, and equilibrium. *The Review of Economics and Statistics, 94*(4), 1025-1044.

Borch, K. (1962). Equilibrium in a Reinsurance Market. *Econometrica, 30*(3), 424-444.

Bresnahan, T.F. (1982). The Oligopoly Solution Concept is Identified, *Economic Letters* 10, 87-92.

Cabon-Dhersin, M.L., and Drouhin, N. (2014). Tacit Collusion in a One-Shot Game of Price Competition with Soft Capacity Constraints. *Journal of Economics & Management Strategy, 23*, (2), 427–442.

Cabon-Dhersin, M.L., and Drouhin, N. (2020). A general model of price competition with soft capacity constraints. *Economic Theory, 70*, 95–120. https://doi.org/10.1007/s00199-019-01203-w

Cournot, A. (1838). Recherches sur les Principes Mathématiques de la Théorie des Richesses; English Edition, in Researches into the Mathematical Principles of the Theory of Wealth, N. Bacon, ed., Macmillan, New York, 1879.

Dastidar, K. G. (1995). On the existence of pure strategy Bertrand equilibrum. *Economic theory,5*, 19–32.

Dastidar, K. G. (1997). Comparing Cournot and Bertrand in a Homogeneous Product Market. *Journal of Economic Theory 75*, 205-212.

Dionne, G. (2000). Handbook of Insurance, Kluwer Academic Publishers, 2000, https://doi.org/10.1007/978-94-010-0642-2.

Directive 2009/138/EC Solvency II (2009). Directive 2009/138/EC of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II).

Dixon, H. (1990). Bertrand-Edgeworth equilibria when firms avoid turning customers away. *The Journal of Industrial Economics*, 131–146.

Doff, R. (2016). The Final Solvency II Framework: Will It Be Effective? *The Geneva Papers, 41*(4), 587-607.

Dusek, T. (2022). A valószínűség és a kockázat tudományos és hétköznapi értelmezései. Scientific and common concepts of probability and risk. *Statisztikai Szemle, 100*(9), 873-895. https://doi.org/10.20311/stat2022.9.hu0873.

Dutang, C., Albrecher, H. and Loisel, S. (2013). Competition between non-life insurers under solvency constraints: a game-theoretic approach. *European Journal of Operational Research, 231*(3), 702-711.

Edgeworth, F. (1897). La Teoria Pura del Monopolio, Giornale degli Economisti 40; English Version, The pure theory of monopoly, in *Papers Relating to Political Economy, Vol. 1,* F. Edgeworth, ed., Macmillan, London, 1925.

Escobar, M., Kriebel, P., Wahl, M. and Zagst, R. (2019). Portfolio optimization under Solvency II. *Annals of Operations Research, 281*(1), 193-227.

Espeli, H. (2020). Insurance cartels and state policies in Norway, 1870s–1990s. *Scandinavian Economic History Review, 68*(3), 222-238.

Fog, B. (1956). How are Cartel Prices Determined? *The Journal of Industrial Economics, 5*(1), 16-23.

Goddard, J. and Wilson, J. O. S. (2009). Competition in banking: A disequilibrium approach. *Journal of Banking & Finance, 33*, 2282-2292.

Gollier, C. and Pratt, J. W. (1996). Risk Vulnerability and the Tempering Effect of Background Risk. *Econometrica, 64*(5), 1109-1123.

Guidi, F. (2021). Concentration, competition and financial stability in the South-East Europe banking context. *International Review of Economics & Finance, 76*(C), 639-670.

Gyetvai, M. (2022). Optimization of Bonus-Malus Systems. Doktori (PhD) értekezés, Doctoral (PhD) thesis, Budapesti Corvinus Egyetem, Közgazdasági és Gazdaságinformatikai Doktori Iskola. http://doi.org/10.14267/phd.2022041.

Hardelin, J. and Lemoyne de Forges, S. (2012). Raising Capital in an Insurance Oligopoly Market. *The Geneva Risk and Insurance Review, 37*(1), 83-108.

Iwata, G. (1974). Measurement of Conjectural Variations in Oligopoly. *Econometrica, 42*, 947-966.

Joskow, P. J. (1973). Cartels, competition and regulation in the property-liability insurance industry. *Bell Journal of Economics and Management Science, 4*, 327-427.

Kimball, M. S. (1993). Standard Risk Aversion. *Econometrica, 61*, 589-611.

Kovács, E. (1996). Újjáéledő hivatás és képzés: az aktuárius. Reviving profession and training: the actuary. *Szigma, 27*(4), 221-226.

Kovács, N. (2011). A piaci erő közvetett mérése a biztosítási piacon, Indirect measurement of market power in the insurance market, Széchenyi István Egyetem Regionális és Gazdaságtudományi Doktori Iskola doktori értekezés.

Kovács, E. (2023). Hagyomány és innováció a biztosításban, Traditions and innovations in insurance. *Biztosítás és Kockázat 10*, 38-55.

Kouwenberg, R. (2018). Strategic asset allocation for insurers under Solvency II. *Journal of Asset Management 19*(7), 447-459.

Kreps, D. and Scheinkman, J. (1983). Quantity precommitment and Bertrand competition yield Cournot outcomes. *Bell Journal of Economics 14*, 326-337.

Lagerlöf, J. N. M. (2016). Bertrand under uncertainty: private and common costs. *Discussion Papers 16-02, University of Copenhagen.* Department of Economics.

Nash, J. (1951). Non-Cooperative Games. *Annals of Mathematics, 54*(2), 286-295. https://doi.org/10.2307/1969529

MABISZ. (2023). Magyar Biztosítók Évkönyve. Budapest: Magyar Biztosítók Szövetsége (Yearbook of Hungarian Insurance Companies: Hungarian Insurance Association).

MAT. (2024). Magyar Aktuárius Társaság, Hungarian Actuarial Society. https://www.actuary.hu/weblap3/, Download: 2024.03.20.

Matsumoto, A., Merlone, U. and Szidarovszky, F. (2009). Some notes on applying the Herfindahl–Hirschman index. *Working paper*

Mouminoux, C., Dutang, C., Loisel, S. and Albrecher, H. (2022): On a Markovian Game Model for Competitive Insurance Pricing. *Methodology and Computing in Applied Probability, 24*, 1061-1091.

Mossin, J. (1968). Aspects of rational insurance purchasing. *Journal of Political Economy, 79*, 553-568.

Neumann, Von and Morgenstern, O. (1947). Theory of Games and Economic Behavior. Princeton University Press.

OECD. (2023). Global Insurance Market Trends 2022, https://www.oecd.org/daf/fin/insurance/globalinsurancemarkettrends.htm, Download:2024.03.20.

Osborne M. J. and Rubinstein A. (1996). A course in game theory. The MIT Press, Cambridge MA.

Panzar, J. C., and Rosse, J. N. (1987). Testing For 'Monopoly' Equilibrium. *The Journal of Industrial Economics, 35*(4), 443-456.

Pauly, M. (1968). The economics of moral hazard: Comment. *American Economic Review, 58*, 531-536.

Polborn, M. K. (1998). A Model of an Oligopoly in an Insurance Market. *The Geneva Papers on Risk and Insurance Theory, 23*, 41-48.

Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica 32*, 122-136.

Pratt, J. W. and Zeckhauser, R. J. (1987). Proper Risk Aversion. *Econometrica, 55*(1), 143-154.

Prayoonrattana, J., Laosuthi, T. and Chaivichayachat, B. (2020). Empirical Measurement of Competition in the Thai Banking Industry. *Economies, 8*(2), 1-20.

Raviv, A. (1979). The Design of an Optimal Insurance Policy. *The American Economic Review, 69*(1), 84-96.

Rothschild, M. and Stiglitz, J.(1976). Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. *The Quarterly Journal of Economics, 90*(4), 629-649.

Sanchez-Cartas, J. M. (2020). The Panzar-Rosse H statistics and Monopoly Issues on its Use as a Market Power Measure. *The B.E. Journal of Economic Analysis & Policy. 20*(4), 1-11.

Schlesinger, H. and Graf von der Schulenburg, J.M. (1991). Search costs, switching costs and product heterogeneity in an insurance market. *The Journal of Risk and Insurance 8*, 109-119.

Shaffer, S. and Spierdijk, L. (2015). The Panzar-Rosse Revenue Test and Market Power in Banking. *Journal of Banking and Finance. 61*, 340-347.

Somogyi, R., Vergote, W. and Virag, G. (2023). Price competition with capacity uncertainty - feasting on leftovers. *Games and Economic Behavior, 140*, 253-271.

Somogyi, R. (2024). Monopoly pricing with dual-capacity constraints. *Journal of Economics & Management Strategy, 33*(1), 155-174.

Sonnenholzner, M. and Wambach, A. (2004). Oligopoly in Insurance Markets. In: Teugels, J. and Sundt, B. (eds). Encyclopedia of Actuarial Science. Wiley, Chichester, UK (2004).

Stiglitz, J.(1977). Monopoly, Non-Linear Pricing and Imperfect Information: The Insurance Market. *The Review of Economic Studies, 44*(3), 407-430.

Szentpéteri Sz. (1980). Gazdasági döntések bizonytalanság esetén. Economic decisions under uncertainty. Közgazdasági és Jogi Könyvkiadó, Budapest

Szepesváry, L. (2022). Informatikai kihívások a modern aktuáriusi modellezésben: Kvantitatív módszerek és gépi algoritmusok alkalmazása a pénzáram modellezésben és az árazásban. Doktori (PhD) értekezés. IT challenges in modern actuarial modelling: Application of quantitative methods and machine algorithms in cash flow modelling and pricing. Doctoral (PhD) thesis Budapesti Corvinus Egyetem, Közgazdasági és Gazdaságinformatikai Doktori Iskola. https://doi.org/10.14267/phd.2022057.

Tasnádi, A. (2001). A Bertrand–Edgeworth-oligopóliumok Irodalmi áttekintés. Bertrand–Edgeworth oligopolies Literature review. *Közgazdasági Szemle, 48*, 1081–1092.

Tipurić, D., Pejić Bach M. and Pavić, T. (2008). Concentration of the insurance industry in selected transition countries of Central and Eastern Europe, 1998–2006. *Post-Communist Economies, 20*(1), 97-118.

Uhrin G. (2010). A verseny intenzitásának mérhetősége, Measurability of competition intensity. Szakdolgozat, Budapesti Corvinus Egyetem. https://szd.lib.uni-corvinus.hu/2413/, Download: 2021.06.20.

Varian, Hal R. (2014). Intermediate microeconomics: a modern approach. New York: W.W. Norton & Company.

Vaskövi, Á. (2024). Fejezetek az idősödő társadalmak nyugdíjtudatosságáról, Chapters on the pension awareness of ageing societies. Doktori (PhD) értekezés, Doctoral (PhD) thesis, Budapesti Corvinus Egyetem, Közgazdasági és Gazdaságinformatikai Doktori Iskola. https://doi.org/10.14267/phd.2024003.

Vékás, P. (2017). Az élettartam-kockázat modellezése, modelling longevity risk. Doktori (PhD) értekezés, Doctoral (PhD) thesis, Budapesti Corvinus Egyetem, Közgazdaságtani Doktori Iskola. https://doi.org/10.14267/phd.2017018.

Wambach, A. (1999). Bertrand competition under cost uncertainty. International, *Journal of Industrial Organization, 17*, 941-951.

Wilson, R. (1968). The theory of syndicates. *Econometrica, 36*, 113-132.

Winston, W. L. (2004). Operations Research Applications and Algorithms. 4th Edition, Duxbury Press, Pacific Grove, CA.

Zhang, N., Jin, Z., Li, S. and Chen, P. (2016). Optimal reinsurance under dynamic VaR constraint. *Insurance: Mathematics and Economics 71*(C), 232-243.

Zhang, T. and Matthews, K. (2019). Assessing the degree of financial integration in ASEAN—A perspective of banking competitiveness. *Research in International Business and Finance, 47*(C), 487-500.

List of publications

Publications included in the Ph.D. thesis

Ágoston, K. Cs. and **Varga, V.** (2020). Bertrand-árverseny állománypreferenciák mellett a biztosítási piacokon (Bertrand price competition with substance preferences in insurance markets). *Szigma, 51*(2), 149-167. https://journals.lib.pte.hu/index.php/szigma/article/view/3261/3066

Varga, V. and Madari, Z. (2023). The Hungarian insurance market structure: an empirical analysis. *Central European Journal of Operations Research 31*(3), 927-940. https://doi.org/10.1007/s10100-023-00842-8

Varga, V. and Ágoston, K. Cs. (2024). A biztosítási piac modellezése tőkekövetelmény korlát mellett (Modelling insurance market under solvency capital requirement). *Szigma 55*(2-3), 239-255.

Publications that are not included in the Ph.D. thesis

Kovács, E. and Varga, V. (2019). Adathullámok egészségről, idősödésről, nyugdíjba vonulásról. (Data about health, aging and retirement.) *Biztosítás és Kockázat 6*(4), 42-55.

Varga, V. (2019). Külföldi hallgatók a magyar felsőoktatásban – Az elmúlt évek tendenciái. (Foreign students in Hungarian higher education – Trends of recent years.) *In: Nagy, Szabolcs; Salamin, Géza (szerk.) Geopolitikai tényezők hatása a Kárpát-medencei magyar oktatási struktúrákra Budapest, Magyarország : Budapesti Corvinus Egyetem (2019).* 141–157.

Madari, Z. and Varga, V. (2021) Empirical Analysis of the Hungarian Insurance Market. *In: Drobne, Samo; Stirn, Lidija Zadnik; Kljajić, Borštnar Mirjana; Povh, Janez; Žerovnik, Janez (eds.) Proceedings of the 16th International Symposium on Operational Research in Slovenia : SOR'21 in Slovenia*

Szádoczkiné Varga, V., and Madari, Z. (2022). The Hungarian bank market structure – An empirical analysis *In: Dankó, Dóra; Dióssy, Kitti; Horváth, Viola; Nagy, Orsolya; Varga, Gábor (eds.) Embracing change and transformation: Conference proceedings Budapest, Hungary: Corvinus University of Budapest (2022)* 81–90.

Szádoczkiné Varga, V. and Madari, Z. (2024). Empirical analysis of the market structure of the Hungarian bank market. *Central European Journal of Operations Research 32*(4), 1013-1033. https://doi.org/10.1007/s10100-024-00916-1.

Working papers that are not included in the Ph.D. thesis

Varga, V. and Ágoston, K. Cs. (2024). Insurance market under solvency capital requirement with ex ante capital decision. *Working paper.*

Mágó, M. L., Ágoston, K. Cs. and Varga, V. (2024). Modelling insurance market with a sequential game. *Working paper.*

2 Study I: Bertrand price competition with substance preferences in insurance markets

Authors: Kolos Csaba Ágoston and Veronika Varga

Published in Hungarian in SZIGMA, 51(2):149-167. (2020)

https://journals.lib.pte.hu/index.php/szigma/article/view/3261/3066

Abstract

One can think that a larger portfolio is preferred to the insurance company (referring to the law of large numbers). However, it may be worth examining the question from a decision theoretic perspective. We discuss the case of insurance oligopolies using the Bertrand model. We show that the relation between insurance companies and portfolio size is crucial to study the structure of the market, and the equilibrium can be different from the traditional product market equilibrium. We define the concepts of substance aversion, substance neutral and substance seeking behavior, and we illustrate the market equilibriums in these three cases through various examples. Assuming substance neutral insurers, we can see the traditional product market equilibrium emerging in the insurance market. Otherwise, we may face some market anomalies, insurers may realize extra profit, or there may be only one insurance company in the market if we assume that the insurers are substance averse or substance seeking. Furthermore we examine the connection between the substance preference and the concept of absolute risk aversion and proper risk aversion in our examples.

2.1 Introduction

The attitudes of insurance companies to the size of their portfolio is not researched in many details in the literature of insurance modelling. People, who are familiar with insurance pricing, but not proficient in utility theory, tend to think of the problem in terms of probability theory, that a larger portfolio is preferred for the insurer (by referring to the law of large numbers, not always accurately). However, it may be worth examining the question from a decision theoretic perspective. Introducing the concept of substance preference, we present the implications of this attribute for the potential market equilibrium. Assuming substance neutral insurers, we can see the traditional product market equilibrium in the insurance market. Otherwise, we may face some market anomalies, insurers may realize extra profit, or there may be only one insurance company in the market if we assume substance averse or seeking.

The search for a model deriving the best market form for the insurance sector is a challenging research question. The study of insurance (and risk sharing in general) has played an important role in the development of economics. Several phenomenons were first described through insurance market examples (e.g., antiselection, moral hazard). The examination of the structure of the market is less prevalent. The early insurance literature focused on risk sharing and assumed typically perfect competition or monopoly (see e.g., (Rothschild and Stiglitz, 1976), (Stiglitz, 1977)).

Although the number of insurance companies is transparent, the number of competing firms is larger than in other markets (e.g., telecommunication sector, fuel suppliers). On the other hand there are not significantly more financial institutions, although financial markets are often referred to the one's coming closest to the ideal of perfect competition. Even if the number of insurance institutions is not small, market concentration is typically higher than on other markets (Sonnenholzner and Wambach, 2004); the largest 3-5 insurers cover approximately the half of the insurance market in Europe.

Significant barriers to entry also point in the direction of oligopolies, there are personal constraints and also high capital requirements. In addition to the capital requirements, the acquisition of special knowledge is necessary for the operation of an insurer, e.g., public data is limited for several types of insurance. The insurer refers to its own past experience, which is a disadvantage for new entrants. It can also be shown empirically that older stocks can be run more profitably (D'Arcy and Doherty, 1990). Sonnenholzner and Wambach (2004) even mentions product differentiation in insurance markets, which could also lead to less competition and thus the oligopolistic nature is greater.

There are some former researches, which dealt with this question in connection with the insurance markets in several time periods and different countries. Numerous empirical studies in the literature argue that the competition in the insurance market has a limited and oligopolistic nature, including the American (Mondal, 2013), the Dutch (Bikker and van Leuvensteijn, 2008), the Swedish (Lindmark et al., 2006), the Turkish (Kasman and Turgutlu, 2008), the Taiwanese (Wang et al., 2003) and the Central and Eastern European (Tipurić et al., 2008) markets. Most cases seem to support the oligopoly model.

In economic theory, the study of oligopolies appeared relatively early. When studying insurance markets, it may be reasonable to use the Bertrand model, since insurers decide on prices (Sonnenholzner and Wambach, 2004). In the classic Bertrand model two companies are already enough to achieve the same price and output as in the perfect competition, this phenomenon is also called Bertrand paradox. Polborn (1998) and Wambach (1999) have shown that the Bertrand paradox does not hold up in insurance markets, it provides companies a positive profit. These phenomena are attributed to the specialities of the insurance market, mainly to the risk aversion of the firms. Oligopolistic insurance models explicitly mention risk aversion (Powers et al. (1998), Wambach (1999)), sometimes even assume the decreasing absolute risk aversion (Hardelin and de Forges, 2012), but it is unclear how the risk attitude contributes to the results.

In uncertain situations there are several ways to describe the behavior of the decision maker, from which the theory of expected utility is the most widespread to this day. The decision maker has a utility function $u(w)$ to evaluate certain wealth (which is monotonically increasing) and to maximize the expected utility in uncertain situations.

The risk aversion of the decision maker corresponds to the concavity of the utility function u. Arrow and Pratt measured risk aversion with the help of absolute risk aversion (Pratt, 1964). In economic models it is assumed to be either constant (Constant Absolute Risk Aversion, CARA) or decreasing (Decreasing Absolute Risk Aversion, DARA). Around the millennium the problem was examined, that if the decision maker faces more risks, then the previous classification is not sufficient. Gollier and Pratt (1996) describes a paradoxical phenomenon, where in the case of a risk averse (even DARA) utility function, the decision maker due to additional risk, begins to prefer a previously unpreferred risk. It is concluded that the utility function must meet more strict requirements in order to avoid similar situations. Several attempts are known (Gollier and Pratt (1996), Kimball (1993)), in this article we cover the proper risk aversion in more detail (Pratt and Zeckhauser, 1987). We show how the examples used in the presentation of the different substance preferences relate to the absolute risk aversion and the proper risk aversion.

The rest of this paper is organized as follows. Section 2.2 presents the model and the definition of the substance preference. We describe the market equilibrium for the three categories (substance neutral, averse and seeking insurance companies) through examples. Section 2.3 discusses the connection between substance preference and the measure of absolute risk aversion and the proper risk aversion property. Finally, Section 2.4 concludes.

2.2 The oligopolistic model of the insurance market

There are a few (I) insurers in the market, that $(i = 1 \dots I)$ are homogeneous; have the same amount of capital $(w_i = w)$ and have the same risk preference. The behavior of insurers is characterized by their utility functions. Profit maximization of insurers (risk neutrality) is often assumed in economic models, but typically these models focus on the relationship between the insurer and the insured person. The insurer's risk neutrality expresses the fact that the insurer's risk aversion is much lower, than the insured person's. But in some frameworks, there is a profit maximizing insurer, see e.g., Raviv (1979). In case of modelling the interaction between insurers, a profit-maximizing insurer (Borch, 1962) is more accepted.

Our model also assumes utility maximizing insurers with identical utility function, i.e., $u_i(w) = u(w)$. In our model, we only want to focus on the role of risk so the costs are zero. The customers have loss K with the probability of q, which the insurer fully refunds at price P.

Insurers typically have many contracts at the same time, usually we refer to this as the substance of contracts, the expected utility in the case of price P and n contracts:

$$U(w, P, n, q, K) = \sum_{k=0}^{n} \binom{n}{k} q^k (1-q)^{n-k} u(w + nP - kK) \; .$$

It is essential for the model that we distinguish insurance companies with respect to their substance preferences. That's why we intodoce the concept of substance neutrality, substance aversion and substance seekingness similarly to risk aversion.

Definition 1. *Let $P_n(q, K)$ be the price, when the insurance company is indifferent to not selling any contracts or selling n contracts.*
$U(w, P_n(q, K), n, q, K) = u(w)$ *We say, that the utility function u is* ***substance averse****, iff*

$$U(w, P_n(q, K), n + 1, q, K) < U(w, P_n(q, K), n, q, K) \; , \qquad \forall n \in \mathcal{Z}^+, q \in (0, 1), K \in \mathcal{R}^+ \; , \tag{1}$$

where $\mathcal{Z}^+$ is the set of the positive integers, $\mathcal{R}^+$ is the set of the positive real numbers.

If in (1) there is equality, then the utility function is ***substance neutral****, if there is greater relation, then it is* ***substance seeking****.*

It is easy to see that not all utility functions can be classified as substance neutral, seeking or averse but the situation is similar to risk-averse, risk-neutral and risk-seeking categories. In examining the market equilibrium, it is essential whether the insurer is substance neutral, seeking or averse.

In this article, we do not undertake to provide necessary and/or sufficient conditions for substance neutrality, seeking or aversion, we only undertake to show how this concept is related to the theory so far, and through concrete examples we illustrate the market equilibrium for all three categories.

Let $P_1, P_2, \ldots$ be the prices at which the insurer is indifferent to have no substance or to have $1, 2, \ldots$ contracts. In the P, n plane, the points $(P_1; 1), (P_2; 2), \ldots$ define a 'curve', each point of this curve represents the same expected utility level to the insurer, so on the analogy of consumption decisions we call it an indifference curve. The utility of the initial wealth is the initial utility that the insurance company achieves even if it does not sell any contracts. This curve has a special significance, in case of substance aversion the curve has a positive slope (see Figure 4), in case of substance neutrality it is a vertical line (see Figure 3) and in the case of substance seeking it has a negative slope (see Figure 7).

We assume furthermore that, there is a function $D(P)$, which is like a probability between 0 and 1. It shows the probability that one consumer will buy an insurance at price P, of course it is decreasing. There are N potential consumers on the market so the expected demand is $ND(P)$. The consumers buy from the company, which quoted the lowest premium. It is assumed that this insurer cannot refuse to sell to a consumer who wants to buy for the quoted premium. If several insurers quote the same lowest premium, they share the market equally. We examine the equilibrium price in an oligopolistic market.

2.2.1 Substance neutral insurers

In this subsection we will see that, the exponential utility function, which is the case of constant absolute risk aversion leads to substance neutrality. The utility function of the insurers is the following: $u(w) = -\exp(-rw)$, where r shows the measure of risk aversion, which is a positive constant. If the company quotes price P and n customer buy the insurance, then the expected utility of the company is:

$$\begin{aligned} U(w, P, n, q, K) &= \sum_{k=0}^{n} \binom{n}{k} q^k (1-q)^{n-k} (-\exp(-r(w + nP - kK))) \\ &= -\exp(-rw)[\exp(-rP)(q\exp(rK) + (1-q))]^n \quad . \end{aligned} \tag{2}$$

It can be seen, that if the expression in the square bracket in equation (2) is 1, then for every arbitrarily chosen n the expected utility of the insurance company is $-\exp(-rw)$. This expression is 1 if the company is indifferent to sell a contract or not ($P = \frac{1}{r}\ln(q\exp(rK) + (1-q))$). It follows that the company is indifferent to sell n contracts or not, that is what we call substance neutral behavior. If this expression is higher than 1, the expected utility of the company is increasing in n for a fixed P, so selling more contracts is preferred.

Let's see what happens in the market if I insurance companies are selling the insurance. If the price is higher than the indifferent price ($\frac{1}{r}\ln(q\exp(rK) + (1-q))$), one of the companies can achieve the whole market with an infinitesimally smaller price, which gives a higher expected utility. So the only equilibrium price is ($\frac{1}{r}\ln(q\exp(rK)+(1-q))$), which is really similar to the traditional product market, where the companies produce on the marginal cost and the profits are zero, even if there are only two companies on the market. This is the case of Bertrand paradox, where no extra utility is available in equlibrium, the expected utility is equal to the utility of the initial wealth.

We can solve this case analytically. But in further examples it can become difficult, so it worths to plot the indifference curves. We fix the utility level at $\overline{u}$, and give the (P, n) values, which give the expected utility $\overline{u}$. This indifference curves are on Figure 3, the lighter the line, the higher the level of expected utility. It can be seen that the higher number of contacts or higher prices also lead to higher expected utility.

The demand function on the plot is straight, but only the decreasing property matter. The dotted line shows the half of the market demand.

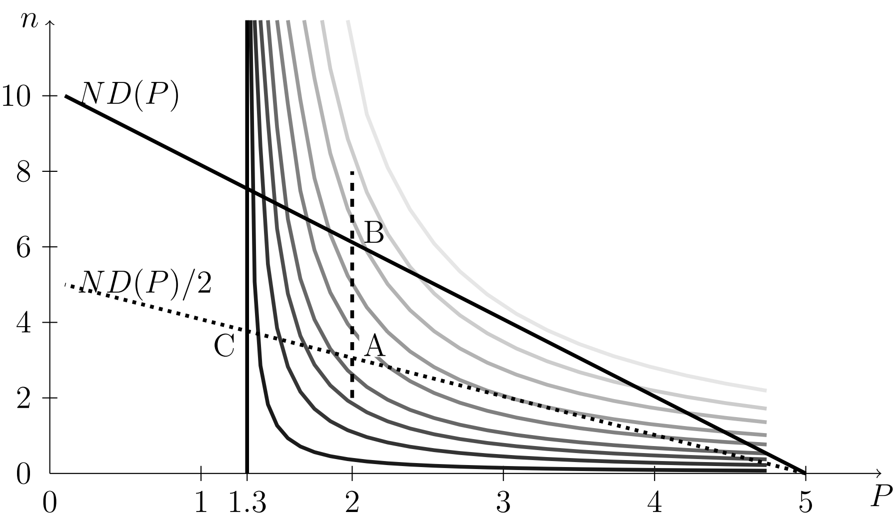


Figure 3: Oligopoly market in the case of substance neutral insurance companies. The wealth of the companies w is 0, but in the case of constant absolute risk aversion this fact does not have significance. The amount of claim K is 100, the probability q is 0.001. The parameter of the risk aversion measure r is 0.04. The demand is strictly decreasing on $[qK, 5]$.

To examine the market mechanism assume that there are two companies on the market. If they quote a higher price than the indifferent price ($\frac{1}{r}\ln(q\exp(rK)+(1-q))$), for instance the price noted by the dashed line on the Figure 3, then the two insurers share the market equally, they are at point A. One of the companies with an infinitesimally smaller price can own the whole market and achieve a higher level of expected utility at point B. So decreasing the price is preferred, this mechanism leads the market to point C, which is on the first indifference curve, that gives the same expected utility as the utility of the initial wealth.

2.2.2 Substance averse insurers

The market equilibrium in the case of substance neutrality is very similar to the product markets, insurance companies do not achieve extra utility. In this subsection we will see that, examining the indifference curves of substance averse insurers, it can be said that the curve with the initial utility level has positive slope. The equlibrium is not unique and asymmetric market share can also be an equilibrium in the market. The following are two examples of a utility function that leads to substance aversion.

Proposition 1. *The mixed exponential utility function* $u(w) = aw - \exp(-rw)$, *where* $a > 0$ *is substance averse.*

Proof. In the case of the mixed exponential utility function, the expected utility can be written in a closed form:

$$\begin{aligned} &U(w,P,n,q,K) = \\ &\qquad aw + an(P - qK) - \exp(-rw)[\exp(-rP)(q\exp(rK) + (1-q))]^n \ . \end{aligned} \tag{3}$$

At P_n price the company is indifferent between selling n contracts or to selling nothing:

$$\begin{aligned} aw + an(P_n(q,K) - qK) - \exp(-rw)\cdot \\ [\exp(-rP_n(q,K))(q\exp(rK) + (1-q))]^n \\ = aw - \exp(-rw) \ . \end{aligned} \tag{4}$$

The expression (4) can be transformed:

$$1 + \exp(rw)an(P_n(q,K) - qK) = [\exp(-rP_n(q,K))(q\exp(rK) + (1-q))]^n \tag{5}$$

Thus

$$
\begin{aligned}
&U(w, P_n(q,K), n+1, q, K) \\
&\quad = aw + a(n+1)(P_n(q,K) - qK) + \\
&\qquad\qquad - \exp(-rw)[\exp(-rP_n(q,K))(q\exp(rK) + (1-q))]^{n+1} \\
&\quad = aw + a(n+1)(P_n(q,K) - qK) + \\
&\qquad\qquad - \exp(-rw)[1 + \exp(rw)an(P_n(q,K) - qK)] \cdot \\
&\qquad\qquad\qquad [1 + \exp(rw)an(P_n(q,K) - qK)]^{1/n} \\
&\quad = aw + \exp(-rw) \cdot \\
&\qquad \Big\{ \exp(rw)a(n+1)(P_n(q,K) - qK) - \\
&\qquad\qquad\qquad [1 + \exp(rw)an(P_n(q,K) - qK)]^{\frac{n+1}{n}} \Big\} .
\end{aligned}
$$

Let us consider the expression $(n+1)x - (1+nx)^{\frac{n+1}{n}}$, where $n > 0$. This is at the point $x = 0$ equal to -1. For positive values of x it is monoton decreasing, so from this we can conclude that:

$$
(n+1)x - (1+nx)^{\frac{n+1}{n}} < -1 \qquad \text{if } n > 0,\ x > 0 . \tag{6}
$$

Using expression (6) the utility of $U(w, P_n(q,K), n+1, q, K)$ can be furthermore transformed:

$$
\begin{aligned}
&U(w, P_n(q,K), n+1, q, K) \\
&\quad = aw + \exp(-rw) \cdot \\
&\qquad \Big\{ \exp(rw)a(n+1)(P_n(q,K) - qK) + \\
&\qquad\qquad\qquad -[1 + \exp(rw)an(P_n(q,K) - qK)]^{\frac{n+1}{n}} \Big\} \\
&\quad < aw + \exp(-rw) = u(w) .
\end{aligned}
$$

Summarizing:

$$
u(w) = U(w, P_n(q,K), n, q, K) > U(w, P_n(q,K), n+1, q, K) ,
$$

which means, that the statement has been proven. □

Figure 4 presents the demand function and the indifference curves in the case of the mixed exponential utility function for different levels: moving to the lighter curves the level of the expected utility is higher. The first curve (black) is the indifference curve, which gives the same expected utility as the utility of the inital wealth, and it has a positive slope. The lowest potential equlibrium price is P_0, at which the companies are on the initial utility indifference curve at point A, they share the market equally. In this case all the two companies are indifferent to have a substance or not.

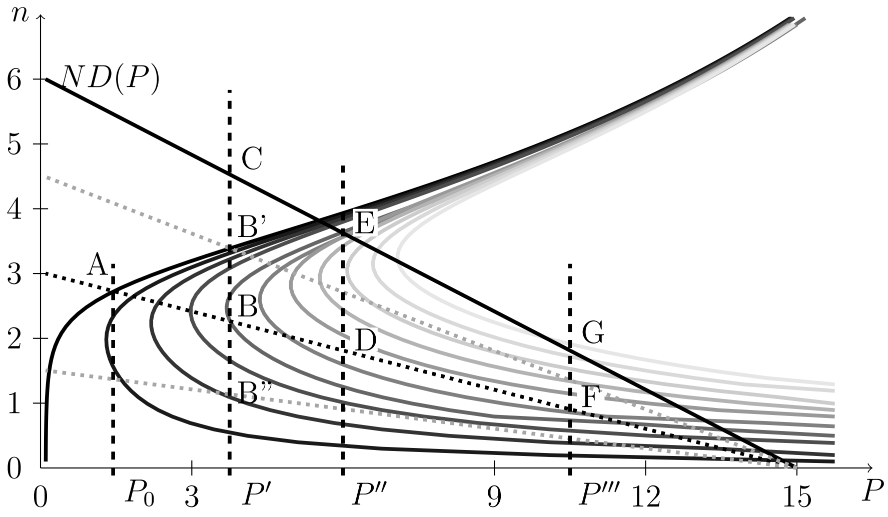


Figure 4: Oligopoly market in the case of substance averse insurance companies with mixed exponential utility function. The wealth w is 0, the amount of claim K is 100, the probability q is 0.001, $a = 1000$, $r = 10$. The demand is strictly decreasing on $[qK, 15]$.

It is also possible that a higher price will be the equilibrium price on the market. Assume that both companies quote price P'. They share the market equally, so they are at point B on Figure 4, where decreasing the price is not preferred. In this case the firm, with the lower price owns the whole market, it is in point C, which gives a lower level of expected utility than point B. P' can be an equilibrium price, and in this case the companies are on a higher indifference curve, they achieve some extra utility in addition to the utility of initial wealth.

P'' is the price, when owning the half of the market (D) and owning the whole market (E) have the same expected utility. In this case the companies with the same price P'' are in point D, where decreasing the price is indifferent (point E). Let see a higher price, P'''. This can not be an equilibrium price, because with a smaller price we could go to point G, where owning the whole market gives a higher level of expected utility compared to point F. We can say that in interval $[P_0, P'']$ every price can be an equilibrium price, and except for P_0 these prices give some extra utility in addition to the initial utility, but there is no market mechanism that would deduct the price to the point P_0.

We may see another interesting phenomenon at price P'. It is easy to see that with a price of P_0, only half of the market can be the equlibrium share, because a larger substance gives a lower level of utility. But at the price P', it is not necessary to get half of the market, non-symmetric market share may also be equilibrium. Suppose one company owns a quarter of the demand (point B") and the other one three-quarters (point B'). The situation is a bit paradoxical, but who has the higher rate is indifferent between whether having a substance or not, while the one with a smaller market share is clearly better of than having no stock. Thus, for a price of P', the market share can be arbitrary in the range of (0.25; 0.75) (although these are only approximate values). It is interesting that with the price P'' only the 50-50% distribution can be equilibrium again.

Finally in Figure 5 a case is shown which is similar to the substance neutral case. The above mentioned effetcs are still meet, but the interval of the potential equilibrium prices is smaller, under normal market conditions, it may become unnoticed.

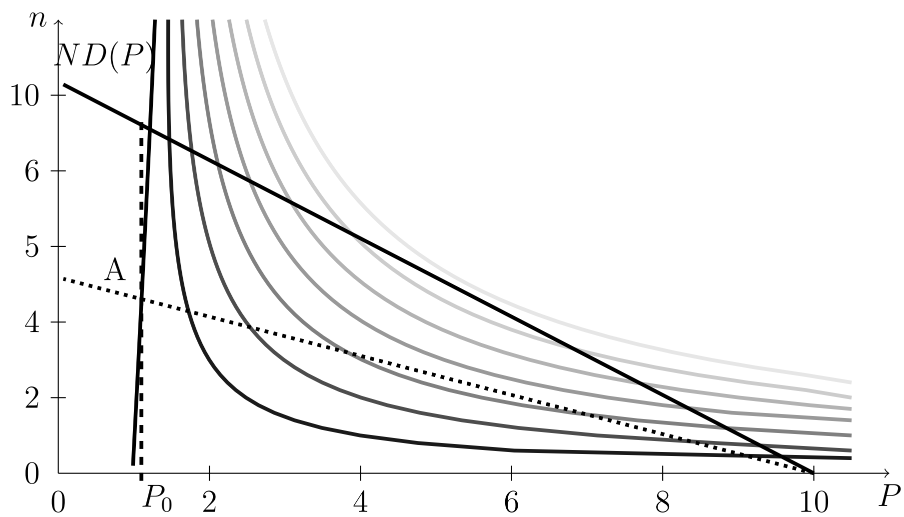


Figure 5: Oligopoly market in the case of substance averse insurance companies with mixed exponential utility function. The wealth w is 0, the amount of claim K is 100, the probability q is 0.001, $a = 0.1$, $r = 20$. The demand is strictly decreasing on $[qK, 10]$.

In addition to the mixed exponential utility function, the quadratic utility function also leads to substance aversion, in which case the possible market equilibria are similarly.

Proposition 2. *The quadratic utility function* $u(w) = w - bw^2$, $b > 0$, $w \leq \frac{1}{2b}$ *is substance averse.*

Proof.

$$\begin{aligned} U(w, P, n, q, K) = \\ \sum_{k=0}^{n} \binom{n}{k} q^k (1-q)^{n-k} ((w + nP - kK) - b(w + nP - kK)^2) = \\ w + n(P - Kq) - b(w + n(P - Kq))^2 - bK^2 nq(1-q) \ . \end{aligned} \tag{7}$$

At price P_n the insurance company is indifferent to sell n contracts or to sell zero:

$$w + n(P_n(q,K) - Kq) - b(w + n(P_n(q,K) - Kq))^2 - bK^2 nq(1-q) = w - bw^2 \ ,$$

So:

$$n(P_n(q,K) - Kq) - b(w + n(P_n(q,K) - Kq))^2 + bw^2 = bK^2 nq(1-q) \ . \tag{8}$$

The expected utility is the following:

$$\begin{aligned} U(w, P_n(q,K), n+1, q, K) = \\ w + (n+1)(P_n(q,K) - Kq) - b(w + (n+1)(P_n(q,K) - Kq))^2 + \\ -bK^2(n+1)q(1-q) \ . \end{aligned} \tag{9}$$

Expression (9) can be tranformed using equation (8):

$$\begin{aligned} U(w, P_n(q,K), n+1, q, K) = \\ w - bw^2 - b(n+1)(P_n(q,K) - qK)^2 = \\ u(w) - b(n+1)(P_n(q,K) - qK)^2 \ . \end{aligned} \tag{10}$$

The expression $-b(n+1)(P_n(q,K) - qK)^2$ is always negative, which means that we proved the statement. □

On Figure 6 one can see the indifference curves of the quadratic utility function.

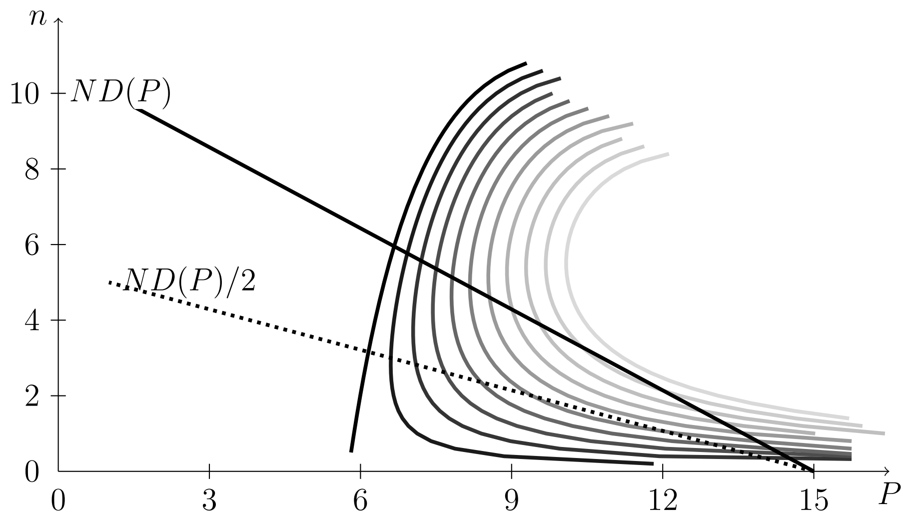


Figure 6: Oligopoly market in the case of quadratic utility function, the parameter b is 0.1, the wealth of the companies w is -100, the amount of the claim K is 1000, the probability q is 0.01. The demand is strictly decreasing on $[qK; 15]$.

2.2.3 Substance seeking insurers

The case of the substance seeking insurer is definitely interesting based on the insurance premium calculation, where a larger stock is favorable to the insurer. The case of substance seeking (or a similar concept) is not discussed in the literature (in utility theory), thus, no necessary and/or sufficient condition is known for the property. We also illustrate this case with an example.

Let's see the following utility function:

$$u(w) = \begin{cases} 300[-\exp(-1 - \frac{x-100}{140}) + \exp(-1) + \frac{x-100}{1000000}] & \text{if } x \leq 100 \\ \\ 750[-\exp(-1 - \frac{x-100}{500}) + \exp(-1) + \frac{x-100}{1000000}] & \text{if } x > 100 \end{cases} \tag{11}$$

Figure 7 presents the indifference curves of the utility function given by expression (11) and the demand function. The first curve, giving the same utility as the initial weath has a negative slope, this is the case of substance seeking.

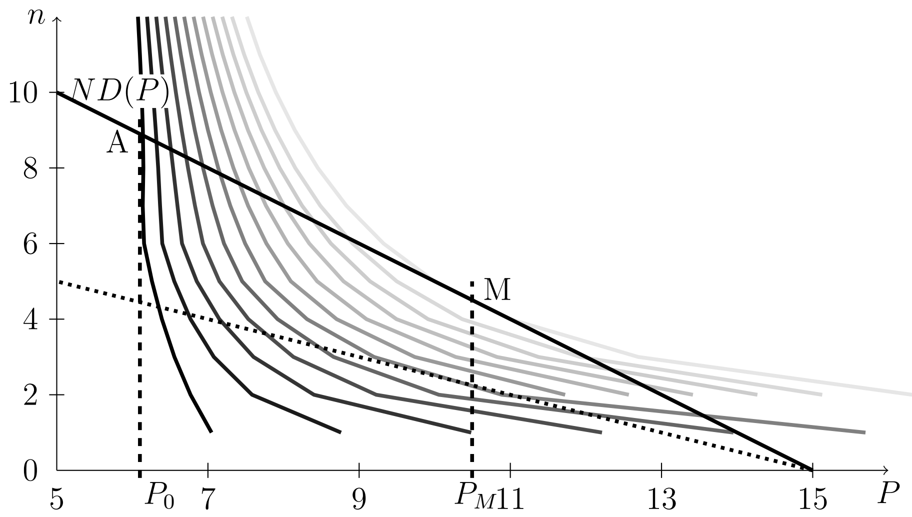


Figure 7: Oligopoly market in the case of substance seeking (with utility function (11)). The wealth of the companies w is 111, the amount of claim K is 50, the probability q is 0.1. The demand is strictly decreasing on $[qK, 15]$.

On Figure 7 there can be seen another type of market equlibrium, in which there is only one company in the market in point A. Larger substance allows lower prices, so it is optimal to owning the whole market. But in this case the price is lower, than the monopoly price P_M. The equilibrium price P_0 is much lower than P_M, and we simply face a situation where an insurer can set a better price than several companies together. But potential competitors pose a threat, if the insurer presents a higher price, then he or she will be immediately replaced by someone else with a lower price.

On Figure 8 we can see the same utility function as in Figure 7 but with other risk parameters, i.e., the probability and the amount of claim is changed to q =0.03 and K =10. In this case the first indifference curve hase positive slope, this is not a substance seeking utility function. Only with some q and K parameters can we show this substance seeking property.

2.3 Connection between substance and risk preferences

The concept of substance aversion was defined in the same pattern as risk aversion. We assumed risk aversion through the model. In the following, we examine the risk attitudes in the examples mentioned earlier from the point of absolute risk aversion and proper risk aversion property in more detail.

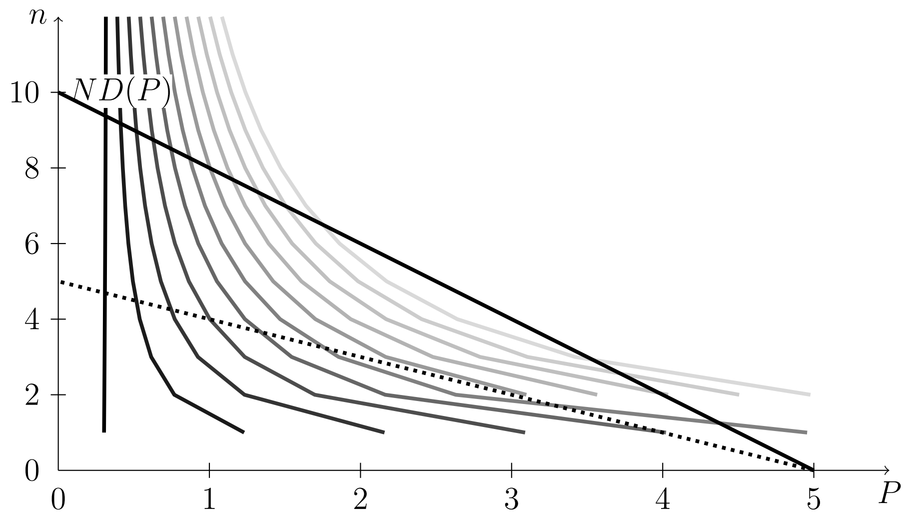


Figure 8: Oligopoly market in case of substance seeking (with utility function (11)). The wealth of the companies w is 111, the amount of claim K is 10, the probability q is 0.03. The demand is strictly decreasing on [qK,5].

The measure of absolute risk aversion is defined as the second derivative of the utility function u divided by the first derivative of it, multiplied by minus one ($-\frac{u''(w)}{u'(w)}$) (Pratt, 1964). Based on this, we distinguish utility functions as constant (CARA), decreasing (DARA) and increasing (IARA) absolute risk averse. According to this definition, the exponential utility function (that leads to substance neutrality) is a constant absolute risk averse utility function. The substance averse examples are the mixed exponential and the quadratic utility functions. The mixed exponential utility function is decreasing abolute risk averse, while the quadratic utility function is increasing absolute risk averse. The utility function given by (11) has decreasing absolute risk aversion, and with some K and q parameters it leads to the substance seeking property. However, it may be worthwhile to examine risk aversion in a deeper way.

2.3.1 Proper risk aversion

The early insurance literature has defined the concept of risk aversion and the measure of absolute risk aversion. This theory can be used to model a single risk, but it is not sufficient if there are multiple risks in the model (Gollier and Pratt, 1996), a more precise description is needed. Therefore, Pratt and Zeckhauser (1987) introduced the concept of proper risk aversion: a non-preferred risk may not become preferred due to an unpreferred background risk.

Definition 2. *Let W be the initial wealth of the insurance company (can be certain or uncertain), let R_1 and R_2 be two independent risks (random variables). (If the wealth is a random variable as well, then W, R_1 and R_2 are independent.) None of the risks are preferred to the company:*

$$\mathbb{E}u(W + R_1) \leq \mathbb{E}u(W) \ , \tag{12}$$

$$\mathbb{E}u(W + R_2) \leq \mathbb{E}u(W) \ , \tag{13}$$

The utility u satisfies the proper risk aversion, if (12) and (13) ensure that:

$$\mathbb{E}u(W + R_1 + R_2) \leq \mathbb{E}u(W + R_1) \tag{14}$$

The substance preference is a similar concept to the proper risk aversion, but they are not exactly the same. The company has n contracts and quotes the price P. Then the insurance payment is $nP - LK$, where L is distributed binomially with paramters n and q. Let $R_1 = nP - L_1K$. Let R_2 be the payment of another substance with the same number of contracts. Then R_1 and R_2 are independent and $R_1 + R_2 = 2nP - (L_1 + L_2)K$, where $L_1 + L_2$ is distributed binomially with the parameters $2n$ and q. This shows the insurance payment in the case of $2n$ contracts at price P. If P is the price, when the copmany is indifferent to have n contracts or none, then at this price it is not preferred to double the substance. The proper risk aversion leads to:

$$U(w, P_n(q, K), 2n, q, K) \leq U(w, P_n(q, K), n, q, K) \ , \quad \forall n \in \mathcal{Z}^+, q \in (0, 1), K \in \mathcal{R}^+ \tag{15}$$

property. This is not the definition of the substance aversion, but mostly proper risk aversion leads to substance aversion as well. Pratt and Zeckhauser (1987) give necessary and sufficient conditions for proper risk aversion, but in general it is difficult to decide whether proper risk aversion is satisfied for a utility function or not. This property holds for the frequently used utility functions (e.g., logarithmic, root,

Utility functions	The number of contracts				
	1	2	3	5	10
$\ln(w)$	1.05299	1.05300	1.05300	1.05301	1.05303
$\sqrt{w}$	1.02604	1.02605	1.02605	1.02605	1.02606
$1000w - \exp(-w/10)$	0.1217	0.3468	2.2017	9.7895	19.2057

Table 4: The indifferent price for different amount of substances. The initial wealth w is 1000 for the logarithmic function and root function and 0 for the mixed exponential utility function; the amount of claim K is 100 and the probability q is 0.001.

mixed exponential function). Table 4 illustrates the proper risk aversion property. For the mixed exponential utility function we have derived its substance aversion in Proposition 1.

2.3.2 Improper risk aversion

In the previous subsection, we have seen that proper risk aversion (ignoring some extreme cases) leads to substance aversion. But the condition of substance aversion may also be satisfied in the case of improper risk aversion. An example of this is the quadratic utility function.

The quadratic utility function typically plays an important role in finance, but theoretical arguments can be made against it, where the most significant is the increasing absolute risk aversion measure. In the case of increasing absolute risk aversion the risk aversion can not be proper. The simplest way to see this in the case, when in the definition of proper risk aversion (Definition 2) R_2 is a certain loss in wealth. With lower wealth the measure of risk aversion is lower, thus it can occur that a not preferred risk (with higher wealth) becomes preferred because of a wealth loss. Despite of the increasing risk aversion, substance aversion can be shown. So substance aversion is a more general concept, it can be met with proper and improper risk aversion as well.

The study of improper risk aversion is barely mentioned in the literature, but in the case of risk aversion it is treated as a paradox and trying to be excluded. There are some improper cases mentoined, this only means that (14) does not hold everywhere, and that, it is not the case of substance seeking. The utility function given by (11) is based on the counterexample in Gollier and Pratt (1996). The utility function given here consists of two linear sections and it is broken at 100. The degree of risk aversion is incomprehensible in this example, therefore, we applied a small curvature to the function to achieve decreasing absolute risk aversion.

It can be verified that the utility function given by (11) is strictly concave, and its absolute risk aversion is decreasing. The degree of absolute risk aversion at $x = 100$ has a rupture; if it were essential not to have such, then the function should be changed in the small environment of 100 to have continously decreasing absolute risk aversion, this change does not affect the numbers in the numeric example.

It can be shown that the utility function given by (11) does not satisfy the proper risk aversion property, because on Figure 7 the first indifference curve, giving the utility of the initial wealth has a negative slope.

2.4 Conclusions

Insurance companies try to establish risk pools and collect the customers with homogenous risks. This can be a motivation to create larger substances. However, the risk aversion assumption in the models may imply that substance growth is not preferred by the insurers. To study this phenomenon, we defined the concepts of substance neutralilty, aversion and seeking, and then examined the market equilibrium for the three categories using various examples.

The exponential utility function is a constant absolute risk averse function, which is substance neutral. The equilibrium price in this case is the indifference price, which ensures the utility of the initial wealth for the company. This case is very similar to the case of the traditional product market. In the case of substance averse insurers, the market equilibrium is not unique, insurers may achieve higher expected utility than the utility of their initial wealth, and an asymmetric distribution of the market may be an equilibrium too. In the third case, where there are substance seeking insurers, a set of parameters can be given when there is only one company in the market equilibrium, but the price is lower than the monopoly price. In the case of utility functions with different properties, we can obtain different results from the product market.

One can see that substance neutrality can be achieved by the constant absolute risk aversion. As well as the fact that proper risk aversion usually leads to substance aversion, however, substance aversion is a more general concept, it can also occur in the case of improper risk aversion. The utility function used to illustrate substance seeking appears as a counterexample in the literature, where proper risk aversion is not satisfied, this possibility is usually tried to be ruled out if risk aversion is assumed.

Let us mention a few extensions of the current work. An issue that requires further research is a more general exploration of the relationships between substance aversion and risk aversion categories, and the formulation of necessary and/or sufficient conditions. Also, the inclusion of insurers with different attitudes in a model can lead to an interesting result, in addition to the type of risk aversion, insurers may also differ in terms of risk aversion parameters.

References

Bikker, J. A. and van Leuvensteijn, M. (2008). Competition and efficiency in the Dutch life insurance industry. *Applied Economics, 40,* 2063–2084.

Borch, K. (1962). Equilibrium in a Reinsurance Market. *Econometrica, 30(3),* 424–444.

D'Arcy, S. P. and Doherty, N. A. (1990). Adverse Selection, Private Information, and Lowballing in Insurance Markets. *The Journal of Business, 63(2),* 145–164.

Gollier, C. and Pratt, J. W. (1996). Risk Vulnerability and the Tempering Effect of Background Risk. *Econometrica, 64(5),* 1109–1123.

Hardelin, J. and de Forges, S. L. (2012). Raising Capital in an Insurance Oligopoly Market. *The Geneva Risk and Insurance Review, 37,* 83–108.

Kasman, A. and Turgutlu, E. (2008). Competitive Conditions in the Turkish Non-Life Insurance Industry. *Review of Middle East Economics and Finance, 4(1),* 1–16.

Kimball, M. S. (1993). Standard Risk Aversion. *Econometrica, 61,* 589–611.

Lindmark, M., Andersson, L-F. and Adams, M. (2006). The Evolution and Development of the Swedish Insurance Market. *Accounting, Business & Financial History, 16(3),* 341–370.

Mondal, W. I. (2013). The Health Insurance Exchange: An Oligopolistic Market In Need Of Reform. *Journal of Business & Economics Research, 11(12),* 569–576.

Polborn, M. K. (1998). A Model of an Oligopoly in an Insurance Market. *The Geneva Papers on Risk and Insurance Theory, 23,* 41–48.

Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica 32,* 122–136.

Pratt, J. W. and Zeckhauser, R. J. (1987). Proper Risk Aversion. *Econometrica, 55(1),* 143–154.

Powers, M. R., Shubik, M. and Yao, S. T. (1998). Insurance Market Games: Scale Effects and Public Policy. *Journal of Economics, 67(2),* 109–134.

Raviv, A. (1979). The Design of an Optimal Insurance Policy. *The American Economic Review, 69(1),* 84–96.

Rothschild, M. and Stiglitz, J.(1976). Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. *The Quarterly Journal of Economics, 90(4),* 629–649.

Sonnenholzner, M. and Wambach, A. (2004). Oligopoly in Insurance Markets. In: Teugels, J. and Sundt, B. (eds). Encyclopedia of Actuarial Science. Wiley, Chichester, UK (2004)

Stiglitz, J.(1977). Monopoly, Non-Linear Pricing and Imperfect Information: The Insurance Market. *The Review of Economic Studies, 44(3),* 407–430.

Tipurić, D., Bach, M. P. and Pavić, T. (2008). Concentration of the insurance industry in selected transition countries of Central and Eastern Europe, 1998–2006. *Post-Communist Economies, 20(1),* 97–118.

Wambach, A. (1999). Bertrand competition under cost uncertainty. *International Journal of Industrial Organization, 17,* 941–951.

Wang, J. L., Tzeng, L. Y., and Wang, E-L. (2008). The Nightmare of the Leader: The Impact of Deregulation on an Oligopoly Insurance Market. *Journal of Insurance Issues, 26(1),* 15–28.

3 Study II: Modelling insurance market under solvency capital requirement

Authors: Veronika Varga and Kolos Csaba Ágoston

Published in Hungarian in SZIGMA, 55(2-3):239-255. (2024)

Abstract

Since 2016 the operation of insurance companies in the European Union is regulated by the Solvency II directive. According to the EU directive the capital requirement should be calculated as a 99.5% of Value at Risk. In this study, we examine the impact of this capital requirement constraint on equilibrium premiums and profits. We discuss the case of the oligopoly insurance market using Bertrand's model, assuming profit maximizing insurance companies facing Value at Risk constraints. In our model the companies have the same level of capital and they set premiums simultaneously. We introduce the capital requirement, thus the equilibrium premiums can be higher than the expected amount of damage (net premium) and the insurers may even earn a positive profit. Under certain parameters it can occur, that fewer companies with larger level of capital or even a monopoly market allow lower equilibrium premiums.

Keywords: Insurance market, Bertrand model, Capital requirement, Solvency II, Value at Risk

JEL Classification: D43, G22

3.1 Introduction

After the financial crisis of 2008 the regulation of financial institutions became a central issue. Since 2016 the operation of insurance companies in the European Union (EU) is regulated by Solvency II. This EU directive ensures the same regulation for all member states and the protection of policy holders and beneficiaries. According to Solvency II directives the solvency capital requirement of insurance companies should ensure that bankruptcy occurs not more often than once in every 200 cases (Solvency II , 2009). So capital is able to cover losses in 99.5% of cases. The level of the capital requirement plays a crucial role in the operation of insurance companies (premium and reserve calculation).

This capital level is a value at risk (Value at Risk, VaR) with a confidence level of 99.5%. According to the limit analyzed in the article, the insurers' capital and income from sales must cover the expected claim payments with a probability of 99.5%. In our model, the capital level of insurers is considered the same and given, and there is a cost of holding capital.

The main objective of this paper is to analyze the effect of the solvency capital requirement on equilibrium premiums and profits, and to determine which factors influence equilibrium premiums. The natural approach is to model the sector as a Bertrand oligopoly since risk neutral insurers decide on premiums and maximize their expected profit.[1]

In the classic Bertrand model two companies (with the same costs) are already enough to achieve the same premium and output at equilibrium which coincides with those in perfect competition. This phenomenon is called the Bertrand paradox. There are several modifications of the Bertrand model when companies can achieve positive profits and equilibrium prices are higher. Assuming premium matching guarantees in a Bertrand model leads to a continuum of symmetric Nash equilibria (Dixit and Nalebuff, 2008), and in several cases companies achieve positive profits. Wambach (1999) in the case of uncertain costs, Polborn (1998) and Ágoston and Varga (2020) in the case of risk averse companies showed example that positive profits can be realised even in the price competitive insurance markets.

Banyár and Regős (2012) analyzes an insurance market within the framework of an oligopoly model . In their study, they draw attention to the fact that the intermediary system can play an important role in the case of insurance products. Banyár and Regős (2012) shows in a theoretical framework that price increases can be experienced even with a high degree of competition. Based on the findings of the theoretical model, the model is also tested on the available data. It is interesting

[1] This analytical framework also occurs in other studies, see for instance Schlesinger and Graf von der Schulenburg (1991).

that the conclusion of the study coincides with the results of this research at certain points, but it is also important to emphasize that the assumptions of the two models are fundamentally different.

There are also empirical arguments for using the Bertrand model. One of the established frameworks for the empirical investigation of market structures is the Panzar and Rosse model (Panzar and Rosse, 1987). This model has been applied to insurance markets around the world (Coccorese, 2010; Camino-Mogro et al., 2019), Varga and Madari (2023) tested the model on Hungarian data, according to their results, the Hungarian insurance market is characterized by monopolistic competition.

The capital requirement can also be interpreted as a kind of capacity constraint, the literature of which is very extensive. The insurance company can sell a sufficient number of contracts and does not face any serious physical obstacles. [2] However, it can only sell as many contracts as the capital level allows. If the company sells more contracts than that, supervisory sanctions are triggered. In our model, insurers face a fixed amount of penalty in case of violation of the constraint, which is large enough to avoid these cases. On the one hand, the analogy between the capital and capacity constraint is obvious, but on the other hand there is also a significant difference, the number of contracts allowed by the capital requirement depends on the premium of the insurance, which is not typical for product markets.

Solvency II criteria have an extensive literature (Doff, 2016). Some related papers deal with the question of portfolio optimization and asset allocation under Solvency II (Kouwenberg, 2018; Escobar et al., 2019), and some study the investment and reinsurance strategies under VaR constraints (Bi and Cai, 2019; Zhang et al., 2016). Although the literature on solvency regulations is abundant, the impact of the VaR constraint on the market equilibrium is rarely studied. Dutang et al. (2013) built a non-cooperative game for studying market premium, solvency level, market share and underwriting results of non-life insurance companies. Mouminoux et al. (2021) study a similar repeated game and determine long run market shares, leadership and ruin probabilities and the effect of deviaton from the regulated market.

[2] There are also physical limitations in the case of insurance companies, such as the number of employees, but the number of contracts sold is typically not limited by these factors.

The contribution of this paper is the inclusion of the solvency capital requirement in the model. Without the constraint, insurers sell the contracts at net premium, but introducing the solvency capital constraint leads to the existence of a continuum of symmetric Nash equilibrium premiums, or it can occur that there is only one company in the market. In some cases the companies can achieve positive profits. Further interesting result is that a decrease in the number of companies (lower level of competition) leads to lower possible equilibrium premiums. Fewer companies with larger capital or even a monopoly can set lower premiums, which is a more advantageous situation from the point of view of consumers.

The paper is organized as follows. Section 3.2 presents the basic model. In Section 3.2.1 we describe the capital requirement constraint. In section 3.3 we present the main results, the equilibrium premiums and expected profits and their comparative static analysis. Finally, section 3.4 concludes and contains further research opportunities.

3.2 Model

We model the situation as a non-cooperative game. In this game the set of players I are the insurance companies ($i = 1, \ldots, I$). We assume Bertrand premium competition, companies (players) decide on the premium level (P_i) simultaneously. [3] All insurance companies have the same level of capital, this is a given size C, this is not a decision variable.

In our model the customers are homogeneous with respect to risk, but their reservation premiums are different. Each customer incurs a financial loss K ($K > 0$) with probability q ($0 < q < 1$), furthermore we assume that claims are independent. The risks of the customers are independent random variables $K \cdot \kappa_j$, where κ_j is a random variable with Bernoulli distribution with parameter q. Customers can cover their losses by buying an insurance policy. If someone buys an insurance policy and financial loss occurs, then the insurance company will cover it completely (full coverage). Insurance contracts are homogeneous products, thus customers are indifferent between buying policies from any of the insurers.

[3] In insurance models, the price of insurance is called premium, so we use this terminology Banyár and Vékás (2016).

Customers' intentions to buy are different, they are represented by a demand function $D(P)$, which shows how many people buy insurances at premium P. The most simple –and most usual– case, if we assume linear demand function (see for instance Mas-Colell et al. (1995), in insurance context Kliger and Levikson (1998)). Unfortunately, in our case the linear demand function would lead to a quartic expression, which would result very cumbersome formulas. We specify the demand curve as $D(P) = \frac{\alpha^2}{P^2}$ (, if $qK \leq P$ and $\alpha > 0$). This function belongs to the family of iso-elastic demand curves which is also frequent both in theory (Tramontana et al. (2010)) and practice (Huang et al. (2013)) and also in insurance models (Hao et al. (2018)).

According to Mossin at net premium every consumers buy an insurance (Mossin, 1968), thus at net premium (qK) the demand reaches the maximum size, $N_{max} = \frac{\alpha^2}{q^2K^2}$. The inverse demand function is $D^{-1}(n) = \frac{\alpha}{\sqrt{n}}$ the number of policies sold n is less than N_{max}.

Insurance companies cannot choose between customers, at premium level P_i they have to serve all potential customers [4].

Denote $n_i(P_i, \boldsymbol{P_{-i}})$ the number of customers buying from insurer i at premium P_i. The premium vector $\boldsymbol{P_{-i}}$ contains the premiums of all insurers except that of company i. Customers buy insurance coverage from the cheapest available company. If it is not unique, i.e. more companies offer the same level of premium, each company gets equal share of the customers. $P_{\min}$ denotes the smallest premium and $\mathcal{M}$ is the minimum set $P_i = P_{\min}$, and $|\mathcal{M}|$ stands for the cardinality of set $\mathcal{M}$, showing the number of such insurers. Thus, the number of contracts sold by company i is as follows:

$$n_i(P_i, \boldsymbol{P_{-i}}) = \begin{cases} \frac{1}{|\mathcal{M}|} D(P_i) & \text{if } i \in \mathcal{M} \\ 0 & \text{if } i \notin \mathcal{M} \end{cases}$$

[4] In several member states insurance companies are obliged to serve their customers at the quoted premium (92/49/EEC , 1992), this is important to avoid discrimination for instance in the case of health insurance and compulsory motor third party liability insurance. There is a time period of the year, when people can choose or change their insurance ('open enrollment') and the insurance company must insure them. Thus, we assume in the model, that companies cannot reject customers, they must serve all of them at the given premium. See also in Polborn (1998).

3.2.1 The capital requirement

Because insurance is a risky business not everybody is eligible to be part of it. Every insurance company faces a solvency capital requirement, in other words, they have to have enough capital to cover unexpected losses. The main concept of Solvency II framework is that any insurance company can go bankrupt no more than once in every 200 cases. From a mathematical viewpoint the solvency capital requirement is a one-year 99.5% Value at Risk (VaR) constraint.

VaR_β is a risk measure, which shows the maximum level of loss over a given time period, at a given confidence level. If X is a continuous random variable with distribution function $F(.)$, then $\mathrm{VaR}_\beta(X) = \inf\{x|F(x) \geq \beta\}$ at confidence level β i.e. insurance company i fulfills the solvency capital requirement if its capital C plus premium incomes (P_i per policy) cover losses with probability 0.995:

$$\mathrm{Prob}\left(\sum_{j=1}^{n_i} K\kappa_j > C + n_i P_i\right) < 0.005 \ , \tag{16}$$

where n_i is an integer, and stands for the integer number of policies the insurance company i has. If n_i is sufficiently large, then the distribution of the sum $\sum_{j=1}^{n_i} K\kappa_j$ can be approximated by a normal distribution with mean $n_i qK$ and standard deviation $\sqrt{n_i q(1-q)}K$. Using the normal distribution approximation, equation (16) can be reformulated in a way that for given P_i and n_i the minimum capital requirement (MCR) is:

$$\mathrm{MCR}(n_i(P_i, \boldsymbol{P_{-i}}), P_i) = n_i(qK - P_i) + \sqrt{n_i}\phi\sqrt{q(1-q)}K \ , \tag{17}$$

where $\phi = \Phi^{-1}(0.995)$ and $\Phi(x)$ is the cumulative distribution function of the standard normal distribution.

Remark 1. *It is easy to see that serving completely twice as many customers requires less than twice as much capital at the same premium level. In general, the increase of the number of customers to* $(1+a)n$ *raises the solvency capital requirement to less then* $(1+a)MCR(n_i(P_i, \boldsymbol{P_{-i}}), P_i)$ *(,where a>0)i.e. the minimum capital requirement has a decreasing return to scale in* n*.*

The companies' payoff is their expected profit. Let us suppose that company i has capital C, which is a given parameter, not a decision variable. Holding capital has some costs, because the insurance company loses the interest rC on this capital. This is a fixed cost, it must be paid even if the insurer does not sell any contracts. Without this capital, the insurer would not be allowed to enter the market. The expected profit of company i is $n_i(P_i, \boldsymbol{P_{-i}})(P_i - qK) - rC$, if it fulfills the solvency capital requirement (i.e. $\mathrm{MCR}(n_i(P_i, \boldsymbol{P_{-i}}), P_i) \leq C$). If it is not satisfied, the company faces a penalty A, which is partly due to a financial penalty levied by the insurance supervisor, and to the decreased reputation of the firm.[5] The penalty is so great that selling no policies is preferred to selling many (profitable) with penalty. The expected profit of company i can be written as:

$$\pi_i(P_i, \boldsymbol{P_{-i}}) = \begin{cases} n_i(P_i, \boldsymbol{P_{-i}})(P_i - qK) - rC \ , & \text{if} \quad \mathrm{MCR}(n_i(P_i, \boldsymbol{P_{-i}}), P_i) \leq C \\ n_i(P_i, \boldsymbol{P_{-i}})(P_i - qK) - rC - A \ , & \text{if} \quad \mathrm{MCR}(n_i(P_i, \boldsymbol{P_{-i}}), P_i) > C \end{cases}$$

The companies' payoff is the expected profit, which we briefly refer to as profit from now on. Referring to Remark 1 we can state, that twice as large company's profit is more than twice as large at the same premium level. Part of the profit is the technical result which is the profit without the interest loss and financial penalty: $\mathrm{TR}_i = n_i(P_i, \boldsymbol{P_{-i}})(P_i - qK)$. We show that insurance companies' short term behaviour depends on the technical result.

Equation (16) can be rearranged in a way that it shows for given n and C the minimum premium levels needed to fulfill the solvency capital requirement. It is the minimum premium requirement

$$\mathrm{MPR}(n_i, C) = qK - \frac{C}{n_i} + \frac{\phi\sqrt{q(1-q)}K}{\sqrt{n_i}} \ . \tag{18}$$

The first part of equation(18) can be interpreted as the net premium, this is the

[5] If the insurer fails to meet the solvency capital requirement or is at risk of doing so, the insurer shall immediately notify the supervisor authority and submit a realistic recovery plan within two months. The supervisor authority shall require the insurer to make up the level of the solvency capital requirement or to reduce its risk profile to ensure compliance with the solvency capital requirement. If according to the supervisor authority the financial situation of the obligation concerned will deteriorate further, it can also restrict or prohibit the free disposal of the assets of that liability (DIRECTIVE 2009/138/EC, 2009).

reference point. The second part is an increasing term in n_i, meaning that the more policies are taken by the company, the less portion of the capital can be allocated to a single policy and therefore the insurance company becomes riskier (which can be covered by an increased premium level)[6]. The third part is a decreasing term meaning that even if the variance is increasing for the whole portfolio but not in a linear way, less risk margin is needed for a greater portfolio. [7]

The greatest premium (for a given capital) can be calculated, if we maximize $\mathrm{MPR}(n_i, C)$ in n_i. The maximum is reached at $\hat{n}_i = \frac{4C^2}{\phi^2 q(1-q)K^2}$, while the maximal premium is:

$$\mathrm{MPR}_{\max} = qK + \frac{\phi^2 q(1-q)K^2}{4C}. \tag{19}$$

It is easy to see that for smaller values than $\frac{4C^2}{\phi^2 q(1-q)K^2}$ the MPR curve is increasing in n_i, and for higher values of n_i it is decreasing. It is also obvious from expression (18) that for a fixed capital the MPR curve tends to qK as n_i tends to infinity.

Proposition 1. *Let's assume that the premiums of insurance companies in the market are as high that company i can sell n_i quantities in all cases if it chooses premium level $P_i = \mathrm{MPR}(n_i, C)$. Assuming a given capital level, $n_i(\mathrm{MPR}(n_i, C) - qK)$ increases in n_i, so the expected profit increases along the* MPR *function.*

Proof. Using expression (18) for $\mathrm{MPR}(n_i, C)$, the profit can be written:

$$n_i(P_i - qK) - rC = \Bigg(\mathrm{MPR}(n_i, C) - qK\Bigg) n_i - rC =$$

$$\left(qK - \frac{C}{n_i} + \frac{\phi\sqrt{q(1-q)}K}{\sqrt{n_i}} - qK\right) n_i - rC = -(1+r)C + \sqrt{n_i}\phi\sqrt{q(1-q)}K$$

The expected profit is increasing in n. □

[6]If the company has big enough capital (and small number of policies) even negative premiums could be determined, but it is only a technical mathematical assumption.

[7]Formula (18) can determine a higher level of MPR than K under certain parameters. This level of the MPR is meaningless, because it is higher than the maximal level of claim, at this premium, consumers do not buy insurance This is due to the inaccuracy of the approximation with the normal distribution, for meaningful parameters we get lower levels of MPR than K, see Appendix A.

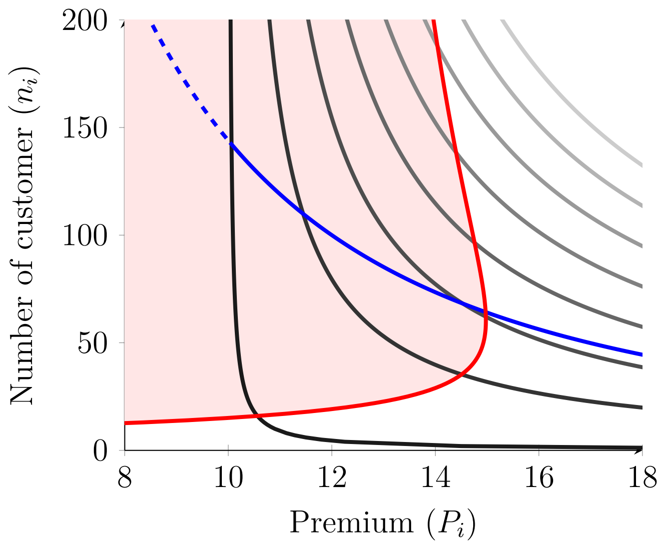


Figure 9: Illustration of the demand function, the isoprofit curves and the MPR capital requirement constraint in the (P_i, n_i) plane. $q = 0.1, K = 100, C = 300, r = 3\%, \alpha = 120$

We put the isoprofit, demand and MPR curves into a common plot as can be seen in Figure 9. The plot is more readable if the horizontal axis is the premium and the vertical axis is the number of policies. The MPR curve does not have an inverse function. We refer to it as MPR curve (red line). The shaded pink area is not available for companies, these pairs of premium and number of policies do not fulfill the solvency requirements. So companies can compete in the area right to the red curve. The gray curves are isoprofit curves, the lighter the curve the greater the expected profit.

Proposition 2. *The inverse demand function and the MPR curve (for a fixed capital) has exactly one intersection point on interval* $(0, \infty)$ *at*

$$\tilde{n_i} = \frac{\left(-(\phi\sqrt{q(1-q)}K - \alpha) + \sqrt{(\phi\sqrt{q(1-q)}K - \alpha)^2 + 4qKC}\right)^2}{4q^2K^2},$$

and the premium at the point of intersection is

$$P_U = \frac{2\alpha qK}{-(\phi\sqrt{q(1-q)}K - \alpha) + \sqrt{(\phi\sqrt{q(1-q)}K - \alpha)^2 + 4qKC}}. \tag{20}$$

This is the lowest premium level such that an insurer can cover the whole market alone without penalty.

Proof. See in Appendix B . □

It is not profitable running the insurance company at a premium lower than the net premium level.

Remark 2. *The premium at the intersection point is higher than qK if*

$$\frac{\alpha}{C}\phi\sqrt{\frac{1}{q}-1} > 1 \ .$$

If the premium at the intersection point is less than qK the solvency capital requirement does not have an important role in the market, companies compete as if there were no capital requirements.

Remark 3. *The point of intersection is left to the maximum of the MPR curve (or in other words it is on the increasing part of the MPR curve) if the following condition holds:*

$$1 > \frac{\phi^2(1-q)\left(-(\phi\sqrt{q(1-q)}K-\alpha)+\sqrt{(\phi\sqrt{q(1-q)}K-\alpha)^2+4qK\overline{C}}\right)^2}{16qC^2} \ . \tag{21}$$

The fact that the point of intersection is on the increasing or the decreasing part of the MPR curve plays an essential role in determining the type of the equilibrium.

3.3 The equilibrium

For a benchmark we consider the case without solvency capital requirement. In this case in a Bertrand oligopoly game every company sets premium qK and the technical result is zero, the negative expected profit is the fixed cost.

In the following, we assume a capital requirement in the model and examine the equilibrium premium and profit, as well as the effect of different parameters on it. An insurance company is selling contracts until the technical income, $n_i(P_i, \boldsymbol{P_{-i}})(P_i - qK)$ is non-negative. The equilibrium on the market is influenced by whether the intersection of the demand and the capital requirement is located in the increasing or decreasing part of the curve, so we examine these two cases separately.

3.3.1 Equilibrium on the increasing part of the MPR curve

First we consider the case when the increasing part of the MPR curve is relevant, the point of intersection is on the increasing part of the MPR curve. Expression (21) gives a condition for it. The point of intersection of the demand function and the MPR curve is denoted with P_U and defined by (20). The Ith part of the demand is $D_I(P) = \frac{D(P)}{I} = \frac{\alpha^2}{IP^2} = \frac{\alpha'^2}{P^2}$, its inverse is $D_I^{-1}(n) = \frac{\alpha}{\sqrt{I}\sqrt{n}}$. Thus, it can be considered as an inverse demand function with a modified parameter $\alpha' = \frac{\alpha}{\sqrt{I}}$. Therefore the premium P_L at the point of intersection of functions $D_I(n)$ and the MPR is the lowest premium such that an insurer who serves 1/I of the market fulfils the capital requirement.

$$P_L = \frac{2\frac{\alpha}{\sqrt{I}}qK}{-(\phi\sqrt{q(1-q)}K - \frac{\alpha}{\sqrt{I}}) + \sqrt{(\phi\sqrt{q(1-q)}K - \frac{\alpha}{\sqrt{I}})^2 + 4qKC}} \tag{22}$$

Proposition 3. *If $P_U > qK$, then there exists a continuum of symmetric Nash equilibria in the interval* $[\max(qK, P_L), P_U]$.

Proof. Since the inverse demand function intersects the MPR curve in its increasing part we can state that $P_L < P_U$.

Let us suppose that all companies set premium level $P_E \in [\max(qK, P_L), P_U]$. None of the companies intends to deviate from the equilibrium level. If $P_E \in [\max(qK, P_L), P_U]$, the technical result is nonnegative. Setting a higher premium results in no customers and so the technical results are 0. However, setting a lower premium means that the insurance company should serve the whole market alone, but in the premium interval covering the whole market, does not satisfy the solvency capital requirement. Thus the company should pay a penalty, meaning that the profit is$n_i(P_i, \boldsymbol{P_{-i}})(P_i - qK) - rC - A$, which is worse than selling no contract at all according to our initial assumption.

Premiums higher than P_U cannot be equilibria, because in this case decreasing the premium and covering the whole market would lead to a higher level of expected profits.

Premiums lower than qK cannot be Nash equilibria, since in this case the technical result is negative, setting higher premium would mean zero technical result.

Similarly, lower premium than P_L cannot be a Nash equilibrium point, since all of the companies face a penalty, and by setting a higher premium this penalty could be avoided. □

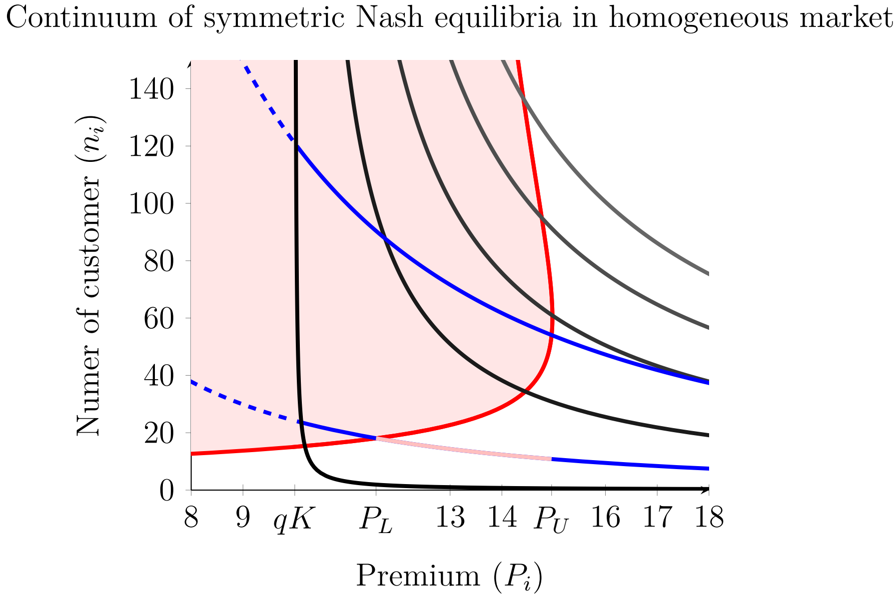


Figure 10: $q = 0.1, K = 100, C = 300, r = 1\%, \alpha = 110, I = 5$

An illustration of a continuum of Nash equilibria can be found in Figure 10. The red line is the solvency capital requirement, the blue lines are the demand functions and the Ith part of them. The gray lines are the isoprofit curves. P_L is the intersection point of the MPR curve and the Ith part of the demand function, and P_U is the intersection of the MPR and the demand function. Every premium between P_L and P_U can form an equilibrium. In these cases the market is somewhere on the pink line. The I companies share the market equally, and they satisfy the solvency capital requirement.

In this concrete case the lowest possible equilibrium premium is P_L is higher than the net premium. This is the lowest equilibrium premium most customers buy an insurance at. The highest possible equilibrium premium is P_U. At that premium even one company could serve the whole market without the possibility of paying a penalty.

An important question is the profit of the insurance companies. We see that the technical result is nonnegative in every Nash equilibrium point, but the companies incur loss of interest as well. In Figure 11 the solid black curves are the zero-profit curves. Left to it the profit is negative, right to it is positive. It is easy to see that if the interest rate is high enough, then all Nash equilibria give negative expected profits, see the c) part in Figure 11. On the other hand if $P_L > qK$ and interest rate is small enough, then all Nash equilibria give positive profit (extraprofit), see part a) in Figure 11. It can happen that some Nash equilibria give positive profit while others give negative profit, see part b) in Figure 11.

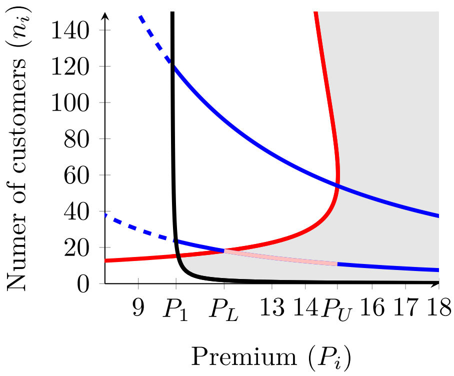
In short, an equilibrium may exist where the companies have negative profits, because by selling some contracts they can reduce the loss of capital. But in the long run, the negative profit means that the sector is not profitable. On the other hand, the possibility of extra profit draws further companies to the market.

The equilibrium premium interval is influenced by the parameter values included in the model. It is worth examining how these variables affect equilibrium premiums. Lower level of confidence leads to lower equilibrium premiums. Expression (18) shows that a decrease in the value of ϕ moves the function downward which means that the MPR curve moves to the left in Figure 10 which results in lower P_U and P_L values. So there is a trade-off between safety and equilibrium premium.

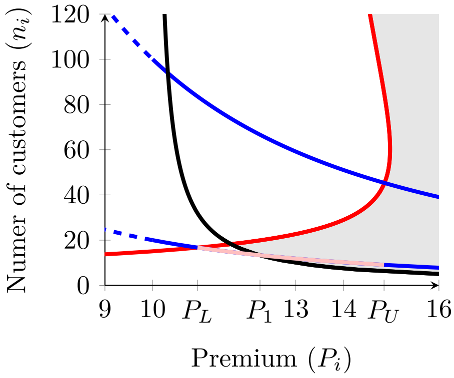
Further important question is the effect of the number of the companies (the level of the competition). For a higher number of companies (I) P_U remains unchanged while P_L decreases. P_U remains unchanged, since this is the (lowest) premium at which an insurance company can insure the whole market. This can be seen in expression (20) which does not contain variable I.

Decrease of P_L is quite trivial. Increasing the number of insurance companies leaves the MPR curve unchanged, while the Ith part of the inverse demand curve moves downward. Since we are in the increasing part of the MPR curve, the point of intersection has to move to the left, which means lower premium. Crucial in the argument is that we are in the increasing part of the MPR curve, in the decreasing part exactly the opposite is true.

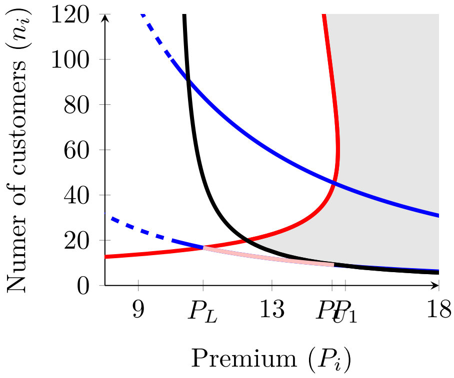
So the set of equilibrium premiums entails lower premiums as the number of insurance companies increases, which agrees with economic intuition. However, as the number of companies increases, the aggregate level of solvency capital is increasing. One can ask whether the increased number of companies alone has a reduction effect. To put it differently, what is the effect of increasing the number of



(a) $q = 0.1, K = 100, C = 300, r = 1\%, \alpha = 110, I = 5$



(b) $q = 0.1, K = 100, C = 300, r = 10\%, \alpha = 100, I = 5$



(c) $q = 0.1, K = 100, C = 300, r = 15\%, \alpha = 100, I = 5$

Figure 11: Illustration of the cases of positive expected profit.

companies if the aggregate level of the solvency capital is fixed?

Proposition 4. *Assuming fixed total capital level in the market (each firm has capital level $\frac{C}{I}$) by increasing the number of companies both P_U and P_L increase.*

Proof. For a fixed level of aggregate capital, definition of P_U changes to:

$$P_U = \frac{2\alpha qK}{-(\phi\sqrt{q(1-q)}K - \alpha) + \sqrt{(\phi\sqrt{q(1-q)}K - \alpha)^2 + 4qK\frac{C}{I}}} \ ,$$

which is an increasing function in I.

Similarly, the definition of P_L changes to:

$$P_L = \frac{2\frac{\alpha}{\sqrt{I}}qK}{-(\phi\sqrt{q(1-q)}K - \frac{\alpha}{\sqrt{I}}) + \sqrt{(\phi\sqrt{q(1-q)}K - \frac{\alpha}{\sqrt{I}})^2 + 4qK\frac{C}{I}}} \ .$$

With some algebra we get that $P_L(I)$ is also an increasing function of I, see Appendix C.

In this situation, both the MPR curve and the Ith part of inverse demand function change. This statement is true generally (i.e. in the decreasing part of the MPR curve as well). □

The result of Proposition 4 is quite interesting: the more concentrated the market is the lower is the equilibrium premium. This result is quite unusual in economic models. It suggests that a ban of mergers of big companies would be disadvantageous for customers. Going further with this idea: is a monopoly market better for customers than an oligopoly market?

In case of monopoly $P_L = P_U = P_s$. Since a monopolist does not face competition can set higher premium than P_s. Let P_M be the premium which ensures the highest technical result for the insurance company (without solvency capital requirement). By doing a little algebra one can derive that premium $P_M = 2qK$ will give the highest technical result. So the monopoly would like to set premium P_M, but it is not possible if $P_M < P_s$. The monopoly sets premium $\max\{P_s; P_M\}$ in equilibrium with solvency capital requirement. If $qK < P_L$ and $P_M < P_L$, then a monopoly determines a premium lower than any insurance companies in an oligopoly market. Thus, a monopoly market can lead to a lower premium than an oligopoly market, which is advantageous for customers.

3.3.2 Equilibrium on the decreasing part of the MPR curve

In this section we investigate the case, when the point of intersection is on the decreasing part of the MPR curve. We will see, that in this case different equilibria can appear.

The fact that the point of intersection of the MPR curve and the inverse demand curve is on the increasing part of the MPR curve does not necessarily mean that the point of intersection of the MPR curve and the Ith part of the inverse demand curve is on the decreasing part of the MPR curve either. On the contrary, if we look at expression (22), value P_L tends to zero as the number of companies (I) tends to infinity. Therefore we have to be in the increasing part of the MPR curve because we know that the limit of the MPR curve at infinity is qK. For large enough I there is a continuum of Nash equilibrium points, since $P_L < P_U$.

Proposition 5. *Consider an oligopoly market with I companies and suppose that in this market $P_L > P_U$. If $P_M \leq P_U$ then there is only one type of Nash equilibrium point: one arbitrary company sets premium P_U, the others set higher premium.*

Proof. Let us suppose that companies set premiums $P_1, \ldots P_I$, the minimum of its is $P_{\min}$. There are two cases:

a) $P_{\min} > P_U$, in this case it is advantageous for any company to set premium a bit lower than $P_{\min}$. At this premium the company gets the whole market ensuring higher profit than before.

b) $P_{\min} = P_U$ and more than one company sets this premium. Than it causes loss for them since they cannot fulfill the solvency capital requirement. If only one company sets premium P_U and $P_M \leq P_U$, then this is a Nash equilibrium. If $P_M > P_U$ than this cannot be a Nash equilibrium, since setting a premium a little bit higher than the minimum would increase the company's profit. □

If the condition $P_M \leq P_U$ is not fulfilled, there is no pure equilibrium on the set of continuous strategies.

We know from Proposition 4 that for a fixed level of aggregate capital the increased number of insurance companies increases both P_U and P_L values. So again, low equilibrium premium can be achievable if few big companies are in the market and the merger of companies (even big companies) is advantageous for customers in many occasions.

We can also state that monopoly can set a lower premium than competitive companies in an oligopoly situation. The condition changes a bit: $P_M < \min\{P_L; P_U\}$.

3.4 Conclusion

The specific objective of this paper was to study the effect of the Solvency II directive's capital requirement in the market equilibrium in a Bertrand oligopoly market. In the traditional Bertrand game the profit of the symmetric companies is zero, even if there are only two companies in the market. However, when modeling the insurance sector, higher premiums and profits may arise due to the uncertainty of the service. This research has shown that introducing the solvency capital requirement constraint also can lead to different market anomalies.

As a result of the capital constraint, two types of equilibria may arise during the simultaneous pricing of insurers with the same capital. A continuum of many symmetric equilibrium premiums or only one insurer in the market. In both cases, companies may achieve a positive technical result, if the premium level is higher than the net premium. If the interest rate is low enough, this is higher than the fixed costs, so the expected profit can also be positive.

Lowering the confidence level and increasing the capital level of insurers makes lower equilibrium premiums available. While the increase in the number of companies, with fixed level of the total capital on the market, causes higher level of premiums. In such cases, the level of capital per company decreases as the number of companies increases, so consumers may prefer a market with less bigger (higher capital) companies. In extreme cases, this can also be achieved in a monopoly market. The obtained results depend strongly on the assumptions of the model, according to which we assumed insurers making simultaneous price decisions in an oligopoly market and maximizing their expected profit. Another interesting research question could be the examination of insurers with different capital levels, and the endogenization of the capital decision, as this also greatly influences , how much cost a company faces and how low premiums it can charge.

References

Ágoston, K. Cs. and Varga, V. (2020). Bertrand-árverseny állománypreferenciák mellett a biztosítási piacokon (Bertrand price competition with substance preferences in insurance markets). *Szigma, 51*(2):149-167.

Avis, D., Rosenberg, G.D., Savani, R. and von Stengel, B. (2010). Enumeration of Nash equilibria for two-player games. *Economic Theory, 42*, 9-37.

Banyár, J. and Regős, G. (2012). Paradoxical price effects on insurance markets. *Economic Modelling* 29: 1399–1407.

Banyár, J. and Vékás, P. (2016). A pénzügyi termékek ára. The premium of the insurance products. *Közgazdasági Szemle* 63: 380–406.

Bi, J. and Cai, J. (2019). Optimal investment–reinsurance strategies with state dependent risk aversion and VaR constraints in correlated markets. *Insurance: Mathematics and Economics 85*(C), 1-14.

Camino-Mogro, S., Armijos-Bravo, G. and Cornejo-Marcos, G. (2019). Competition in the insurance industry in Ecuador: An econometric analysis in life and non-life markets. *The Quarterly Review of Economics and Finance, 71:* 291–302.

Coccorese, P. (2010). Information Exchange as a Means of Collusion: The Case of the Italian Car Insurance Market. *Journal of Industry Competition and Trade, 10:* 55–70.

Council Directive 92/49/EEC of 18 June 1992 on the coordination of laws, regulations and administrative provisions relating to direct insurance other than life assurance and amending Directives 73 /239/EEC and 88/357/EEC (third non-life insurance Directive).

Dixit, A.K. and Nalebuff, B.J. (2008). The Art of Strategy. W. W. Norton & Company New York London, 2008.

Doff, R. (2016). The Final Solvency II Framework: Will It Be Effective? *The Geneva Papers, 2016, 41*(4), 587-607.

Dutang, C., Albrecher, H., Loisel, S. (2013). Competition between non-life insurers under solvency constraints: a game-theoretic approach. *European Journal of Operational Research, 231*(3), 702-711.

Escobar, M., Kriebel, P., Wahl, M. and Zagst, R. (2019). Portfolio optimization under Solvency II. *Annals of Operations Research 281*(1), 193-227.

Hao, M., Macdonald, A.S., Tapadar, P. and Thomas, R.G. (2018). Insurance loss coverage and demand elasticities. *Insurance: Mathematics and Economics, 79*(1), 15-25.

Huang, J., Leng, M. and Parlar M. (2013). Demand functions in decision modeling: A comprehensive survey and research directions. *Decision Sciences, 44*(3), 557-609.

Kliger, D. and Levikson, B. (1998). Pricing insurance contracts - an economic viewpoint. *Insurance: Mathematics and Economics 22*(3), 243-249.

Kouwenberg, R. (2018). Strategic asset allocation for insurers under Solvency II. *Journal of Asset Management 19*(7), 447-459.

Mas-Colell, A., Whinston, M. and Green, J. (1995). Microeconomic Theory, Oxford University Press.
Insurance loss coverage and demand elasticities. *Insurance: Mathematics and Economics, 79*, 15-25.

Mossin, J. (1968). Aspects of Rational Insurance Purchasing. *Journal of Political Economy, 76*(4), 553-568.

Mouminoux, C., Dutang, C., Loisel, S. and Albrecher, H. (2022): On a Markovian Game Model for Competitive Insurance Pricing. *Methodology and Computing in Applied Probability, 24*, 1061-1091.

Panzar, J. C, and Rosse, J. N. (1987): Testing for 'monopoly' equilibrium. J Ind Econ 35(4):443–456 *The Journal of Industrial Economics, 35(4):* 443–456.

Polborn, M. K. (1998). A Model of an Oligopoly in an Insurance Market. *The Geneva Papers on Risk and Insurance Theory, 23*(1), 41-48.

Schlesinger, H. and Graf von der Schulenburg, J.M. (1991). Search costs, switching costs and product heterogeneity in an insurance market. *The Journal of Risk and Insurance 58*, 109-119.

Solvency II (2009). Directive 2009/138/EC of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II). *Official Journal of the European Union*, L 335/1.

Sonnenholzner, M. and Wambach, A. (2004). Oligopoly in Insurance Markets. In: Teugels, J. and Sundt, B. (eds). Encyclopedia of Actuarial Science. Wiley, Chichester, UK (2004).

Tramontana, F., Gardini, L. and Puu, T. (2010). New properties of the Cournot duopoly with isoelastic demand and constant unit costs. WP-EMS Working Papers Series in Economics, Mathematics and Statistics. 2010/06.

Varga, V. and Madari, Z. (2023). The Hungarian insurance market structure: an empirical analysis. *Central European Journal of Opeartions Research 31 : 3.* 927–940.

Wambach, A. (1999). Bertrand competition under cost uncertainty. *International Journal of Industrial Organization, 17*(7), 941-951.

Zhang, N., Jin, Z., Li, S. and Chen, P. (2016). Optimal reinsurance under dynamic VaR constraint. *Insurance: Mathematics and Economics 71*(C), 232-243.

Appendix A

A higher level of MPR than K is meaningless. This happens if

$$qK - \frac{C}{n_i} + \frac{\phi\sqrt{q(1-q)}K}{\sqrt{n_i}} > K \ .$$

The condition is satisfied if:

$$Kn_i(q-1) + \phi\sqrt{n_i}\sqrt{q(1-q)} - C) > 0 \ ,$$

which necessarily will not be true if n_i is large enough.

Appendix B

The proof of Proposition 2. The intersection of the inverse demand function and the MPR curve:

$$\frac{\alpha}{\sqrt{n_i}} = qK - \frac{C}{n_i} + \frac{\phi\sqrt{q(1-q)}K}{\sqrt{n_i}}$$

After rearranging the term, we get:

$$0 = qKn_i + \sqrt{n_i}(\phi\sqrt{q(1-q)}K - \alpha) - C \ ,$$

which is a quadratic expression in $\sqrt{n_i}$. Using the quadratic formula for roots:

$$(\sqrt{n_i})_{1,2} = \frac{-(\phi\sqrt{q(1-q)}K - \alpha) \pm \sqrt{(\phi\sqrt{q(1-q)}K - \alpha)^2 + 4qKC}}{2qK}$$

The discriminant is always positive, so there are two roots. However, from expression

$$(\sqrt{n_i})_{1,2} = \frac{-(\phi\sqrt{q(1-q)}K - \alpha) - \sqrt{(\phi\sqrt{q(1-q)}K - \alpha)^2 + 4qKC}}{2qK}$$

we get a negative value for $\sqrt{n_i}$, which is out of context. So the intersection point of the two function is at

$$n_i = \frac{\left(-(\phi\sqrt{q(1-q)}K-\alpha)+\sqrt{(\phi\sqrt{q(1-q)}K-\alpha)^2+4qKC}\right)^2}{4q^2K^2}$$

The premium at the intersection point is

$$P = \frac{2\alpha qK}{-(\phi\sqrt{q(1-q)}K-\alpha)+\sqrt{(\phi\sqrt{q(1-q)}K-\alpha)^2+4qKC}}$$

Appendix C

Complete proof of Proposition 4:

Multiplying both the numerator and denominator of (22) by $\sqrt{I}$ we get:

$$P_L = \frac{2\alpha qK}{-(\sqrt{I}\phi\sqrt{q(1-q)}K-\alpha)+\sqrt{(\sqrt{I}\phi\sqrt{q(1-q)}K-\alpha)^2+4qKC}}\ , \tag{23}$$

The numerator of (23) does not depend on I, it is enough to consider the denominator. Denote it by $D(I)$. The derivative of $D(I)$ is

$$\frac{dD(I)}{dI} = -0.5\frac{\phi\sqrt{q(1-q)}K}{\sqrt{I}} + 0.5\frac{2(\sqrt{I}\phi\sqrt{q(1-q)}K-\alpha)}{\sqrt{(\sqrt{I}\phi\sqrt{q(1-q)}K-\alpha)^2+4qKC}}0.5\frac{\phi\sqrt{q(1-q)}K}{\sqrt{I}} = 0.5\frac{\phi\sqrt{q(1-q)}K}{\sqrt{I}}\left(\frac{(\sqrt{I}\phi\sqrt{q(1-q)}K-\alpha)}{\sqrt{(\sqrt{I}\phi\sqrt{q(1-q)}K-\alpha)^2+4qKC}}-1\right)\ .$$

It is easy to see that

$$\frac{(\sqrt{I}\phi\sqrt{q(1-q)}K-\alpha)}{\sqrt{(\sqrt{I}\phi\sqrt{q(1-q)}K-\alpha)^2+4qKC}} < 1\ ,$$

implying that $\frac{dD(I)}{dI} < 0$, so $D(I)$ is a decreasing function of I. If $D(I)$ is a decreasing function then P_L is an increasing function of I; as the number of insurance companies increases, so does P_L.

4 Study III: The Hungarian insurance market structure: an empirical analysis

Authors: Veronika Varga and Zoltán Madari

Published in Central European Journal of Operations Research 31(3):927-940. (2023)

https://doi.org/10.1007/s10100-023-00842-8

Abstract

This paper analyzes the market structure of the Hungarian insurance market, which operated as a monopoly market until 1986. After the regime change this sector started to develop rapidly. But the Hungarian insurance market has a strong oligopolistic character, and thus raises an interesting question as to how close the market is to a state of perfect competition. Based on the Panzar and Rosse (1987) methodology we estimate the elasticity of total revenues with respect to changes in input prices, so that we can determine the market structure. The estimation of input price elasticity is made with a static and a dynamic panel model. According to research the structure of the Hungarian insurance market in most cases significantly differs from the perfect competition case between 2010 and 2019. The market is in long-run equilibrium, and the hypothesis of the monopolistic competition case cannot be rejected. The market structure of a sector is important for modelling phenomena and new regulations effectively, which is relevant for insurance and competition supervision in the protection of customers.

Keywords: Hungarian insurance market, Market structure, Panzar–Rosse model, Dynamic panel model

4.1 Introduction

Modelling a sector plays a crucial role in the preparation of new regulations and supervisor decisions. Knowledge of the market structure has a critical role in the maintenance of modelling Systematically Important Financial Institutions (SIFIs).

The insurance market is a large and risky sector with many clients all over the world. When market competition rises, the situation of consumers improves as well. On the other hand, in case of a monopoly, customers are completely vulnerable. Competition supervision also seeks to curb excessive market power of each firm. Insurance is a trust transaction in which complex financial assets are sold, thus the role of customer protection and supervision are of utmost importance. Consequently, an important factor is to determine how strong competition in the insurance market is. This paper addresses the question concerning the market structure of the Hungarian insurance sector. In studying the companies of the Hungarian insurance market, the answer is not clear, so it is worth examining the problem more thoroughly. The objective of the research is to determine whether the monopoly or the perfect competition case fits better for the balance sheet data between 2010 and 2019.

The history of Hungarian insurance dates back a long time. In the 1800s, many domestic and foreign insurance companies operated in the country. However, most of them were destroyed because of World War II. Under socialism in Hungary, as in several Eastern European countries (Tipuric et al., 2008), insurance operated as a state monopoly from 1952. In 1986, the only insurer company split into the new State Insurer and the Hungária Insurer, and it was allowed to establish new companies. The market started to develop rapidly, foreign companies appeared in the market, and in parallel the supervisor also evolved. For these reasons, after the change of regime the market underwent significant transformation.

The structure of the current Hungarian insurance market cannot be clearly defined. According to the Association of Hungarian Insurers in 2019 not less than 31 insurance companies were present in the market (MABISZ, 2019). Breaking out of the monopoly position, the market has now undergone a major transformation towards perfect competition. On the other hand, in terms of premium income, the top 10 companies cover more than 80% of the market. The position of the market-leading insurers is stable, and their ranking has changed only slightly in recent years, which does not confirm the hypothesis of perfect competition.

Previous researchers have established several methods which can be used for empirical analyses of the market structure, such as the Panzar and Rosse model

(Panzar and Rosse, 1987) or the Iwata model (Iwata, 1974). The Panzar and Rosse method uses the sum of the factor price elasticities of the reduced form revenue equation to create testable hypotheses about the market structure. Studies over the past decades have provided important information on market structures mainly in the bank industry based on this method. Results from studies of small, medium-sized and large banks around the world show monopolistic competition and stronger competition on the international market than at the local level (Bikker and Haaf, 2002). Monopolistic competition is noted in Canada's (Nathan and Neave, 1989) and Italy's (Coccorese, 1998) financial markets. The Panzar—Rosse model can be used also in the insurance sector, see Kasman and Turgutlu (2008), Coccorese (2010), Murat et al. (2002), Jeng (2015), Uddin et al. (2018), Alhassan and Biekpe (2017), Camino-Mogro et al. (2019), Todorov (2016).

The efficiency and concentration of the insurance market is often the focus of the research. Bikker and Leuvensteijn (2008) analyze the competition and efficiency in the Dutch life insurance market via different indicators. Knezevic et al. (2015) makes a data envelopment analysis about the efficiency of the Serbian insurance market, which shows that the market is not as developed as in neighbouring countries. Some articles examining European countries also contain results about the Hungarian market (Tipuric et al., 2008; Kramaric and Kitic, 2012; Kozmenko et al., 2009). The Hungarian insurance market changed markedly after the regime change, with concentration ratios decreasing between 1998 and 2006 (Tipuric et al., 2008). Research in the sphere of the new European Union countries, including Hungary, shows that key insurance indicators are below EU averages, while concentration ratios decreased between 2000 and 2010.

The main research question in articles dealing specifically with the Hungarian insurance market does not usually concern the market structure. Szüle (2017) compares the relationship of taxation and solvency between the bank and insurance industry. The two Hungarian sectors are quite similar, but the two markets are not homogenous. Banyár and Turi (2019) give an overview of consumer protection rules in the country. In a study by Kovács (2011) the main indicators of market power are described in the insurance market by using the Herfindahl-Hirschman Index or the Markov chain model. As the Hungarian insurance market structure is indeed rarely studied empirically, this article seeks to fill in this gap by using the Panzar—Rosse method in the case of the Hungarian insurance market. According to Goddard and Wilson (2009) the factor price elasticity should be estimated with a dynamic panel model, because the static model can cause biased and inefficient coefficients. We used a static and a dynamic panel approach and two different dependent variable to assess the robustness of the results.

The analyses shows that the structure of the Hungarian insurance market in most cases differs significantly from the perfect competition case between 2010 and 2019. But the hypothesis of the monopolistic competition case cannot be rejected. During this time period the insurance market was in long-run equilibrium according to the Panzar—Rosse methodology. The monopoly market means in this case that the insurer companies' decisions do not depend on other companies, which suggests high market power.

The rest of the paper is organized as follows. Section 4.2 presents the methodology of the Panzar—Rosse model and the dynamic panel data approach. In Section 4.3 we describe the dataset on the Hungarian insurance market that we used for the analysis, and some further indicators concerning the market structure of the Hungarian sector. Section 4.4 focuses on the results, and finally Section 4.5 concludes.

4.2 Methods

The analysis of the insurance market is based on the Panzar—Rosse model, which gives testable implications of profit maximizing companies in different market structures. The great advantage of the Panzar and Rosse model is the limited data requirement, its large literature, and easy interpretability. Only revenues and factor prices of the companies are required. There is no need for explicit information about the structure of the market. The reduced form revenue equation is the following:

$$\pi = R\left(y, z\right) - C\left(y, w, t\right)$$

Where $R(y, z)$ is the reduced form revenue function, y is the decision variable and z are further exogenous variables which influence the revenue function. $C(y, w, t)$ is the cost function, where w is the vector of exogenous factor prices and t is the vector of additional exogenous variables that influence cost. This simple model assumes profit maximizing companies. The testable expression is the sum of the factor price elasticities of the reduced form revenue equation:

$$H = \sum_i \frac{\partial R^*}{\partial w_i} \frac{w_i}{R^*}$$

where * indicates the profit maximizing values.

This paper offers different theorems about the value of the sum of elasticities of gross revenue with respect to input prices (denoted H) for competitive and monopolistic markets to be able to distinguish these models. In the case of a neoclassical monopolist or collusive oligopolist, the elasticity is nonpositive ($H \leq 0$). It is equal to unity in the case of a competitive price-taking insurance in long-run competitive equilibrium ($H = 1$). For a monopolistic competitor the factor price elasticity is between 0 and 1 ($0 < H < 1$). An assumption is that in the case of perfect competition and monopolistic competition the companies are observed in long-run equilibrium and entry and exit are free. In long run equilibrium the return rates are not correlated with input prices. To test the long run equilibrium empirically, return on assets (ROA) can be estimated with the same independent variables used in the estimation of the factor price elasticity. In long-run competitive equilibrium, same of the factor price elasticities is zero ($E = 0$).

In Kasman and Turgutlu (2008) the following equation is estimated with a panel dataset:

$$lnTR_{i,t} = \alpha + \beta_1 lnPL_{i,t} + \beta_2 lnPBS_{i,t} + \beta_3 lnPFK_{i,t} + \gamma lnTA_{i,t} + \delta lnETA_{i,t} + \epsilon lnLTA_{i,t} + \zeta$$

where TR = total revenue, PL = unit price of labor, PBS = unit price of business services, PFK = unit price of financial capital, TA = total assets, ETA = the ratio of equity capital to total assets, LTA = ratio of losses paid to total assets and index i shows the insurance company and index t is the time. These values can be calculated using the financial report of the companies. To determine the market

Competitive test	
$H \leq 0$	Monopoly or collusive oligopoly
$0 < H < 1$	Monopolistic competition
$H = 1$	Perfect competition
Long-run equilibrium test	
$E = 0$	Long-run equilibrium
$E \neq 0$	Disequilibrium

Table 5: Interpretation of the tests for the market structure and the long run equilibrium based on the Panzar and Rosse methodology Source: Simpasa (2013)

structure, we need to test the hypothesis of factor price elasticity (H), which can be calculated as the sum of the coefficients of the factor prices ($\beta_1 + \beta_2 + \beta_3$).

The Panzar and Rosse approach is used in several studies in different countries and time periods, a summary of which is listed in Table 6. Some of the studies focus on the whole sector, but Camino-Mogro et al. (2019) and Uddin et al. (2018) distinguished the life and the non-life sector. Kasman and Turgutlu (2008) concentrated on the non-life sector, but they apply the data to three different sub-periods in their article in order to observe changes on the market. The insurance market operated in a perfect competition environment in Nigeria, Ecuador and in the case of not-fined Italian companies. In most cases where monopolistic competition or a monopoly characterizes the market structure, the hypothesis of perfect competition can be rejected. Most of the studies use the static panel data approach to estimate factor price elasticity. Alhassan and Biekpe (2017) used the dynamic panel analyses for the estimation.

We also use two approaches of panel modelling, namely, static and dynamic. The static approach means that we do not use any autoregressive, lagged variables. The easiest way to estimate a pooled OLS model is a simple OLS for panel data. There could be one serious problem, however, which is the unobserved effect which violates the exogeneity assumption. In that case the goal of the estimation is to eliminate the unobserved effect. We can make a within transformation or fixed effects transformation in that case. It means that we take the average of cross-section observations over time and then subtract it from the original equation. In this way all the time constant effects disappear (unobserved effect and all explanatory variables which are constant over time) (Wooldridge, 2012).

The dynamic approach uses an autoregressive model, the lag of the dependent variable as an explanatory variable. In that case several problems occur during estimation. When the lagged value of the dependent variable correlates with the error term, the fixed effect estimation could not solve the problem of endogeneity. Arellano and Bond (1991) use Generalized Method of Moments (GMM) estimation, in which they use first differences to eliminate individual effect. They solve the endogeneity problem by using all the lagged values of dependent variables as instruments. The method is also called one-step GMM in case of panel modelling. The hypothesis of factor price elasticity (H) could be tested in this specification because the lag of dependent variables and the instruments belong to control variables.

4.3 Data

To test whether the market is competitive or monopolistic we built empirical models. From the Hungarian insurance market, we chose the ten biggest companies and collected the required information about them between 2010 and 2019. In this way we had the opportunity to build a balanced panel dataset with 10 cross-section observations and 10 time periods.

Table 7 shows the number of the insurance companies in Hungary between 2011 and 2019. Insurance companies can operate under various forms of organisation, the most significant of which are formed as corporations. Our most recent analyses contain only corporations. Although there are several smaller firms on the market, their activity is difficult to review. The Hungarian Insurance Association (Mabisz) also tracks companies with different forms of operation. In total, there are more insurers in the Hungarian market than indicated in Table 7, but they are not significant in terms of revenue and size.

The most important indicators of market concentration are given in Table 8. The Herfindahl-Hirschman Index used to calculate gross premium income was 948 in 2011 and began to decrease until 2015 after which HHI increased to 922 by 2019. These

Author	Country	Time period	Method	Result
Alhassan and Biekpe (2016)	South-Africa non-life	2007-2012	Static panel	Monopolistic competition
Alhassan and Biekpe (2017)	South-Africa non-life	2007-2012	Dynamic panel	Monopolistic competition
Camino-Mogro et al. (2019)	Ecuador	2001-2016	Static panel	Life: perfect competition Non-life: perfect competition
Coccorese (2010)	Italy	1998-2003	Static panel	Not fined: monopolistic comp. or perfect competition Fined: monopoly
Jeng (2015)	China	2001-2009	Static panel	Life: monopolistic competition property-liability insurance: monopoly
Kasman and Turgutlu (2008)	Turkish non-life	1996-1998 1999-2001 2002-2004	Static panel	Monopoly Monopoly Monopolistic competition
Murat et al. (2002)	Australia	1998	Static cross-sectional data	Monopolistic competition
Todorov (2016)	Bulgaria	2005-2014	Static panel	Monopoly
Uddin et al. (2018)	Nigeria	1999-2008	Static panel	Life: perfect competition Non-life: perfect competition
Madari and Szádoczkiné Varga (2021)	Hungary 10 biggest companies	2010-2019	Static and dynamic panel	Monopoly or monopolistic competition in long run equilibrium

Table 6: Summary of the Panzar and Rosse methodology in the insurance market

Year	**Number of members in MABISZ**	**Number of insurance corporations based in Hungary**
2011	31	31
2012	33	29
2013	31	26
2014	32	27
2015	31	26
2016	27	24
2017	27	22
2018	27	22
2019	23	22

Table 7: Number of insurance companies according to Mabisz between 2011 and 2019

values are below the commonly used limit of 1,500, which indicates that the market is not concentrated. The market share of the top three, five and ten companies also decreased until 2016, after which it increased. The ten highest-income companies accounted for 83% of the market in 2011, but by 2019 they already accounted for 87%. The top five insurance companies cover more than half of the market. The C3 concentration was 40% in 2019. In 2018, Aegon became the third highest-income insurer after Allianz and Generali, ahead of Groupama. The market leader in the country was Allianz throughout the whole period, with a share of around 15% (MABISZ, 2019).

There is no general threshold to determine whether the market is too concentrated or not (Kovács, 2011). However, in the case of a merger, the aim is to avoid excessive dominance. Therefore, the competition authorities cover the extent to which market concentration may be considered too high as a result of a merger. According to the European Union Council Regulation (EC) No 139/2004 of 20 January 2004 on the control of concentrations between undertakings if the market share is lower than 25%, than the fusion does not ruin the competition. A merger of companies with a larger market share constitutes too much market dominance (Council Regulation, 2004). According to the European Guidelines on Horizontal

	HHI	C3	C5	C10
2011	948.56	43.18%	62.45%	83.52%
2012	926.84	42.55%	61.33%	83.10%
2013	894.08	40.70%	59.50%	83.41%
2014	853.92	39.14%	58.03%	82.57%
2015	829.09	38.16%	57.70%	81.08%
2016	831.94	38.34%	57.19%	81.27%
2017	896.38	38.87%	58.77%	86.44%
2018	925.34	40.11%	58.92%	87.69%
2019	922.77	39.96%	58.56%	87.84%

Table 8: Concentration indicators of the Hungarian insurance market between 2011 and 2019 according to data released by Mabisz

Mergers, there is no negative effects if the HHI is below 1000, as the market is not concentrated. Between 1000 and 2000 basis points, it is moderately concentrated, if the HHI is above 2000, then the market is highly concentrated (Csorba, 2007).

4.4 Results

We summarise the results of the static and dynamic panel models below (see Table 9). As factor prices we used the unit price of labour (PL), business services (PBS) and capital (PFK) and as control variable the ratio of losses to total assets (LTA) and the ratio of life insurance portfolio to total portfolio (Life), ratio of equity capital to total assets (ETA), claim costs to technical provisions (PD) and outward reinsurance premiums to earned premiums (Reinsurance). According to Biker et al. (2012) only an unscaled revenue equation can give unbiased result, so the dependent variable is not scaled and the model does not contain the total asset as a control variable. To provide positiveness in case of taking a logarithm we shifted the values of PBS, PFK and ROA with a unique constant value above zero. The parameters did not show a significant difference after this change-over. The results seem robust. This calculation is similar to the approach taken by Alhassan and Biekpe (2017). The final equation contains only the significant control variables (LTA and Life) as

	Fixed effects model			GMM model		
	Coefficient	Standard error	P-value	Coefficient	Standard error	P-value
Constant	25.597	0.425	0	0.059	0.009	0
$\ln PL_{i,t}$	0.002	0.018	0.907	0.005	0.012	0.658
$\ln PBS_{i,t}$	0.042	0.061	0.496	-0.036	0.074	0.622
$\ln PFK_{i,t}$	0.037	0.043	0.39	0.025	0.018	0.162
$LTA_{i,t}$	-4.536	3.621	0.214	13.129	2.88	0
$Life_{i,t}$	-0.802	0.467	0.09	1.035	0.342	0.003
$\ln Y_{i,t-1}$				0.698	0.198	0
n	100			80		
t	10			10		
Instruments	-			42		
Sargan test	-			X^2=53.561 and p-value=0.023		
AR(2) test	-			z=0.211 and p-value=0.833		
$\beta_1 + \beta_2 + \beta_3 = 0$	F=1.049 and p-value=0.308			F=0.006 and p-value=0.937		
$\beta_1 + \beta_2 + \beta_3 = 1$	F=136.349 and p-value=0.000			F=177.729 and p-value=0.000		

Table 9: Results of fixed effects and GMM models

the following equation shows.

$$TR_{i,t} = \alpha + \beta_1 lnPL_{i,t} + \beta_2 lnPBS_{i,t} + \beta_3 lnPFK_{i,t} + \gamma LTA_{i,t} + \delta Life_{i,t} + \epsilon TR_{i,t-1} + \zeta$$

The first model is the fixed effects panel model. The first obvious thing is that none of the variables, parameters are significant in the model ($\alpha = 5\%$). We checked heteroskedasticity. It could be the case, the standard errors are biased, in this the t-tests are not consistent. We used heteroskedasticity and autocorrelation corrected standard errors, but the results remained the same. It means that, the parameters of logarithm of PL, PBS and PFK are zero separately. In this way the sum of these parameters should be zero too. To test it, below the results of the model we report the two parameter tests. These are simple linear parameter restrictions, so we could implement an F test for the sum of coefficients.

In the case of monopoly, elasticity is nonpositive ($H \leq 0$). In this case it means that the revenue function does not depend on the decision of the rivals. The value of the appropriate test is 1.049 with a 30.8% p-value. This means that we cannot reject the null hypothesis; we accept a monopoly market. In a monopolistic competition $H \leq 1$. In long-run competitive equilibrium, the elasticity is unique ($H = 1$). The value of the second appropriate test is 136.349 and the p-value is near 0. This means that we reject the null hypothesis, so there is no perfect competition. The result of the two tests shows that the insurance market is a monopoly or a monopolistic competition. Goddard and Wilson (2009) found that, the estimator of H-statistics in fixed effects model could be biased towards to zero, which could be a limitation here. But they also proved that, the GMM estimator is more efficient in case of H statistics.

It is rational and realistic to make the model dynamic. In the one-step GMM model we use the lag of the dependent variable as an explanatory variable. This is significant and our choice seems appropriate. The model should meet some requirements. The first is the AR(2) test, which tests the number of lags and model specification. The null hypothesis states that the first lag of Y is enough. The p-value of the test is 83%, so more lags are not needed in the model. The second requirement is the Sargan over-identification test. Due to huge number of instruments over-identification could occur in the model. In our model the p-value of the test is 2.3%. This is not unambiguous; it is on the edge of acceptance and rejection. Thus, it is a limitation when we discuss the result of GMM model.

The parameter tests ($H \leq 1$ and $H \leq 0$) provide the same result as in the fixed effects panel model. The insurance market in Hungary, in the given time period is monopoly or monopolistic competition.

We would like to see the robustness and variability of results if we modify the definition of dependent variable. In the literature, the dependent variable, revenue is not defined the same way, but most cases focuses on the technical incomes. Alhassan and Biekpe (2017) and Coccorese (2010) used the net earned premiums and investment income, Kasman and Turgutlu (2008) used the sum of financial and technical income, Murat et al. (2002) used the premium revenue and investment income. In the first case we used the total revenue (income from life, non-life and non-technical parts). Table 10 contains the results of those models, in which the dependent variable is the revenue which contains only the premiums and investment incomes from life and non-life sector without the non-technical incomes.

	Fixed effects model			GMM model		
	Coefficient	Standard error	P-value	Coefficient	Standard error	P-value
Constant	27.938	0.533	0			
$\ln PL_{i,t}$	-0.017	0.028	0.559	0.055	0.038	0.148
$\ln PBS_{i,t}$	0.273	0.036	0	0.456	0.134	0
$\ln PFK_{i,t}$	0.234	0.043	0	0.247	0.036	0
$LTA_{i,t}$	-26.522	11.589	0.048	-63.738	10.721	0
$Life_{i,t}$	-2.657	0.466	0	-2.186	1.133	0.054
$\ln Y_{i,t-1}$				0.127	0.081	0.115
$\ln Y_{i,t-2}$				-0.287	0.083	0
n	100			70		
t	10			10		
Instruments	-			40		
Sargan test	-			X^2=58.448 and p-value=0.004		
AR(2) test	-			-		
$\beta_1 + \beta_2 + \beta_3 = 0$	F=103.915 and p-value=0.000			F=33.822 and p-value=0.000		
$\beta_1 + \beta_2 + \beta_3 = 1$	F=112.597 and p-value=0.000			F=3.419 and p-value=0.064		

Table 10: Results of fixed effects and GMM models with redefined revenue

In the fixed effects model we used heteroskedasticity and autocorrelation corrected standard errors again. We got more significant variables in the model. The cause could be the "cleaned", more rational dependent variable. According to the parameter tests, we can reject the monopoly and perfect competition hypothesis too, so the sum parameters is between 0 and 1. It means that the market is monopolistic competition. The specification of GMM model required the second lag of dependent variable based on the result of AR(2) test (p-value ∼0.000). In the GMM model there are also more significant variables. If we see the parameter tests, we find that the $H \leq 0$ hypothesis could be rejected. We should accept $H = 1$ hypothesis on 5%

significance level. According to that the market is perfect competition. We should know that in the GMM model there is over identification problem according to Sargan test. According to the two models in Table 10 the conclusion could be that, the Hungarian insurance market is not monopoly or collusive oligopoly. Panzar and Rosse assume that their estimations are acceptable under the assumption of long-run equilibrium. We can test it because in long-run equilibrium input prices are not expected to be correlated with the rate of returns in the model (Alhassan and Biekpe, 2017). So we built the same models in which the logarithm of ROA is the dependent variable as the following equation shows.

$$ROA_{i,t} = \alpha + \beta_1 lnPL_{i,t} + \beta_2 lnPBS_{i,t} + \beta_3 lnPFK_{i,t} + \gamma LTA_{i,t} + \delta Life_{i,t} + \epsilon ROA_{i,t-1} + \zeta$$

	Fixed effects model			GMM model		
	Coefficient	Standard error	P-value	Coefficient	Standard error	P-value
Constant	1.431	1.397	0.309	0.016	0.015	0.282
$\text{lnPL}_{i,t}$	0.005	0.06	0.933	-0.027	0.031	0.385
$\text{lnPBS}_{i,t}$	-0.003	0.201	0.987	-0.002	0.034	0.951
$\text{lnPFK}_{i,t}$	0.091	0.14	0.518	0.096	0.134	0.473
$\text{LTA}_{i,t}$	0.593	11.892	0.96	-7.771	9.552	0.416
$\text{Life}_{i,t}$	0.105	1.537	0.946	-0.569	0.589	0.334
$\text{lnY}_{i,t-1}$				-0.02	0.012	0.091
n	100			80		
t	10			10		
Instruments	-			42		
Sargan test	-			X^2=6.705 and p-value=0.999		
AR(2) test	-			z=0.603 and p-value=0.547		
$\beta_1 + \beta_2 + \beta_3 = 0$	F=0.128 and p-value=0.721			F=0.295 and p-value=0.587		

Table 11: Results of fixed effects and GMM models for ROA

From Table 11 it is clear that there are only insignificant variables. We test the sum of input parameters. If the sum is equal to zero, then long-run equilibrium exists in the Hungarian insurance market. In both models the value of the F-test for restrictions is low, which implies a high p-value. For this reason, we cannot reject the null hypothesis, so the assumption of long-run equilibrium is valid. The estimations in the previous models are acceptable. All the model diagnostics and tests are reliable. The heteroskedasticity does not affect the results of fixed effect model. There is no autocorrelation and overidentification problem in GMM model. Molyneux et al. (1994) observed banking in European countries, they got similar results, there were no significant variables in the models for ROA in some countries. They drew the similar conclusion, the assumption of long-run equilibrium is valid.

4.5 Discussion and Conlusion

We examined the market structure of the Hungarian insurance sector with the help of empirical analysis. Based on the Panzar and Rosse model, we tested input price elasticity. We accepted the long-run equilibrium assumption which enabled us to perform further estimations. Using a static and dynamic panel model with two different dependent variable we got the results, that the Hungarian insurance sector is monopolistic competition market. We reject null hypothesis about the unit factor price elasticity in most models, the sum of the parameters of a unit price of labour, unit price of business services and unit price of financial capital is not equal one, thus the market is not under perfect competition. Using this model a monopoly market means that the decision of a company does not depend on the decisions of the other participants. In same cases the hypothesis of the monopoly market can be accepted, that fact suggests large market power in the sector. Similar results in the insurance market can be seen in other countries. Methodologically it is important that the estimation of factor price elasticity was made with a static and a dynamic panel model also. Most cases seem to support monopolistic competition. The scope of this study was limited in terms of the time period and the number of companies examined. More extensive research would be needed to work with a larger sample. Greater effort is needed to devide the sector and estimate factor price elasticity for life and non-life separately. A further interesting research question could examine the insurance sector at a regional or even European level. It is important to understand the market structure of Systematically Important Financial Institutions (SIFIs), so a similar study would be worthwhile undertaking for the banking sector, which would allow for a comparison of the two sectors.

4.6 Acknowledgement

The authors would like to express their very great appreciation to Kolos Csaba Ágoston, Magdolna Gyenge, Zsombor Szádoczki and Ferenc Varga and the anonymous referees.

References

Alhassan, A.L. – Biekpe, N. (2016): Competition and efficiency in the non-life insurance market in South Africa. Journal of Economic Studies, Vol. 43 No. 6, pp. 882-909. https://doi.org/10.1108/JES-07-2015-0128.

Alhassan,A. L. – Biekpe, N. (2017): Liberalization Outcomes and Competitive Behaviour in an Emerging Insurance Market, African Development Review, Vol. 29, No. 2, 2017, 122—138.

Arellano, M. – Bond, S. (1991): Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations. The review of economic studies, 58(2), 277-297.

Banyár, J. – Turi, P. (2019): A biztosítási fogyasztóvédelmi szemlélet evolúciója Magyarországon (The evolution of the insurance consumer protection approach in Hungary), Pénzügyi Szemle 2019/2, 191-209.

Bikker, J. A. – Haaf, K. (2002): Competition, concentration and their relationship: An empirical analysis of the banking industry. Journal of Banking & Finance 26, 2191-2214.

Bikker, J. A. – van Leuvensteijn, M. (2008): Competition and efficiency in the Dutch life insurance industry, Applied Economics, 40:16, 2063-2084. DOI: 10.1080/00036840600949298

Camino-Mogro, S. – Armijos-Bravo, G. – Cornejo-Marcos, G. (2019): Competition in the insurance industry in Ecuador: An econometric analysis in life and non-life markets. The Quarterly Review of Economics and Finance, 71, 291–302. https://doi.org/10.1016/j.qref.2018.10.001.

Coccorese, P. (1998): Assessing the Competitive Conditions in the Italian Banking System: Some Empirical Evidence. BNL Quarterly Review, no. 205, 171-191.

Coccorese, P. (2010): Information Exchange as a Means of Collusion: The Case of the Italian Car Insurance Market. Journal of Industry Competition and Trade, 10, 55–70.

Council Regulation (2004): Council Regulation (EC) No 139/2004 of 20 January 2004 on the control of concentrations between undertakings (the EC Merger Regulation) (Text with EEA relevance), Official Journal L 024 , 29/01/2004 P. 0001 - 0022, https://eur-lex.europa.eu/legal-content/EN/TXT/HTML/?uri=CELEX:32004R0139&from=hu, Download: 2022.06.25.

Csorba, G. (2007): A fúziókontroll módszertanáról. Dominancia- vagy versenyhatásteszt? In: Verseny és szabályozás 2007, (On the methodology of merger control. Dominance or competition test? In: Competition and Regulation 2007), http://www.econ.core.hu/kiadvany/vesz.html

Goddard, J. and Wilson, J. O. (2009). Competition in banking: A disequilibrium approach. Journal of Banking & Finance, 33(12), 2282-2292.

Iwata, G. (1974): Measurement of Conjectural Variations in Oligopoly. Econometrica, Vol. 42, No. 5 (Sep., 1974), pp. 947-966.

Jeng, V. S. C. (2015): Competition and Its Variation Over Time: An Empirical Analysis of the Chinese Insurance Industry. The Geneva Papers on Risk and Insurance - Issues and Practice, 40(4), 632–652. https://doi.org/10.1057/gpp.2015.4.

Kasman, A., – Turgutlu, E. (2008): Competitive Conditions in the Turkish Non-Life Insurance Industry. Review of Middle East Economics and Finance, 4(1), 1-16.

Knezevic, S. – Markovic, M. – Brown, A. (2015): Measuring the efficiency of Serbian insurance companies, Acta Oeconomica,Vol. 65 (1), pp. 91–105 (2015) DOI: 10.1556/AOecon.65.2015.1.5

Kovács, N. (2011): A piaci erő közvetett mérése a biztosítási piacon (Indirect measurement of market power in the insurance market), Széchenyi István Egyetem Regionális és Gazdaságtudományi Doktori Iskola doktori értekezés.

Kozmenko, O. – Merenkova, O. – Boyko, A. (2009): The analysis of insurance market structure and dynamics in Ukraine, Russia and European Insurance and Reinsurance Federation (The analysis of insurance market structure and dynamics in Ukraine, Russia and European Insurance and Reinsurance Federation (CEA) member states, Problems and Perspectives in Management, Volume 7, Issue 1, 2009.

Kramaric, T.P. – Kitic, M. (2012): Comparative Analysis of Concentration in Insurance Markets in New EU Member States, World Academy of Science, Engineering and Technology International Journal of Social, Behavioral, Educational, Economic, Business and Industrial Engineering Vol:6, No:6, 2012 1322-1326.

MABISZ. (2019): Magyar Biztsoítók Évkönyve. Budapest: Magyar Biztosítók Szövetsége (Yearbook of Hungarian Insurance Companies: Hungarian Insurance Association).

Madari, Z. – Szádoczkiné V.V. (2021): Empirical Analysis of the Hungarian Insurance Market, In: Drobne, S.– Stirn, L. Z. – Kljajić, B. M. – Povh, J. – Žerovnik, J. (2021). Proceedings of the 16th International Symposium on Operational Research in Slovenia. (S. Drobne, L. Z. Stirn, B. M. Kljajić, J. Povh, & J. Žerovnik, Eds.). Ljubljana: Slovenian Society Informatika. pp. 125-129.

Molyneux, P., Lloyd-Williams, D. M., & Thornton, J. (1994). Competitive conditions in European banking. Journal of banking & finance, 18(3), 445-459.

Murat, G. – Tonkin, R.S. – Jüttner, D.J. (2002): Competition in the General Insurance Industry, German Journal of Risk and Insurance, Vol. 91, No. 3, pp. 453—81.

Nathan, A., – Neave, E. H. (1989): Competition and contestability in Canada's financial system: empirical results. The Canadian Journal of Economics / Revue canadienne d'Economique, Aug., 1989, Vol. 22, No. 3, 576-594.

Panzar, J. C., – Rosse, J. N. (1987): Testing For 'Monopoly' Equilibrium. The Journal of Industrial Economics, 35(4), 443—456.

Simpasa, A. M. (2013). Competition and market structure in the Zambian banking sector. African Development Bank Group, Working Paper 168.

Szüle, B, (2017): Solvency effects of taxation: A comparison of Hungarian banks and insurance companies, Acta Oeconomica, Vol. 67 (1), pp. 63–75 (2017) DOI: 10.1556/032.2017.67.1.4

Tipuric, D. – Bach, M. P. – Pavic, T. (2008): Concentration of the insurance industry in selected transition countries of Central and Eastern Europe, 1998–2006. Post-Communist Economies, 20(1), 97-118.

Todorov, A. B. (2016): Assessing Competition in the Bulgarian Insurance Industry: A Panzar-Rosse Approach, International Journal of Economics and Financial Issues, 2016, 6(3), 872—879.

Uddin, G. – Oserei, K. – Oladipo, O. – Ajayi, D. (2018): Industry Competitiveness Using Firm-Level Data: A Case Study of the Nigerian Insurance Sector (pp. 473–492).
https://doi.org/10.1007/978-3-319-70055-7_37.

Wooldridge, J. M. (2012): Introductory Econometrics a modern approach 5th Edition.

5 Endnotes

In order to ensure that the articles presented in the article-based dissertation are included in their published form, we respond to the comments received from reviewers and the suggestions raised during the discussion of the dissertation draft in this chapter.

The substance aversion introduced in the first study seems to contradict the behavior observed in practice since the insurer wants to sell as many correctly priced insurance products as possible. Commonly used risk-averse utility functions are substance averse. However, if we examine the indifference curves belonging to the mixed exponential utility function even at higher prices in Figure 12, we can see that there is an abscissa at the indifferent price level (P_0) belonging to the exponential term of the function, and the curves tend there. Thus, for prices above this, the indifference curves take the usual 'C' shape, and in this interval, more sales are preferred for the insurer. Thus, substance preference does not contradict the usual market behavior. The indifferent price is always determined by comparing it with the utility of the initial wealth; hence, the definition only applies to the indifference curve that provides expected utility equal to the utility of the initial wealth. In the case of prices smaller than P_0, interesting equilibrium situations can be observed. Our study also draws attention to the differences between actuarial and theoretical economic research.

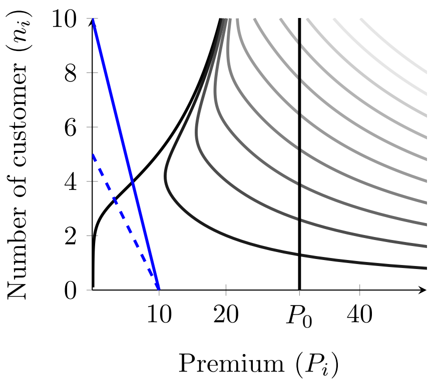


Figure 12: Indifference curves in the case of substance aversion. The blue lines are the demand functions.

The behavior of market participants suggests substance seeking utility functions, so it is important to examine whether such a function exists. The example presented in the first study is substance seeking only for certain parameter combinations, not globally. Since proper risk aversion usually entails substance aversion, improper functions may be worth considering. An example of this can be found in Pratt and Zeckhauser (1987). The utility function $u(w) = a - be^{-sw} + ce^{tw}$, where $s > t > 0$ and $b, c > 0$ is improper on the interval $w < \frac{1}{s+t} ln\frac{b}{c}$. We can see with a numerical example in Table 12 that it is not a substance seeking function, since the indifferent price for two contracts is higher than the one for one contract, so selling two contracts instead of one contract at the indifferent price is not preferred for the insurer. Among the functions used in the economic literature, it is difficult to show an example of a function that fulfills substance seeking.

One review mentions that we defined substance preference using the number of sold contracts. The interesting part of the concept is that in the case of substance aversion, a larger substance (more sold contracts, larger n value) is not preferred. The concept's meaning is also closely related to the number of contracts, and the representation of the function is also in the (P, n) space, so we characterized the concept through this. A planned future research direction could be the determination of necessary and sufficient conditions for the utility function u to be substance averse, neutral, or seeking, and the definition of the concept in the case of other loss distributions (exponential, normal).

In the 1990s, the problem of managing multiple risks together received great emphasis. At that time, three new risk aversion concepts were also introduced, risk vulnerability (Gollier and Pratt, 1996), standard risk aversion (Kimball, 1993) and proper risk aversion (Pratt and Zeckhauser, 1987). Necessary and/or sufficient conditions for the fulfillment of these properties and the relationships between them were only partially determined until today as we know. The results are still cited, but the focus of current research shifted towards finance and portfolio optimization[8].

Although there are no costs in our model, we can show a connection between our results and the models with convex costs. If we assume risk aversion, for a given price level there is one (or maybe more) maximum level(s) of the expected utility. In the local area of this contract number selling more is not preferred similarly to

[8]https://ideas.repec.org

Utility function: $a - be^{-sw} + ce^{tw}$	The number of contracts 1	2
Indifferent price	0.01292807	0.01293218

Table 12: The indifferent price for different amounts of substances. The initial wealth w is 6, the amount of claim K is 1 and the probability q is 0.01, and the parameters of the utility function are the following: $s = 0.5, t = 0.1, b = 5000, c = 1, a = 20000$.

models using convex costs.

A common assumption in market structure models is that the profit function is quasi-concave (Caplin and Nalebuff, 1991) and the demand function is log-concave (Cowan, 2007). The question arises whether these properties are fulfilled in the models we present. When analyzing substance aversion, we plotted the indifference curves of expected utilities. These are quasi-concave in n for a given p, but this property raises the issue of interpreting fractions as a technical difficulty. The expected utility as a bivariate function is not quasi-concave. Figure 13 shows the upper-level set of the expected utility corresponding to the utility of the initial wealth, which is not convex.

The demand function in the first study is linear. The shape of the function does not significantly affect the results, only its decreasing property is important, which is a common assumption in economic models. However, in the insurance sector, there are mandatory insurances (motor vehicle liability insurance, liability insurance for various jobs), which are required to be taken year after year to perform a given activity, so in this case, consumers may not be as price-sensitive.

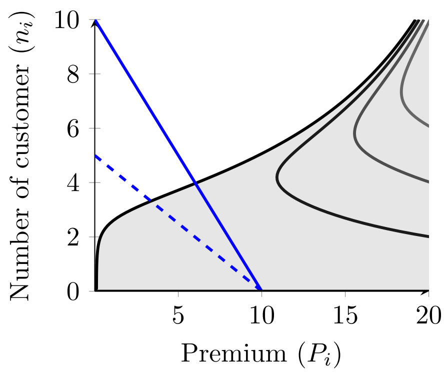


Figure 13: Upper-level set of the expected utility at the level of utility of the initial wealth highlighted by gray. The blue lines are the demand functions.

In the second study, the demand function chosen for the analysis of the capital constraint is of greater importance. With a demand function of this shape, the demand curve and the VaR curve intersect at a single point, and this intersection point can be defined in closed form. In the case of functions with other shapes, there may be multiple intersection points, which also affect the resulting equilibria.

The unique intersections of the capital constraint and the demand curves are important in determining the equilibrium, as they form the two endpoints of the equilibrium price interval in the case of the increasing phase. Another important statement of the article is that the expected profit increases along the VaR curve. If the companies always set the lowest price, at which the capital limit is met, then their expected profit increases as the number of customers increases. However, the VaR curve has a maximum in n, so the price increases with the increase of n below n_{max}, but decreases above the threshold value. That is, in the case of a smaller portfolio, each new contract increases the risk, but above a certain level, a larger portfolio results in a more accurate estimate. This is consistent with the phenomenon of risk dispersion experienced in the market. Insurers strive to form large homogeneous risk communities, but this is only beneficial above a certain size.

An interesting result of the study is related to the market power of companies with higher level of capital. In the case of less symmetrical companies with a fixed total market capital level, premiums decrease. In this case, each player has more capital and can provide the same level of safety at a lower price. If two companies merge, they can free up solvency capital or sell insurance policies cheaper. An extreme example of such a case can also be shown. In Figure 14, two insurers operate with the same level of capital (so both face the capital constraint on the right-hand side), the equilibrium interval is $[P_L, P_U]$, which exceeds the monopoly price ($2qK = 40$, the green point) with such a parameter combination. The figure shows that the monopoly price does not meet the insurers' solvency capital requirement. If the two insurers merge and their capital doubles, there is only one player in the market, who sets the monopoly price with which the expected profit is maximized, and due to the higher capital level, the capital constraint is also respected (it moves to the left side).

In the model, the phenomenon can be traced back to the subadditivity of the

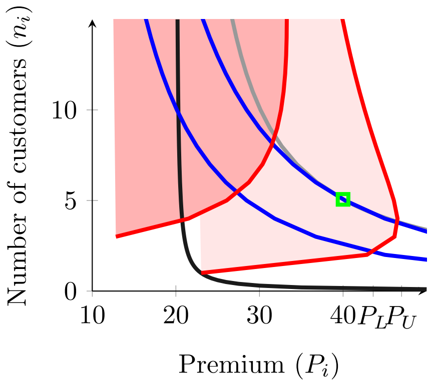


Figure 14: Merger of insurers in the capital constraint model, where $q = 0.2$, $K = 100$, $C = 100$, $r = 3\%$, $\alpha = 90$, $I = 2$.

VaR risk measure. If X and Y are normally distributed random variables, then the value at risk is subadditive: $VaR_{99.5\%}(X+Y) \leq VaR_{99.5\%}(X) + VaR_{99.5\%}(Y)$. Due to new regulations affecting the insurance and banking sectors, the examination of risk measures (Value at Risk, Expected Shortfall, Conditional Value at Risk (CVaR)) and their various properties has recently received much attention. A property often expected from a risk measure is coherence. A risk measure is coherent when it satisfies monotonicity, subadditivity, positive homogeneity and translation invariance (Artzner et al., 1999). A disadvantage of VaR is that it is not a coherent risk measure in general. Therefore, another risk measure, the Expected Shortfall (ES), which has definitions that are coherent regardless of the distribution of losses, is often analyzed (Acerbi and Tasche, 2002; Jadhav and Variyam, 2023). Different risk measures can lead to similar results under certain assumptions and distributions. Optimizing the VaR in the special cases when it is coherent, leads to the same problem as minimizing the variance. Another risk measure is CVaR, Rockafellar and Uryasev (1999) mention that a market equilibrium with low CVaR necessarily means low VaR as well. Since the Solvency II regulation (Directive 2009/138/EC, 2009) requires insurance companies to calculate VaR, we worked with this risk measure, but due to the approximation with the normal distribution, the VaR is coherent in our model.

In the case of analytical models, the question arises that how the results resonate with the phenomena experienced in practice. We used several simplifying assumptions and did not take into account many factors (no costs, symmetric companies, etc.) to examine the effect of the capital requirement. The problem of non-unique equilibria can be solved by using another equilibrium concept (payoff dominated) or introducing a different market dynamic (sequential game). The insurance market has a strong oligopoly characteristic. Strict entry barriers and capital regulation result in a concentrated market. According to the VaR model, fewer but more capitalized companies have a market advantage. Thus, this theoretical model can serve as an explanation for why market concentration can be high in the sector.

In the third study, the sample used for the empirical research contains data from 2010 to 2019. Due to further similar analyses, the question of expanding the time dimension arises. The restrictions introduced due to the pandemic in 2019 also had economic effects. According to OECD (2023), the restrictions affected sales processes and the development of claims' payments, but in 2021 the premium income of insurers increased, especially in the life sector. According to a study specific to the Hungarian sector (Kelemen and Németh, 2020), the effects of the COVID-19 pandemic required flexibility from employees (home office) and insurance agents (restrictions), but the basic insurance processes (pricing, reserve calculations, and capital requirements) were not significantly affected. Since all domestic actors faced the same environmental effects, we do not necessarily consider the extension of the time window problematic. However, if we want to examine how the input price elasticity changed in time due to the impact of the crisis, then the H statistic can be estimated even annually with a cross-sectional model, and we can see some examples of this in the literature (Jeng, 2015). However, other phenomena can cause problems, acquisitions, mergers or possible bankruptcy of the insurers in the sample makes the data unbalanced, so it can cause issues during the estimation.

The hypotheses of frequently used non-structural models, such as the Panzar and Rosse (1987) and the Bresnahan (1982) model, are mainly related to the cases of monopoly market and perfect competition. The intermediate state is usually regarded as monopolistic competition. They cannot be used to distinguish between oligopoly market and monopolistic competition. Possibly, indicators related to the degree of product differentiation can be useful (Howell and Girell-Tatjé, 2022; Hackl et al., 2021), for example, cross-price elasticity estimation.

These were the main points during the defense of the Ph.D. draft, and we are grateful for the useful suggestions of the reviewers. Hopefully, the endnotes presented here make the findings of the studies more nuanced.

References

Acerbi, C. and Tasche, D. (2002) On the coherence of expected shortfall. *Journal of Banking & Finance*, 26, 1487-1503.

Artzner, P., Delbaen, F., Eber, J. and Heath, D. (1999) Coherent Measures of Risk. *Mathematical Finance*, 9:3, Pages 203-228.

Bresnahan, T.F. (1982). The Oligopoly Solution Concept is Identified, *Economic Letters* 10, 87-92.

Caplin, A. and Nalebuff, B. (1991). Aggregation and Imperfect Competition: On the Existence of Equilibrium. *Econometrica* 59, 25–59.

Cowan, S. (2007). The Welfare Effects of Third-Degree Price Discrimination with Nonlinear Demand Functions. *The RAND Journal of Economics*, 38, 419-428.

Directive 2009/138/EC Solvency II (2009). Directive 2009/138/EC of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II).

Gollier, C. and Pratt, J. W. (1996). Risk Vulnerability and the Tempering Effect of Background Risk. *Econometrica*, 64(5), 1109-1123.

Jadhav D., K. and Variyam, R. T. (2023). Modified Expected Shortfall: a Coherent Risk Measure for Elliptical Family of Distributions. *Sankhya B: The Indian Journal of Statistics, Springer;Indian Statistical Institute*, 85(1), 234-256.

Jeng, V. S. C. (2015): Competition and Its Variation Over Time: An Empirical Analysis of the Chinese Insurance Industry. *The Geneva Papers on Risk and Insurance - Issues and Practice*, 40(4), 632–652. https://doi.org/10.1057/gpp.2015.4.

Hackl, F., Hölzl-Leitner, M. and Pennerstorfer, D. (2021). How to Measure Product Differentiation. *Economics working papers* 2021-01, Department of Economics, Johannes Kepler University Linz, Austria.

Howell, C. and Girell-Tatjé, E. (2022). An Index Approach to Measuring Product Differentiation: A Hedonic Analysis of Airfares. *The review of income and wealth*, 69(4):952-974.

Kelemen, K. and Németh, V. (2020). A COVID-19 járvány hatásai a magyar biztosítási piacra. The effect of COVID-19 on the Hungarian insurance sector. *Biztosítás és Kockázat*, 7, 76-89.

Kimball, M. S. (1993). Standard Risk Aversion. *Econometrica*, 61, 589-611.

OECD. (2023). Global Insurance Market Trends 2022, https://www.oecd.org/daf/fin/insurance/globalinsurancemarkettrends.htm, Download:2024.03.20.

Panzar, J. C. and Rosse, J. N. (1987). Testing For 'Monopoly' Equilibrium. *The Journal of Industrial Economics*, 35(4), 443-456.

Pratt, J. W. and Zeckhauser, R. J. (1987). Proper Risk Aversion. *Econometrica*, 55(1), 143-154.

Rockafellar, R. T. and Uryasev, T. (1999). Optimization of Conditional Value-at-Risk. *Implemented in Portfolio Safeguard by AOrDa.com*