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COGNITIVE MATHEMATICAL COMPETENCES AND THE IMPACT OF THEIR DEVELOPMENT ON DECISION-MAKING PROCESSES

CORVINUS UNIVERSITY of BUDAPEST
CORVINUS DOCTORAL SCHOOLS

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DOCTORAL THESIS

Written by: **András Máté Farkas-Kis**

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To my wife Noémi and my children Boldizsár and Olivér,
who have been steadfastly by my side and following my research.

To Zita and András, who have supported my work throughout
and constantly challenged me with their feedback.

And finally, to Dr. Béla Bodosi,
with whom we set out in the field of education to
look deep into the pedagogy of mathematics.

„Because true mathematics is nothing more than pure thinking using a minimal set of tools, the acquisition of non-trivial knowledge without the accumulation of artificial arithmetic."

Rademacher H. & Toeplitz O.

I. INTRODUCTION

1. Actuality of the topic

In our fast-paced lives, more decisions must be made in less and less time, especially in the business world where decision-making and how it is done is a key element of how companies operate. How this is done, through what kind of organisational process, based on what information, with what tools, processed, evaluated and, last but not least, linked to the level of the employee, is crucial for success. However, practice shows that, although formal frameworks for decision-making are established in most places, daily decisions are never taken based on actual situations but perceived situations. In other words, the psychological state and the perception of reality play a major role in overcoming an obstacle and in the final decision. Therefore, a crucial condition for the success of decisions is whether the issue behind the decision has been properly defined, identified and structured at the level of the decision-maker.

However, when we think about decisions, we should also remember that in the vast majority of cases, especially in a corporate environment, decisions are accompanied by information and quantification in some way, represented by numbers. And when we talk about numbers and quantification, it is inevitable to not talk about mathematics, mathematical competences and mathematical literacy.

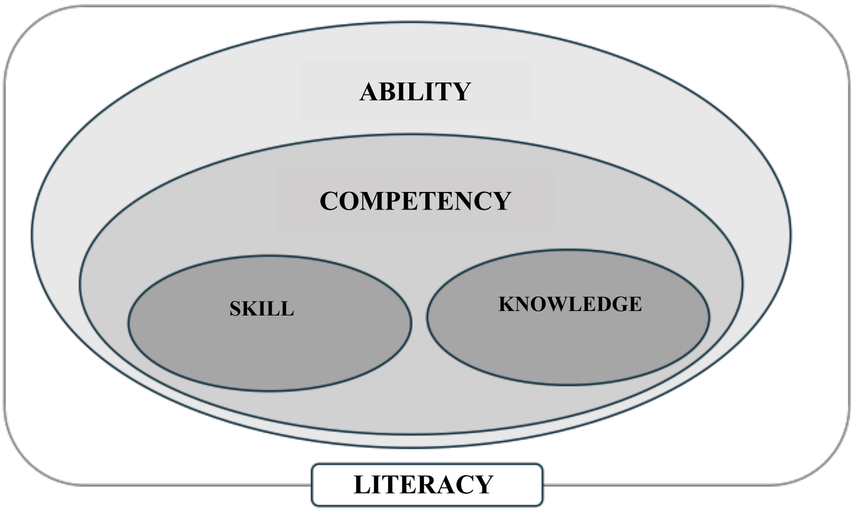
Mathematical literacy is defined by levels of mathematical ability, competence, knowledge and skills. In order to make sense of the complex relationship between these, it is necessary to clarify how the areas discussed below relate to each other in the context of the present research and how these similar concepts, often used synonymously in the literature (Roberts et al. 2007; Csapó 2003; Gardner 1983; Spancer & Spancer 1993), relate to each other:

1. 'Literacy' is a broad, well-organised body of knowledge that represents a holistic approach to a given discipline (in this case, mathematics). It is characterised by critical thinking, the ability to evaluate and to understand cultural contexts. Literacy enables individuals to participate consciously in social and cultural life.
2. 'Ability' refers to the general mental, physical and social potential of an individual to learn, solve problems and adapt to his or her environment. They are innate or developed qualities that determine an individual's performance in different areas.

3. 'Competency' is the ability that encompasses the skills, knowledge and attitudes that enable individuals to perform certain tasks or roles effectively.
4. A "skill" is a specific, automated set of behaviours, developed through skilled activities, that enables individuals to perform their tasks efficiently and smoothly. Skills are acquired through practice and are often linked to specific contexts.
5. "Knowledge" is the set of information, facts, concepts, theories and principles that an individual acquires and understands through education, learning or experience.

The relationship between these concepts is illustrated in Figure 1.

Figure 1: Interpreting the conceptual context (own editing)



Mathematics should be taught from primary school onwards to help students learn to think. The Program for International Student Assessment (hereafter: PISA) assessment, of which one of the key components is the measurement of mathematical competences, defines mathematical literacy as follows (own translation from the Hungarian report (PISA, 2006)):

> *"Applied mathematical literacy means that an individual recognises and understands the role of mathematics in the real world*, makes well-informed decisions, and *uses his or her mathematical knowledge to help him or her solve* real-life problems *and become a constructive, interested, thoughtful member of society."*

By definition, applied mathematical literacy is essential for making informed decisions, solving problems correctly and playing a full role in society. This definition shows that, if we are talking about decisions and decision-making, mathematical literacy is inevitable. The definition above also suggests that at the heart of decisions is always the human being, whose relationship with mathematics (whether conscious or not) is decisive.

When a basketball player shoots a basketball, he is not aware that his brain is measuring the distance, calculating the magnitude of the force and the angle, solving a series of equations and deciding how to execute the move. In the same way, the midfielder can be described as driving the ball and looking up to assess how to proceed and decide on the pass. He sees the movement of the other players and, as if possessed by Laplace's demon (an imaginary being calculating the future of the world using equations of motion), he makes the decision and puts the ball in the right place. These processes simply make it impossible for the players making the decisions to think about what is happening, or to calculate any trajectory data, because of the pace of the game. They simply know, they see, what needs to be done. This line of thinking has led to the need to see mathematics not as a tool for computing, but, like the PISA definition, as a fundamental element and determinant of problem solving and decision making competences. In this way, mathematics is nothing other than an abstract, formalised description of the unconscious processes that are present in all human beings and that fundamentally guide their thinking and feelings.

In recent decades, the fields of economics, decision theory and mathematics education have developed side by side, but largely independently of each other. While all have made significant progress, their common areas have been less at the centre of academic research. The aim of the current research is to summarise the key milestones and show how quantification developed and how it has evolved into mathematical thinking. How it has influenced and connected the disciplines listed above. How mathematics has become the main determinant of economic processes, a symbolic system that influences decisions and plays a central role in education. How it has become a depository of rationality and a way of deceiving the inattentive decision-making mind. And yet how it can mediate between our emotional and rational decisions. And, last but not least, why people are so afraid of mathematics that many develop mathematical anxiety, which influences their decisions.

2. Structure of the doctoral thesis

The doctoral thesis is set in the diverse and wide-ranging field of research described in the Introduction, so it is essential to provide the reader with some reference.

Chapter II of the thesis, the "Panorama", presents the theoretical background of the disciplines of economics, decision theory and mathematics education and the dominant theories and research results in these fields over the last 50-100 years. It covers the relationship between decision making and quantification, and the role of mathematics in decision making. This chapter concludes with a discussion of the communication of quantification, data visualisation and their mathematical interpretation.

Chapter III „Horizon" focuses on research questions and hypotheses. In this chapter, the questions that the research aims to answer after the hypotheses have been formulated are presented. It also sets out the research framework, which defines and describes the spaces, their basic interrelationships and interrelationships of this complex set of questions.

Chapter IV identifies the "Background", i.e. the most recent research that is closest to the aim of the hypotheses. It is also in this chapter that the pedagogical-psychological field that the research intends to explore is discussed.

Chapter V, "Pathways", presents the research planned on the basis of the research questions in order to validate the hypotheses formulated, from the methodological and data collection side on the one hand, and from the data analysis side on the other.

Chapter VI is the "Focus", which, in response to the space constraints and the reviewers' reflections on the draft dissertation, focuses on the detailed presentation of a more narrowly focused, concrete research project and the evaluation of its results, starting from the complex research design presented in the Pathways chapter.

Finally, Chapter VII summarises and concludes the thesis and looks to the future.

II. PANORAMA - THE SOCIAL, ECONOMIC AND ENVIRONMENTAL RELEVANCE OF THE TOPIC

1. The history of the economic role of numbers

For many hundreds of thousands of years, as long as man fought wildly with nature to survive, mathematics and arithmetic did not exist, because they were not needed. Humanity at that time lived by simple rules of survival. Its decisions were based on the need to survive, to stay alive and to keep its head above water. There was no need for calculation. Even if it subconsciously took stock of the possibilities, it was caught or exhausted in the instinct of the moment, in the judgement of what is-not and what is little.

Change came, often simultaneously, when man discovered the first grain-growing landscapes, which allowed him to make his way of life permanent and fixed in a particular place. This change lasted from 10,000 BC to 3,000 BC and significantly transformed the previous way of life and, in this context, presented people/groups of people with completely new decision making and problem solving situations. Man's reactive attitude towards nature has been transformed into an active and increasingly proactive activity. The rye and barley of Mesopotamia, the spelt of Syria, the wheat of the Nile Valley and the rice and millet of the Yangtze-kiang and Hoang-ho valleys offered an opportunity for change: a shift from hunting, gathering and in some places fishing to a more agricultural lifestyle. Since survival was no longer the only goal, there was room for improvement in all areas. Domestic animals, tools, pottery and related crafts appeared. The first settlements were established, and with them began the development of centralised and controlled power and economy/farming leading into the modern age.

Farming required organisation, more detailed observation of nature. The various crafts/sectors had to be coordinated, care had to be taken to protect the settlements and later the state. Although these developing territories were often thousands of kilometres apart and at that time geographically inaccessible, they were all moving in the same direction. Growth, industrial activity, local trade, storage and the observation of nature could no longer be achieved without the units of measurement of length, area, volume and mass. The development of travel and warfare made orientation in time and space indispensable. Production, farming, industry and commerce inevitably began to bring the business mechanisms that are part of modern management policy to life: the need to keep records of the quantities produced, stored and consumed over long periods of time, and to keep a numerical check on goods. Thus, from 4000 BC, the first accounts were written, and from then on, decisions were determined in an organic way.

One of the richest first written remains from antiquity was found in the archives of the royal palace of the city of Mary on the banks of the Euphrates, during an excavation by French archaeologist André Parrot, begun in 1933 (Sain, 1986).20,000 Babylonian cuneiform texts were found: letters, decrees and economic records, the oldest of which date from 2000 BC. From this period, many cuneiform numerals were found designed to speed up counting.

One of the most important finds from the Nile culture, is the Ahmes papyrus (Sain, 1986), which was found (bought) by Henry Rhind of Scotland and is dating back from the Middle Kingdom. Several mathematical problems are formulated and presented, such as methods of calculation, solutions of simple equations, area and volume calculations. The annual discharge of the Nile was crucial for management, so it was necessary to know in advance when the river would flood. The movement of the stars could give a clue. Observations of the sky and the movement of the stars provided the basis for the calendar we still use today.

The first written records were found in the Mycenaean culture in the Aegean. (Sain, 1986). The found clay tablets date from around 2000 BC and 1500 BC and were associated with tax-drinking. Later, with the transformation of the region, the emergence of city-states and, from the 7th to the 5th centuries BC, the emergence of a free bourgeoisie and the rule of the demos, Athens in particular saw a dazzling development, not only economically but also in the sciences and the arts. A lifestyle adapted to the Aegean Sea, as opposed to the static lifestyle of Mesopotamia and Egypt, which cultivated the land, had to be a more dynamic style. This allowed them to increasingly dare to ask not only the 'how' questions about natural phenomena, but also the 'why'.

It is also worth briefly mentioning the other areas where numbers also appeared early on. From the territory of today’s China the first numerals date from the 14th to 11th centuries BC (Sain, 1986). Their purpose was to count and inventory goods. Away from them, the Maya, even if later in time (from 300 AD), were also very close to numbers and mathematically advanced. Although no explicitly mathematical artifacts like the earlier ones have survived, the excavated cities suggest that their knowledge of astronomy and the calendar, for example, was highly developed.

It can be seen, therefore, that the origin of numbers, of mathematics, is to be found in the context of the tasks of practical life and was created to describe and control

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situations in order to support decisions. It was created by human needs and was intended to support and serve practical solutions. There is a particularly interesting element in this service. This is the static nature of lifestyles and social arrangements. In mathematics, in the use of numbers, this has been observed almost everywhere, even in isolated cultures. In all ancient cultures, numbers and counting were stuck in the use of solutions to the problems that life demanded of them, a fact that cannot be blamed on either the Egyptian or the Babylonian empires. The Greeks, on the other hand, were able to move on, perhaps because of the different way of life mentioned earlier: thinking about numbers, mathematical operations, at a certain stage of social development, became detached from direct practice, and self-purpose appeared. The amazing thing about this self-purposing is the way mathematics has evolved and developed over the last 2000 years. Thanks to the Arab culture, it has not been forgotten and has been able to re-emerge in the Enlightenment and eventually form the basis of our modern economic understanding.

Since the birth of civilisations, mathematical thinking and the language of numbers have dominated both individual thinking and managerial decision-making. Its purpose has not changed much, but its method and complexity have. Since the structure of decision-making processes, the range of information used to prepare decisions, and the way and means of using it, are all humanly determined and based on mathematical competences, they are the ultimate key to success in decision-making.

2. Decision-theoretical context

In the 21st century, companies are the unit of the complex economic system. Since the resources necessary for the activities that meet human needs are scarce, the efficiency of a given enterprise plays an important role. In 1776, Adam Smith showed that the social division of labour was the most basic means of increasing efficiency (Smith, 1776). However, this specialisation is not enough. The units that emerge from the division of labour must communicate with each other and coordinate their work. In this coordination, a myriad of decisions have to be taken by the participants, decisions which affect the activities involved in meeting a given need, sometimes in very complex ways.

In this context, it is important to clarify the spirit and purpose of the decisions, which requires an understanding of the concept of the company itself (Chikán, 2021).

- According to the standard microeconomics view of business, business decisions are made rationally, based on complete information, in order to maximise profits.

- Transaction cost theory takes into account additional aspects compared to the previous one, namely that there are also costs associated with each exchange transaction, with the creation of contracts.
- The principal-agent theory approaches the issue from an operational point of view: in our time, the owner (principal) and the management (agent) of a company are not always the same, which in principle can cause conflicts of interest, since profit will not belong to the one who makes the decisions to maximise it. Furthermore, another phenomenon that affects decisions and arises because of the different objectives and information available is information asymmetry.
- According to evolutionary approaches to business theory, the efficiency of a company is determined by the actors involved in it and those who are in contact with it. Therefore, the focus of a decision is on the aspirations of the stakeholders.

As theories of the firm have evolved, managerial decision-making has also been discussed in increasingly complex ways. Human behaviour has become increasingly important in understanding and describing corporate goals. And modern, complex systems imply the emergence of decision mechanisms that result from the limitations of human thinking and information processing capacity: bounded rationality. It can therefore be said (Cristofaro, 2017) that, if human behaviour is taken into account, companies and the decisions they make:

- act as a coalition of stakeholders;
- present many objectives and therefore seek not an optimum but a satisfactory solution;
- assume that the information has a cost;
- are unable to stay within the bounds of rationality.

In addition to the history of the development of the theory of the firm, it is equally important to review the history of the development of different conceptions of decision-making. In ancient times, philosophers told us how to think about decisions. But in the Middle Ages, and especially in the modern era, with the spread of industrial production and the emergence of corporations, a new approach became commonplace, namely that of economics.

As the goals of companies, and therefore of their owners, have not changed over the centuries, i.e. to maximise profit, it has become increasingly common thinking that it is important for the members of management, the people who run the company, to be able to make good decisions. However, the concept of good decision making is relative and determines how we think about the decision situation itself.

The economic approach is to quantify the products and services that appear on the market. In this case, the utility of a given product or service becomes articulable and tangible. Taylor laid down the foundations of decision theory derived from this line of thought (Taylor, 1965), which were based on

- for a given decision, any of the associated outcome events can occur;
- The decision-maker has all the information needed to make a judgement;
- results are interpretable and transparent on a scale;
- The decision is always made in order to maximise the given benefit.

This approach has proved to work at the macroeconomic level and has been able to predict to some extent what some actors in the economy will do. However, recent economic research has revealed that individuals do not always act in accordance with utility maximisation expectations.

A major factor in the development of the administrative model was the observation that individual decisions did not always meet the conditions set out in the economic model. On the one hand, not all the information is available to determine utility, and on the other hand, in many cases there are other motivating factors that take the focus away from utility. The latter has been highlighted by psychological research (Zoltayné, 2005), which has shown that perceptions dominate over reality in the context of a decision. In other words, the perceptions and expectations of the decision-maker have a significant influence on decisions. March and Simon (1965) finally summarized the main features of the administrative model as follows:

- decision alternatives are not available, they must be developed;
- information on the results is incomplete and must be searched for;
- information is expensive to obtain;
- inaccuracy of information leads to uncertainty in the expected value of the results.

Based on the above, two additional phenomena were observed. One is that decision-makers, in dealing with the problem of information uncertainty, tend to focus their attention on outcomes (including rather short-term outcomes) and, in particular, on quantifiable outcomes, and tend to ignore those that are not quantifiable. The other is that they seek to reduce the cost of obtaining information and therefore avoid complex information management systems and settle for information that satisfactorily supports a given decision.

Skinner's model represents that if we want to understand the causes of decisions, we need to explore the mechanisms by which outcomes are confirmed (Skinner, 1971). Thus, the model posits that there are strict reinforcements at work, the foundations of which are well framed by Thorndike's law of effects, which states that "behaviour or conduct that leads to (or appears to lead to) a reward is repeated by people" (Haire, 1974). The four cornerstones of Skinner's decision theory model are as follows:

- The most effective and powerful influencing factors are those that are most directly related to the decision-maker's most basic needs.
- In contrast to continuous reinforcement, intermittent reinforcement keeps a given behavioural preference at a much higher level.
- Positive reinforcement can have a greater impact than negative reinforcement.
- Only empirical evidence can reveal results that reinforce behaviour.

But in the corporate environment, the needs of decision-makers are far from being emotional needs. In most cases, they are objectives motivated by some kind of pay bonus, which can be described in terms of Key Performance Indicators (KPIs) or equivalent measures.

Among the alternative approaches, it is also important to highlight the gradual returns model (Allison, 1969). This model assumes a kind of ordered anarchy, i.e. it argues that in practice, in contrast to how little we know about the relationship between outcomes and action variables, a very large number and complex set of outcomes occur. Consequently, utility-based comparisons are not possible and decision making misses out outcomes in evaluations that may later become highly relevant. Main features of the model:

- The evaluation is based on comparing the identified action variations in a given situation with the actual results of past actions similar to the action variations in question.
- Hence, the decision maker minimises risk by choosing a similar action in a given situation that has worked before. This in turn implies that changes are usually implemented slowly, in small steps, because there is no significant deviation from previous practices.

Another typical corporate practice, which illustrates how this model works, is when managers use the current year's value or a slightly changed value for a given cost line in corporate planning.

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And we are only one step away from anarchy to chaos, which led Cohen, Marchot and Olsen (1972) to create the bin decision model. Based on their observational research, they showed that the time taken to make a given decision typically depends on the following conditions:

- when the problem arrives,
- what other problems they are dealing with,
- what ready-made solutions are available,
- how the settling and rising of problems changes,
- what the decision rituals are,
- the extent to which resources are wasted.

The dustbin model shows the possibility of dragging out the solution to problems for a long time with delayed decision-making, but this can lead to collapse.

From the models presented, it appears that the conceptions of the firm and the approaches to decision theory have evolved and developed together. However, one thing was and is still present in all cases. And that is that both enterprise concepts and decision theory approaches are based on information, which in most cases is numerical or quantified data. It makes no difference whether they are objective measurements or subjective judgements. At the level of mental abstraction, almost everything is expressed in numbers at some point of judgement and decision making. These numbers in turn exert an important and inescapable control and influence on us in our decisions.

3. Mathematics education in a scientific context

Since antiquity, numbers, quantification and accounting have been used to perform the control function of management and to support related decisions. It was therefore necessary for the performance of this service that mathematics, as a discipline dealing with numbers, should be introduced into education, and it has remained so to the present day. Mathematics is the subject that students typically study for the longest period of time, compulsory for 12 years up to the school leaving examination. And many even after that at university, but it is also indispensable for a career in science, if we think of quantitative research methodologies.

Today, the National Core Curriculum[1] (NAT) defines the values and competences in mathematics education as follows:

- Orientation in space and time, in the quantitative relations of the world. In other words, the ability to identify a symbolically encoded address, time, and to choose the most optimal solution from a myriad of possible approaches.
- Cognition, experience, imagination, memory, thinking, organising and using knowledge. It is this inner unconscious representation that enables us to acquire and assimilate new knowledge. It is what enables us to process existing information in a complex way, to think about it and to draw conclusions about the future and make decisions accordingly.
- Applying knowledge, recognising analogies.
- Troubleshooting and solutions. To learn about a particular type of problem through a specific task, the solution of which can be generalised to make it easy to perform similar types of tasks.
- Creativity and creativity. It goes back to the imagination, i.e. by observing the world on the basis of existing information, one makes new observations that did not exist before, but is able to match certain information through analogies that lead to new observations.
- Volitional, emotional, self-development skills and values related to coexistence. These include communication, cooperation, motivation, self-awareness, self-evaluation, reflection, self-regulation. These are perhaps the most important, because in one's daily life and relationships, it is important for one's health to be able to see cause and effect and to be able to think ahead.
- The principles of building mathematics through which analogies can be made with all the competences.

It can be seen from the previous definition that mathematics, in terms of its directions, contributes to the development of countless areas that do not point in the direction of calculation and rationality in the classical sense. In many areas, mathematics is an essential part of personal development and also provides a literate foundation for the ability to think. If we look at it in this way, mathematics is one of the main foundations of intellectual development and of a person's decision-making skills. Thinking skills can

[1] https://www.nefmi.gov.hu/kozoktatas/tantervek/nemzeti-alaptanterv-nat

be classified in a number of ways, but they can be divided into three groups according to which we have measurement information:

(1) operational thinking,

(2) inductive reasoning,

(3) problem-solving thinking.

According to Jean Piaget (Csapó, 2012), the creator of operational thinking, knowledge that underpins intellectual development has its origins in interaction with the environment, in experience gained from active activity. According to his theory, this development can be described as a series of so-called stages, which he links to childhood. In practice, they represent different stages, which he distinguishes according to the means available to the child/adolescent for processing and organising information from the environment. The latter are called schemas and, according to his theory, at a given level we are only able to take in information that is consistent with our schemas. In this sense, a processing process can take two forms:

1. Assimilation, when the processing is successful;
2. Accommodation, when there is a mismatch with our schemas and a so-called cognitive conflict arises, leading to a rearrangement of our current schemas.

The latter can be induced through learning, thus stimulating development.

Two further theories of Piaget are closely related to this line of thought. The first is the theory of stages (Piaget, 1993) in which he distinguishes four main stages. The first is the sensory-motor stage, which covers the first years of life. The second is the pre-operational stage. The third is the concrete operations stage. The fourth is the formal level. The last three levels are all in the period of public education, so that the direct experience of the institutional system is decisive in their experience.

The second theory, known as the theory of structures (Piaget, 1970), is an approach that puts the operational structures of the mind into a system that can be described by mathematical means. That is, it describes the thought processes that occur in everyday activities, learning and creation in everyday life, which can be represented by appropriate applied mathematical domains. These include the theory of relations, set operations, mathematical logic, combinatorics, probability theory and geometry. Piaget's research has shown that there is a high degree of similarity between the basic operational structures of mathematics and reason.

Inductive reasoning theory (Csapó, 2002), unlike operational reasoning, does not rely on a clearly definable precise structure. In terms of its psychological background, several approaches are known and they all have in common that, although the steps of the inductive process lead to new thoughts, the result of the process itself can be disjointed, because different (even contradictory) conclusions can be reached from the same starting point. The interesting thing about inductive reasoning is that its process can be paralleled with deductive reasoning. However, while deduction is a set of well-identifiable operations which, if applied correctly, can also lead to true statements from true premises, this is not necessarily true for induction. Induction demonstrates a kind of intelligence that is intrinsic to the thinking process and to human cognition, but the fact that it links information together does not make it certain that it does so correctly. However, what it undoubtedly has in common with deduction is that it tries to think in terms of some kind of regularity and often uses so-called analogies in the thinking process.

Problem-solving thinking (Csapó, 2012) occurs when a situation arises in which known solution patterns are not applicable or are hindered by certain factors. The decision-maker is faced with a new situation in which his or her previously acquired knowledge and experience cannot provide direct assistance. These are so-called complex or intransparent problem situations, for which several approaches have been developed.

According to the Gestalt psychology approach, humans are able to see through the structure of a problem and then restructure it in order to solve it. In this form, the process itself is both productive and reproductive, where the so-called "ah okay I get it-experiences", which are the main indicators of restructuring and insight, are also of particular importance. György Pólya interprets problem-solving thinking as a two-way or cyclical process (Pólya, 1978). According to him, problem solving of a mathematical nature can be divided into four stages:

(1) Identify and understand the problem,

(2) formulating the problem and making a plan,

(3) the choice of strategy and the implementation of the plan,

(4) testing the solution.

These are the mindsets that most determine our reactions in a situation and the driving forces behind the action process. However, it is true for all of them that it is the educational environment that determines their development and frequency of use.

If we look at domestic studies in the field of mathematics education, four major groups can be identified (Csapó, 2012):

(1) Mathematical knowledge tests,

(2) Mathematical competency tests,

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(3) mathematical problem-solving and problem-solving tests,

(4) testing basic mathematical skills.

For the purposes of the present research, two significant studies should be highlighted.

The first one comes from a set of competency studies (Balázsi et al., 2005), which distinguish four levels of ability:

1. Level 1: The learner is able to solve simple, familiar problems and carry out well-practised calculations;
2. Level 2: The learner is able to understand simple problems, visualise simple data, apply familiar procedures and interpret and perform simple operations on data presented in different ways;
3. Level 3: The learner is able to formulate ideas for the mathematical interpretation of situations, to choose the appropriate strategy for a solution, to apply models and to identify the conditions for their application;
4. Level 4: The learner is able to create an independent mathematical model, to think in a novel way and to communicate the context in which it is solved.

The second relates to the testing of basic mathematical skills (Nagy, 1973). These tests measure students' abilities and performance in the following areas:

- the basic operational count;
- counting and quantitative interpretation;
- inference and relational vocabulary.

A common feature of the tests is that they look at existing levels of knowledge and skills. It is important to underline that, as a result of competition between educational institutions, these studies are also carried out to a significant extent at local level, because the skills with which students leave the institutions concerned are important for the maintainers.

Studies conducted in the field of mathematics in Hungary have revealed that the perception of the subject itself, the mathematical self-image, falls into a strongly negative category (Csapó, 2012). This, together with the additional results observed during the measurements, has pointed to a new direction for research. While previously the study of cognitive domains had been almost dominant, in recent decades research on the influence of affective and motivational domains has also developed (Józsa & Fejes, 2012). These studies should be treated with special attention in the context of the present research, as Aschcraft and Krause (2007) pointed out that when students fail in mathematics class, performance anxiety can immediately appear, blocking the working memory of our brain. And working memory plays a key role in arithmetic skills (Márkus, 2007)

József Nagy, in his complex understanding of motivation, defines its concept in four steps (Nagy, 2000):

(1) to make an interest assessment and interest decision,

(2) interest assessment and interest decision,

(3) indication of interest and

(4) stimulate activity.

The motives that play a role in the initiation, maintenance and performance of motivated behaviour include, among others, different personal goals, attitudes, beliefs, convictions, personal norms and values. The approach that considers two basic motivations is also determinating (D. Molnár, 2013; Linnebrink & Pintrich, 2001; Fejes, 2011), the (1) mastery motivation, which seeks understanding; and the (2) achievement motivation, which focuses on achievement and recognition of others. By arranging the latter two motivations in a two-by-two matrix and aligning them with the orientations, Linnebrink and Pintrich (2001) arrived at the goal categorization presented in Table 1:

Table 1: Motivation and orientation matrix (own editing)

	TRANSMISSION MOTIVATION	PUBLICATION MOTIVATION
AVOIDANCE ORIENTATION	**AVOIDING THE MASTERY TARGET** (AVOID NOT TO UNDERSTAND)	**AVOIDANCE PERFORMANCE TARGET** (TO AVOID A POOR RATING)
APPROXIMATE ORIENTATION	**APPROXIMATE LEARNING TARGET** (THE BETTER TO UNDERSTAND)	**APPROXIMATE PERFORMANCE TARGET** (ACHIEVE A GOOD RESULT)

These orientations can also be well observed in management situations where a data-driven decision is made by looking at how the managers concerned react to the quantified values that support the option. They may simply overlook them, acknowledging that there is analysis behind them, but not risking that they may not understand or understand it. Or they consult with the author of the analysis concerned and ask about any values that are not clear as to how they were arrived at, sometimes even questioning their methodological or ad absurdum relevance to the decision.

One of the most interesting things about learning maths is that it is a subject that has an impact on you for a very long time. If we look at public education, we encounter it almost every day. However, the results of mathematics learning, given the theoretical aims of education, can also support a wide range of areas and challenges in our lives. Therefore, it is interesting to see what mathematics learning achieves, what competences and knowledge it gives to students.

4. The presence of mathematics in the development of decision science[2]

The mindsets described above are the ones that most determine our reactions in a situation and the driving forces behind the action process. For all of them, it is the educational environment that determines their development and the frequency with which they are used. It is in this context that mathematics, which is present in primary and secondary education, has or can have a major impact on our lives in the most unexpected areas, such as management skills. Competitiveness research going back several decades has highlighted (Zoltayné & Szántó, 2011) that financial managers who have a stronger mathematical background tend to be a tad bit better than other managers in terms of their problem-solving skills. The same research also showed that more prepared managers tend to be more proactive in responding to change and perform better than average. In this context of corporate decision making, one of the most dominant, used and monitored attributes, both within the company and by the market and public authorities, is financial/economic performance. In the business world, there is hardly a decision situation that does not involve some form of financial analysis. These analyses are part of the warning systems that operate within the company. As such, analyses that are objective, quantified and therefore rational, play a key role in decision making. At least at first sight, one would think that they are, but the reality is much more complex.

Macroeconomics and microeconomics are the two core subjects for students in economics courses. The former deals with industries and the economy as a whole, the latter with individual economic agents. Mathematicians, statisticians, engineers, physicists and economists are among the leading thinkers in these disciplines. All of them have a strong classical mathematical approach in their thinking and have assumed that

[2] Extract and translated from own study, with minimal modifications (Farkas-Kis, 2022)

the behaviour of the agents in their models is rational and guides their decisions. But what do we mean by rational behaviour? It is commonly believed that a good decision is one that is rational. According to the normative decision theory approach (Neumann & Morgenstern, 1955), the axioms of rationality are:

(1) comparability, i.e. the decision-maker is able and able to choose between two alternatives, based on the information available;
(2) transitivity, i.e. if alternative B is better than alternative A and alternative C is better than B, it follows that alternative C is better than alternative A;
(3) dominance, i.e. if two courses of action are equally preferred by the decision-maker, he or she will choose the one that has at least one state of affairs in which it is more preferred than the other; and finally
(4) independence, i.e. the utility and probability of the outcomes of decisions are independent of each other.

The four axioms show that mathematical perspective, mathematical logic and discrete mathematical concepts are constantly present in rationality approaches.

However, the weakness of the former axioms - and thus the weakness of the feasibility of rationality - is that there are serious doubts about the first axiom. If we think of real life and utility as the central guiding objective of classical decision making, we already run into problems. Who, what and to what extent is considered useful is in many cases a matter of subjective judgement. Because utility is not an intrinsic, objective property of things, independent of circumstances, but depends on the effect it has, i.e. it varies from individual to individual. But if rationality is not satisfied in this respect, the question arises: what is it that guides decisions? Is it really the subject's personal experience, his feelings? How do they relate to each other? In terms of decision-making, this questioning has accompanied the development of human culture throughout the last 6000 years.

Already in antiquity, the classical approach of rationality as a gift from God, separating us from the animal world, appeared. We can rise above our feelings and see through situations and make the right decisions. Plato illustrated this dichotomy (Plato, 1984) by saying that our brain is a toothpick with the reins in its hand. And of the two captured horses, one is a noble, well-behaved horse, the other is a rambunctious, doubtful and hardly obedient one. The driver's job is to tame these horses. This dual mindset is very much ingrained in Western thinking and is still felt in our culture today.

Descartes, in the Age of Enlightenment, took this line of thought further and explicitly criticized the expression of emotions, which he called the mechanical passions of the body (Lehrer, 2012). In his "Discourse on Method", he seeks to present a pure form of rationality with the aim of leading humanity out of this prison and revealing the clear and lucid principles along which we are cleansed of what emotions and intuition hide from our eyes. From the Enlightenment onwards, reason and faith in it became increasingly widespread. Francis Bacon and Auguste Comte believed in a society transformed by reason and science (Lehrer, 2012). Thomas Jefferson hoped that man could live guided only by reason (Jefferson, 1903).

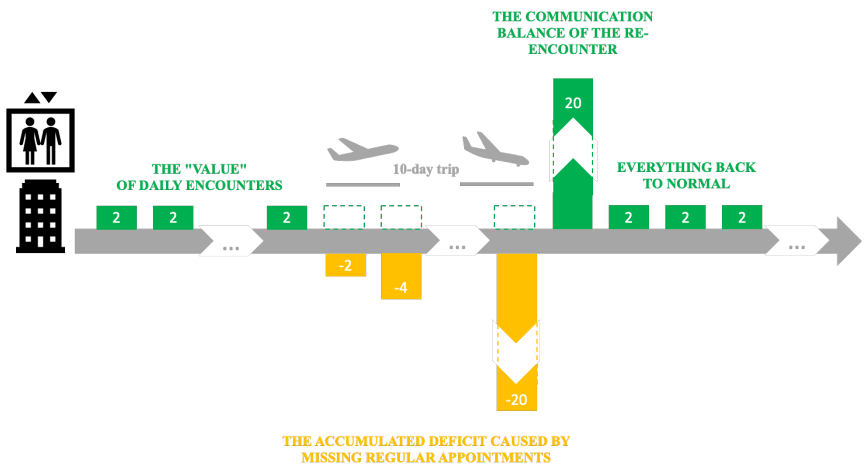
And the 20th century, with the advent of computers, gave a new impetus to this approach to the mind, claiming that our brains are nothing more than a set of programs running on hardware. However, it should be remembered that computers have no feelings for the human brain. In other words, according to the cognitive approach, rationality trumps emotion, and obviously, if our emotions did not interfere with our decisions, we would certainly be much further ahead socially. This idea has been elaborated and used as a premise for a utopian vision in countless pop culture films over the last 20-30 years.

It is also important to stress that when we talk about rational decisions, it is inevitable to mention mathematics. Even if we leave out the IT domain, mathematical logic is almost entirely present in rationality narratives. Moreover, thanks to the development of quantification and computer science, applied mathematics, in addition to being a driving force in the natural sciences, has also become a dominant force in economics. Analysis, statistics, probability theory and stochastics are all mathematical fields whose development and research have led to the emergence of modern economics and management systems. And the increasingly sophisticated computer technology that has accompanied them has made it possible to model economic problems with sufficient complexity, and today there are countless management, analytical and data-processing systems in operation which many people who use them on a daily basis regard as black boxes and simply accept their existence and the results they can produce. After all, they were created rationally, so it is impossible to question them.

A common element of the theories of the classical thinkers is that they start from a view of man that is capable of objectively assessing his environment, of knowing the alternatives. However, this vision ignores the additional factors of human perception and attention, memory, problem solving and decision making. For the perceived world can differ significantly from objective reality. And all such deviations are reflected in omissions and distortions in the practical context of our decisions, which are always based solely on our perception and the conclusions we draw from it. Thus, the basic principle of rationality is already compromised at the time of perception: the perceived reality, the multitude of stimuli/information perceived when comprehending phenomena, passes through the central nervous system and there, as part of an active process, we do not perceive everything, only what comes to our attention. In other words, all information is perceived through the filter of the person, which immediately gives a subjective quality to any problem solving and decision making process.

At the same time, however, a kind of measuring system still seems to operate unconsciously in communicative situations, measuring the value of social interactions, for example. Eric Berne has observed (Berne, 1984) that in some cases peer interactions are highly programmed, using underlying calculations. The communicating parties in a given situation engage in a certain amount / point-value transaction in the process of information exchange. If this occurs on a regular basis, for example because the two parties are neighbours, then it becomes embedded in their daily routine. In this case, for example, a "Good morning!", "You too!" situation on the way to work is considered a subconsciously determined point-value transaction, which should be e.g. 2 points. If one of the parties leaves home for 10 days, when they meet again on the morning after their return, they face the deficit of these 10 days, which is 2x10, i.e. 20 points. This deficit is "accounted for" by both parties unconsciously. Thus, on that morning, instead of the usual 2-point transaction, they will mutually recover the 20 points worth of interaction: "How have the last few days been?", "What was the weather like?", etc. The conversation will continue until the 20 points are mutually credited. Then, from the next day onwards, everything goes back to business as usual, the familiar daily greeting routine (Figure 2).

Figure 2: Transactions during social contact (own editing)



This subconscious functioning, as a kind of internal accounting and controlling, is also reflected in the level of problem solving and alternative reasoning that precedes our decisions. Among others, Thaler observed (Thaler, 1985) that consumers often behave in a way that does not conform to economic models. They often pay attention to sunk costs when they should not, or underestimate benefit costs relative to costs already paid. The phenomenon is called mental accounting and manifests itself in accounting for different events separately, rather than in aggregate.

So people do not follow normative decision-making standards in their daily lives or at work, even when expert, i.e. rational, decision-making is expected. This is a particularly important and interesting issue in business. A study of rational and intuitive approaches to decision-making revealed interesting correlations (Zoltayné, 2010). In a ranking of managerial skills (where 11 skills were examined), the ranking by managers resulted in problem-solving skills typically being in the top three, while analytical skills were in the bottom three. On the other hand, intuition typically appears at several points in managerial decision making when recognising the existence of a problem, in routine situations, when synthesising information and when a workable solution had to be found quickly. Similar experiences have increasingly drawn attention to the importance of psychological and behavioural analysis of decisions. However, in parallel with this research, it has also been observed internationally that the learning of mathematics and people's self-concept about mathematics are strongly negative (Artelt et al., 2003). And if we take the above into account, a closer and closer relationship between mathematics learning, numeracy and rational decision making is beginning to emerge.

5. The link between quantification and decisions[3]

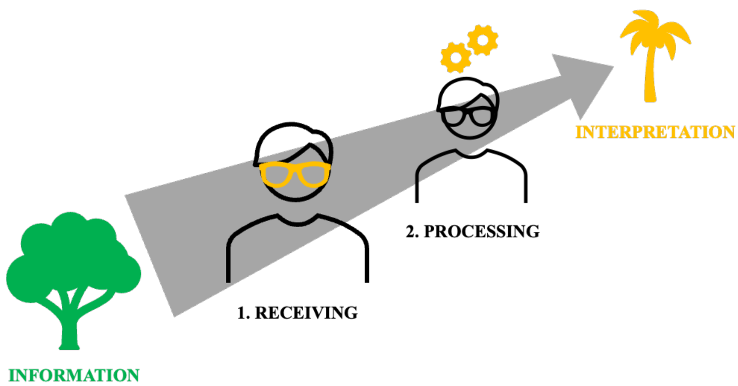
As it gradually became clear that rationality could not be achieved in the context of decisions, the theory of bounded rationality began to gain ground (Zoltayné, 2005). At the heart of this approach is the acceptance that the human brain's capacity to absorb and process information is not infinite. Three things follow from this in the context of choices between alternatives. The first is their sequential treatment, the practice of considering alternatives sequentially, one after the other, and if an alternative is identified that satisfies the criteria, it is retained, and if it is sufficiently satisfactory, the search for an alternative may be stopped. If more than one alternative is identified (in which case it is important

[3] Extract and translated from own study, with minimal modification (Farkas-Kis, 2022)

to emphasise that not all possible alternatives are still possible, only one or more are preferred), then filtering and elimination begins based on the elementary criteria of each alternative. The second is the use of so-called heuristics, rules of thumb, which direct attention in the search for alternatives to areas where there is a high probability of finding a satisfactory solution. The third is satisficing, which means that decision-makers are not looking for the optimal solution, but for the satisfactory one. The main reason for this is that if there are a large number of options to be taken into account, one may, after a while, especially if one does not find a solution close to the optimal one, settle for a satisfactory alternative, which may not be the most satisfactory, but is acceptable.

It can be observed that in all three steps, the focus is on the lowest energy expenditure that can be achieved in solving the problem or making the decision. It is also interesting to note that when we ask decision-makers what decision methods they use, they typically answer that they make rational decisions that maximise benefits. However, practical observations have shown that this group more often follows the path of bounded rationality (Zoltayné, 1999). The main reason for this is that in reality decision-makers are forced to accept satisfactory solutions because of their cognitive limitations. These constraints arise at two significant points in the problem-solving and decision-preparation phases: (1) when information is received and (2) when information is processed. In this way, it can be assumed that the success of decision making, the finding of the best decision, cannot be achieved (Figure 3).

Figure 3: The limits of perception (own editing)



According to Tversky and Kahneman (1991), these barriers are almost insurmountable. At the same time, their work shows that these phenomena often lead us to the right solution, not the wrong one. In fact, according to Tibor Engländer's approach (Engländer, 1999), these characteristic "errors" play an important role in the biological adaptation of the individual. In his view, estimation biases, heuristics, are constantly corrected by thinking (Vranas, 1999). This is because in some situations, the best solution is not the one to be applied, but simply to survive, i.e., a satisfactory solution is actually needed, there is no time to consider, to optimise.

Because of heuristics, our decisions can be manipulated (Thaler & Sunstein, 2008). Since we still want to maintain the appearance of rationality, we almost invariably turn to mathematical solutions, i.e. we quantify everything. We develop and use decision support and data management systems to make decisions legitimate. At the same time, we fail to realise that in many cases we are serving the very heuristics we are trying to avoid. When a phenomenon or a feeling is presented in the form of a number, we look at it in a completely different way. The numerical representation of a probability may immediately lead to an incorrect probability estimate, or the other party may immediately anchor at the amount of bargaining power precisely because of the heuristics. What this means is that if a manager does not understand a particular numerical indicator and its calculation mechanism, then when these values appear in a managerial report, they tend to impede decision-making (Tirnitz, 2012).

To summarise, our decisions can be interpreted along two main dimensions, which also significantly determine the process of their preparation. One dimension is whether the decision is emotional (intuitive) or rational. The other is whether or not quantified values are attached to the decision (Table 2).

Table 2: Quantification of emotional and rational decisions
(translated from own editing; Farkas-Kis, 2022)

APPROACHES	EMOTIONAL	RACIONAL
NOT QUANTIFIED	**DECISIONS BASED ON OUR INTUITION** *EXAMPLE: CORPORATE DESIGN*	**DECISIONS BASED ON EXPERIENCE** *EXAMPLE: COMPLIANCE*
QUANTIFIED	**SELF-CERTIFIED DECISIONS** *EXAMPLE: STRATEGY*	**DISCRETIONARY DECISIONS** *EXAMPLE: ACCOUNTING*

If we try to think through the decisions along these two dimensions, we can identify four main areas, as shown in Table 2:

- The first type includes decisions based on intuition, which we make emotionally without any quantified information. An example is the adoption of a corporate image. Here, the subjective judgement of the decision-maker or decision-makers mostly determines which appearance is adopted.
- The second type is the self-justification decision, where we try to justify a specific emotional decision by assigning a numerical value to it. This is usually the case for strategic decisions, which are often informed by data in such a way as to justify the decision. This type is best characterised by inductive thinking.
- The third type is decisions based on experience. These are typically based on rational reasoning, based on events and conclusions drawn from similar decisions in the past. A typical example of this might be decisions taken in relation to compliance tasks. Processes are developed rationally, but experience is the basis for deciding which steps to follow. This type is closest to problem-solving thinking.
- Finally, in the fourth type, discretionary decisions, decision-makers use well-described, quantified information for each alternative, ranked according to preferences. The best examples of this are the classic areas of corporate governance, finance and controlling, such as accounting. A decision in this category is typically the result of a deductive reasoning process.

A leader cannot think along the lines of just one type of approach. It can be observed that the preferred type in a particular situation depends on the situation or problem. This also means that, to the extent that our particular state of mind determines the quality of our decisions, the circumstances also determine the state of mind we are in at the moment of making a decision. There are a number of approaches behind human behaviour and our understanding and judgement of decisions. However, if we look closely, what they all have in common is that they all involve numbers and mathematical thinking. In the classical approach, rationality, this is not even an issue, since all the models that occur there can be described in mathematical terms. The utility function, the optimal solution are all defined on mathematical grounds. In a broader sense, decision-making is based on weights and probabilities assigned to alternatives. And this is where the way we understand numbers, our relationship to numbers, has been shaped by our mathematics education, becomes of particular importance again. Our perception and ability to interpret probabilities is related to our ability to deal with decision situations (Peters et al., 2006). In a given problem situation, individuals with higher numeracy skills are more likely to search for and use the appropriate numerical principles than those with lower skills. Furthermore, if there is no meaning assigned to a given number, decision makers simply do not use them directly in their decisions (Peters, 2012). This in turn also leads to a positional advantage for managers with higher numeracy.

At the same time, the research findings of bounded rationality make numbers and representation through numbers inescapable. In the case of anchoring heuristics, numbers have a particularly strong influence on the way a decision is made. Kahneman and Tversky's experiments have shown (Kahneman, 2012) that numbers, even when they seemingly have nothing to do with the decision, can influence our decisions. Therefore, numbers can also legitimize intuition-based decisions (Zoltayné & Farkas-Kis, 2021).

6. Interpreting data visualisations from a mathematical perspective

Skovsmose (1994) describes different types of knowledge in terms of the role of mathematics in society: (1) mathematical, (2) technological and (3) reflective knowledge. While reading and comprehension of data visualizations are at the level of mathematical and technological knowledge, reflective knowledge is necessary for critical interpretation of the information in front of us. The latter is key to identifying the role of mathematics in the creation of data visualisations linked to the decision-making process. This includes the assumptions and decision alternatives for a given situation, the embedded uncertainties and the intended or unintended consequences of mathematical inferences. Reading with a critical eye offers a strategy for reflecting on data visualizations through rethinking.

The role and importance of data visualisations can be illustrated by the example of the pandemic COVID-19, where the communication of crisis information was a key factor in influencing both policy and individual decisions, globally. As more and more descriptive/explanatory graphs appeared, they were linked to narratives that could not be understood without mathematics: understanding exponential growth, the importance of modelling, the predictions that follow from statistical analysis, and so on. All of these, by implication, provided opportunities to teach and learn mathematics. And at the same time, one cannot ignore that these data visualisations are also social narratives, created by their creators from a particular perspective with the aim of telling stories (Laurei et al, 2021) and creating impact.

At this point, we must return to where the social role of mathematics began: quantification and our fundamental reliance on mathematics reinforce the social function of data visualisations. People assume this information to be objective, reliable and neutral. Reading data and interpreting graphs, however, requires unpacking the role of mathematics, including how data and variables are represented and how relationships are framed to tell stories from a particular perspective.

Several studies have investigated (from multiple perspectives) how people arrive at interpretations of data visualisations (e.g. Börner et al., 2019; Galesic & Garcia-Retamero, 2011; Glazer, 2011; Lee et al, 2016; Shah & Hoeffner, 2002). Research on mathematics education has highlighted how people learn to identify information from graphs and other representations, discover relationships and trends, make predictions, explore connections with broader contexts, question or make statements about inferences and claims (Ainley, 1995; Ben Zvi & Arcavi, 2001; Curcio, 1987; Friel et al, 2001; Shaughnessy, 2007; Watson, 1997; Watson & Moritz, 1999).

The pandemic has brought this problem and mathematical thinking back into the spotlight of research. Several studies have specifically examined how people interpret data visualisations in light of their behaviour in response to the COVID-19 pandemic (e.g. Rotem & Ayalon, 2021). For example, Romano and colleagues (2021) found that data visualisations presented during the pandemic that used logarithmic scales misled many people and adversely affected individual and community decision-making.

For data visualisations to be understandable and meaningful, it is necessary to be able to provide students with the right level of mathematics education. Gal (2002) points out that when interpreting a figure, one cannot ignore the so-called "reading context", which is linked to the authors and includes the accompanying text and the main title. Laurie et al (2021) have developed an approach to critical reading and interpretation of data. In the context of mathematics education, the interpretation/construction of visualizations relies on three interrelated elements:

1. The mathematical formulation, i.e. what we measure and how, and what we will quantify;

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2. The framing, i.e. how the variables are related to each other and how we will display this;
3. The narrative, i.e. what we want to say with a given data visualisation, whether there are potential effects and constraints that affect the visualisation.

Mathematical formatting and framing is nothing more than organising structures for specific data interpretations. It prepares the narrative for later. The types of data available shape the possible mathematical formulations and framings, setting certain directions. The processes of mathematical formulation, framing and narrative are interactive in the way they interact. Reinterpreting, reframing or re-narrating data visualisations through these can result in what Skovsmose (2018) calls 'hypothetical reasoning': asking active questions that lead to a rethinking of the data visualisation.

"Data cannot speak for themselves", but instead are submerged in narratives that give them form and meaning (Loukissas, 2019), and are woven into a process whereby "at different moments, for different purposes, they tell different stories" (Dourish & Gómez Cruz, 2018). In this way, numbers and their mathematical interpretation, through data visualizations, both "mathematize" reality and transform data into a narrative through interrelated processes of formatting, framing and narration, and are not at all neutral representations.

III. HORIZON - RESEARCH QUESTIONS AND HYPOTHESES

Mathematics is multifaceted and intertwined across disciplines, appearing in almost every theortical field and in our everyday lives, sometimes unnoticed. However, numbers are always present, and the way they are quantified and the way they appear in personal, corporate and consumer decision-making is of particular interest. To make sense of these numbers, we need cognitive mathematical skills. Within this complex, interconnected system, the following research questions arise:

- How does the level of mathematical literacy influence our decision-making approaches?
- Which mathematical areas and competences can be traced back to different decision situations at individual, consumer and company level?
- How does the development of cognitive mathematical skills affect problem solving, decision-making and behaviour?
- How does understanding mathematics affect self-esteem and thus assertiveness in relation to our decisions?

The aim of the research is to understand the relationship between mathematical competences, numerical representations and decision making, with the aim of demonstrating the close relationship between the two, and furthermore to identify a new direction in mathematics education, in the development of mathematical competences, which can provide useful knowledge for everyone. Accordingly, the research also aims to understand and understand the community perceptions of mathematics, which are most affected by time spent learning mathematics. In line with these principles, the research hypotheses are:

H1 Cognitive mathematical skills are related to problem-solving and decision-making skills:

H1a The more educated someone are in relevant mathematical skills related to their profession, the more successful will be as a problem solver and decision maker.

H1b The mathematical competences acquired have an impact on thinking skills and behaviour, thus indirectly leading to more successful decisions.

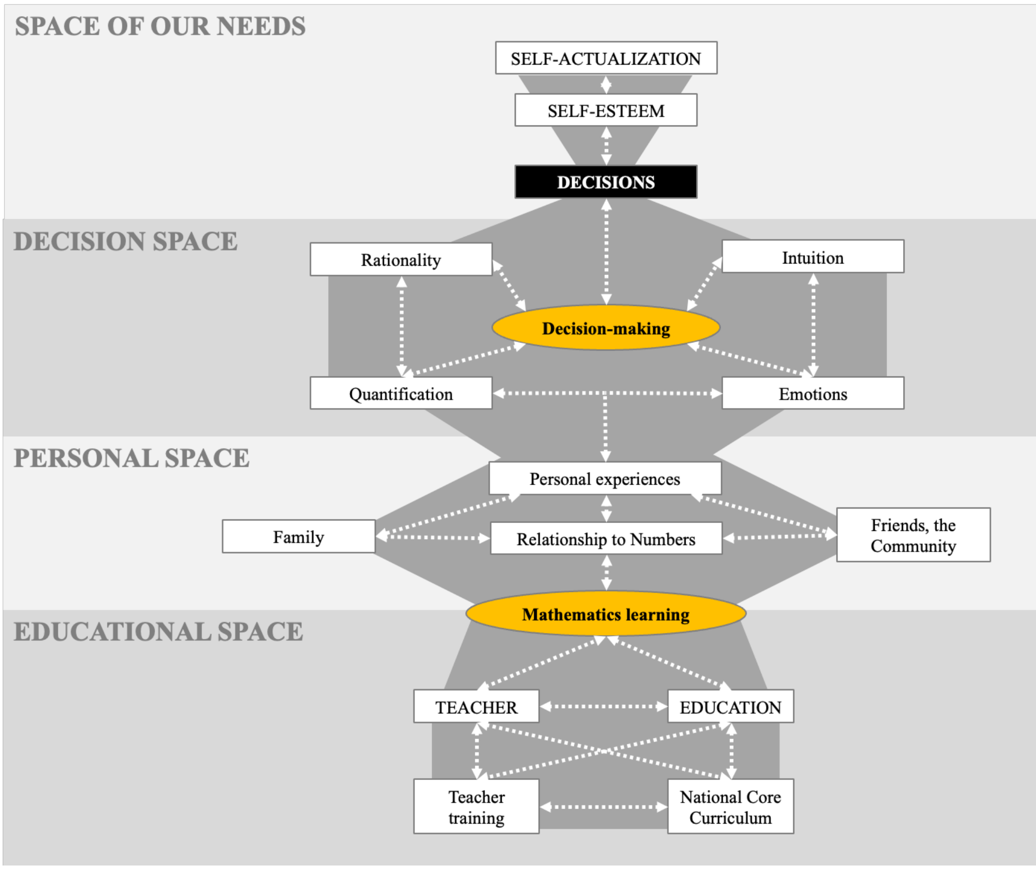
H2 The quality of mathematics education plays a decisive role in the perception of the usefulness of the subject:

H2a Traditional mathematics teaching methodologies are not suited to prepare students in a sustainable way to meet the cognitive challenges of the 21st century.

H2b Innovation in mathematics education can lead to sustainable thinking if it increases the self-esteem of future generations, who will then be able to empower themselves as decision-makers.

The complexity of the research field and the interconnectedness of the elements within it can be assessed through several approaches. In the present doctoral thesis, the focus of the decisions is always on the individual, his thinking, his experiences and his resulting perceptions and actions. To illustrate the holistic and multidisciplinary framework of this research, consider Figure 4. The basic purpose of the figure is to illustrate the interrelationship between the factors that this research - and future research - intends to explore. The direction of the relationships and the extent of their impact are not part of the presentation, only their context is relevant for now, and are therefore shown with double arrows and dotted lines.

Figure 4: The research framework (own editing)



The first four factors under investigation are located in the so-called *Educational Space.* They include:

- The *National Core Curriculum (NAT)* is the legal/regulatory framework that defines the social mission of education, together with its goals and expectations. It defines what students should learn, in what way, and in what structure, according to their age characteristics.
- *Teacher training* is the process of developing the people we later call teachers and trainers. They will be the ones who will interact with students in the context of a given science or subject and support their learning development. Teacher training has two main aims. On the one hand, it is to underpin and develop teachers' subject knowledge. On the other hand, to develop the methodological and psychological competences that are essential for teaching and pedagogical activity.
- The *Teacher is* the educator who directly contributes to the development of the students through his/her personality and skills, while interacting with them in the educational process.
- The *Education is the* space, both physically and in terms of equipment, where the student learns and interacts with the teacher. It includes, therefore, all the professional, methodological implementations that describe the teaching of a given student. For example, the choice of textbook, the furnishing of the classroom, the techniques used by students to solve problems, the way in which they are held to account, etc.

The next four factors and their associated process belong to the so-called *Personal Space.* It is made up of the following:

- *Mathematics learning*, the central process of this space, is the manifestation of the individual in relation to the way he or she approaches the subject, memorises and recalls the material learned in the context of the mathematics subject. This typically includes, for example, whether they memorise formulas, or understand and practise example solving, or whether they struggle or struggle to understand problems.
- The *Relationship to Numbers* collects and describes the feelings that indicate how a person relates to the numbers in different situations in life. The relationship with numbers can be comfortable, but can also become difficult, as their appearance becomes unintelligible or even repulsive.

- *Personal experiences* include all the situations that arise in the context of learning mathematics. It includes the experiences that pupils or students have in the course of example solutions, e.g. in front of the class or in a university examination situation. Also included are all experiences arising from personal interactions with teachers, classmates and friends in the context of mathematics.
- The *Family* and the *Friends, the Community* appear in this space because the experiences they bring, pass on and live have an impact on personal experiences. This is true both ex ante, before a particular event, such as an exam, "don't worry, dad didn't pass, but I'm still somebody"; and ex post, such as "I told you it would be a piece of cake with that super math brain".

The third area, the *Decision Space, is* where people live their daily lives. Here, four factors and one process have also been identified for the research. Its defining elements are:

- *Decision-making*, which is the central process of this space. It essentially includes the events that precede and lead up to the decision as a momentary event, which ultimately result in the decision.
- *Quantification* is presented as a technical process that influences decision-making. It includes all processes whereby, for some reason, numbers are used or called in to influence possible outcomes in the context of a decision before it is taken.
- *Emotions* are the intangible areas that arise in a person in connection with a particular decision, before it is made. They include those feelings that are either based on a past experience or that arise from the situation at the moment. They are typically unjustifiable and are in fact impressions.
- *Rationality* covers all reasoning, thinking and reasoning that is used to support a procedure in a given decision-making process, using logical reasoning and justification, and objective factors.
- *Intuition,* as opposed to rationality, is a way of making decisions that relies on feelings and emotions to justify future choices.

In the model, *Decisions* are self-contained, momentary actions that are the result of past preparation (decision-making) and actions that affect the future.

The fourth and final space in the model is the *Space of Our Needs*. This is the space in which our human needs appear, the two that are the most determinant in the context of human existence and are therefore at the top of the pyramid developed by Maslow (1970), which has been much discussed since then but is essentially unchanged:

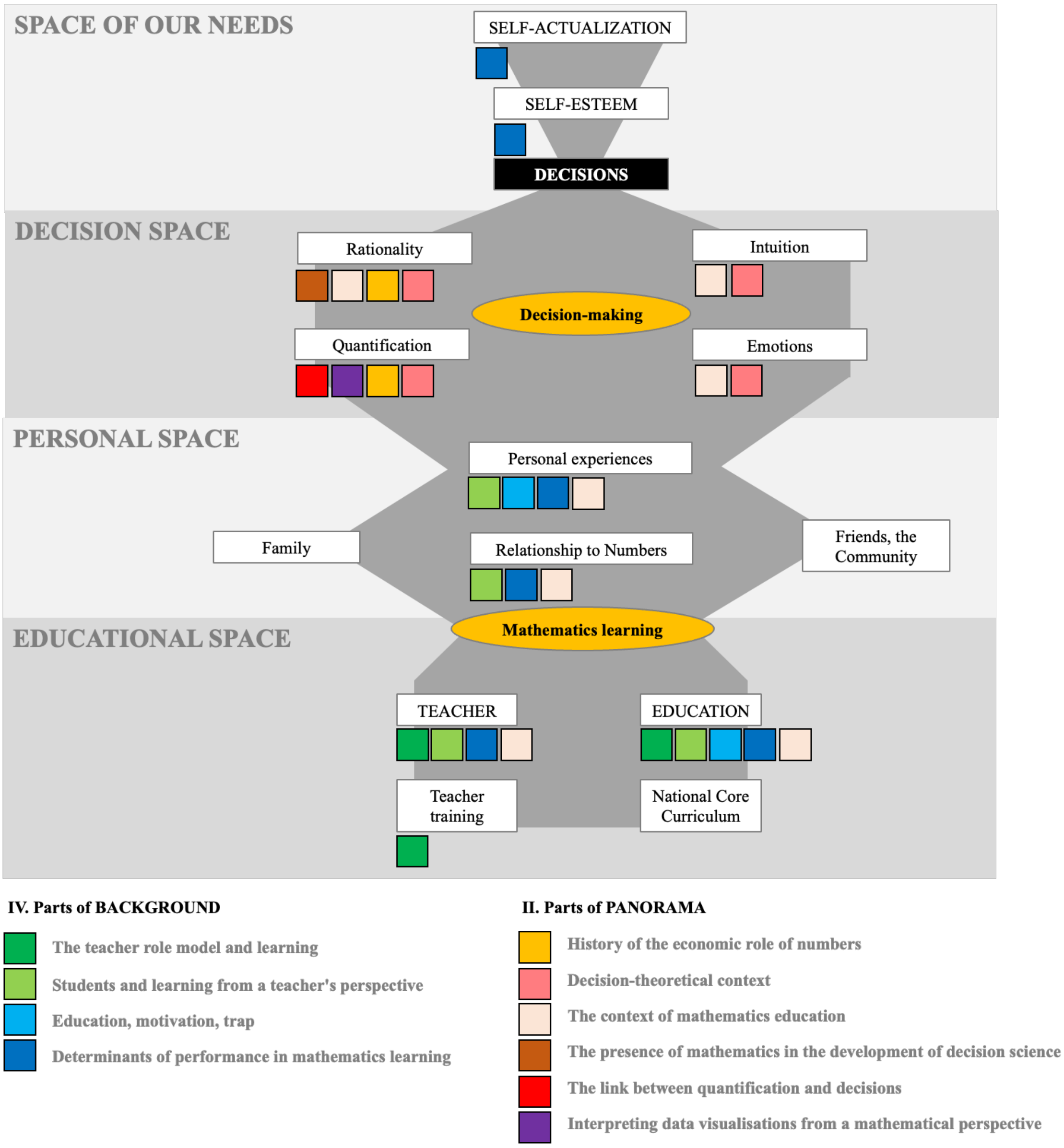
- *Self-esteem*, which is achieved by seeing oneself as a competent and effective decision-maker in problem-solving situations. This leads us to recognise ourselves.
- And *Self-actualization* is the result of the creative force that follows our choices. Those who know and accept themselves are capable of Self-actualization. They are able to respond to community challenges, are open and dare to make choices that are comfortable with the consequences.

The interrelationship between the factors shown in Figure 4 defines the scope of this research and the interrelationship issues that arise in it, which explores the relationship between mathematics and decision making in the context of the experience of individual and community interactions. The research focuses particularly on the impact of past experiences - as determinants of future actions.

IV. BACKGROUND - LITERATURE REVIEW AND ANALYSIS

The exploration of the literature background summarises the dominant theories of the 20th century and the results of relevant research of the 21st century through the themes of the research. The relationship of the sections of this chapter and Panorama II to the research field is illustrated in Figure 5.

Figure 5: Placing the literature background in the research field (own editing)



1. The teacher role model and learning

When people talk about their career choice, you often hear: *'I became an English major because I had a really good English teacher', 'I became a lawyer because I hated maths', 'I became a biologist because I had great biology classes'.*

The choice of career is a crucial - if not one of the most crucial - decisions people make in their lives. It determines the rest of their career. This decision is influenced directly or indirectly by countless factors:

- a closer family community,
- the influence of friends, acquaintances, and last but not least
- the teachers around them.

The latter are the people with whom we have direct daily contact over the years, who introduce us to the world of knowledge and who can therefore have a significant impact on us.

By default, the teacher and the student - whether at primary, secondary or tertiary level - are not in an ordinary relationship. Between them, especially on the part of the instructor, there is a sense of purposeful behaviour. The success of their relationship, their connection, is typically a succession of personal interactions, directed and more or less controlled by the teacher. Moreover, this is in a context in which the level of expectation is extremely high on both sides. On the one hand, the interactants have high expectations of each other, and on the other hand, there are expectations of parents, colleagues, managers, i.e. of those not directly involved in the interactions.

This situation implies that the teaching profession is significantly saturated with internal and inter-role conflicts, which are influenced by two additional factors.

I. The first is the position of power that the teacher is in: his or her decisions and judgements about the student have a significant impact on the student's life and destiny.
II. The second is closely related to the first: the learner is at the mercy of the power exercised by both the personal and the educational system.

Nor should we forget that, despite all this, education is one of the most universal, ancient and defining cultural activities of humanity.

A person's personality is made up of learned patterns of behaviour that are acquired through personal life experience. The patterns he sees around him determine and shape his attitudes. From childhood to young adulthood, most people are obliged to participate in public education. As such, they spend a significant part of their time with educators. The teacher, with his or her whole being and personality, is willingly or unwillingly involved in these educational and training situations, given the time horizon of this interaction. In this way, he or she is also a role model for the students, or, as Gyula Mitrovics puts it, the teacher is a *'role model'* (Zrinszky, 1994).

If we add Thorndike's law to this, it becomes clear that the stimuli that students encounter in the learning process determine their motivation to learn a particular subject. Most of these stimuli come from interactions with the instructor, in the case of mathematics in most cases for a minimum of 8 years and often for at least 12 years.

There is another seemingly small but significant additional effect on role definitions. As a result of social development, the teaching profession has had to face a new phenomenon as disciplines have expanded: the division of its own field. Teachers often joke that their subject is the most important, and give validity to this in various ways, sometimes with tougher examination requirements. This conflict is not only about profound differences arising from differentiation in the field of work, but also about hierarchical conflicts within a subject between teachers from institutions with different sociological characteristics. Although the principles set out in the *'Bologna process' in* higher education seek to eliminate this, we often hear that what is a B in one university or college is a B in another.

Role perceptions are naturally influenced by the structural characteristics of education. The studies of Júlia Kériné Sós draw attention to the particular stratification of the teaching profession (Kériné, 1969). She basically divides teachers into three groups:

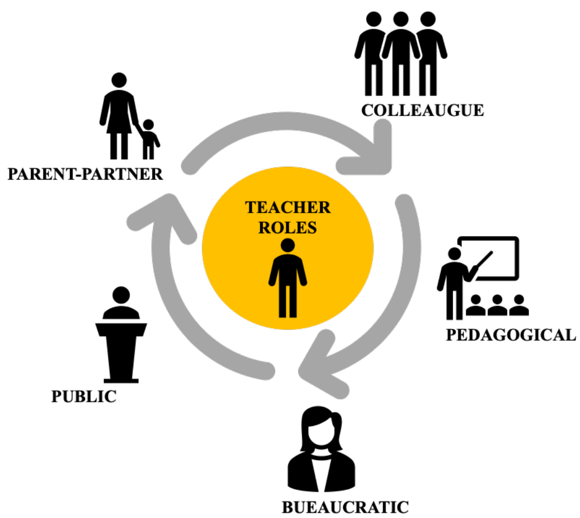
I. The first is the primary school teacher, who is expected to be *"pedagogically"* sensitive and to teach using high-quality methods.
II. The second is the university lecturer, who is expected not only to be a good teacher and examiner, but also to be a high quality representative of his or her discipline. Just think that nowadays you cannot teach in higher education without a doctorate.
III. The third, the secondary school teacher in the middle. He is the bridge between the two.

Given the context of the teaching position, the teacher's position means that he or she transmits values through multiple and complex channels, resulting in multiple ways of being present in everyday situations. Firstly, in individual and collective interactions, as a *"parent"*, as an educator: the older, the more experienced, the wiser, the one who gives advice in conflict situations and to whom students can turn with their problems. On the other hand, he is seen as an expert: someone who knows more than his students in a given field and is thus respected and respected. And thirdly, as an administrator: someone who manages the administration, documents the progress of his students, reviews their level of knowledge and produces statistics.

Teacher roles are and can evolve accordingly. According to Helga Lange-Garritsen's approach, five roles can be identified (Figure 6) in relation to the teaching career based on a structural approach (Lange-Garritsen, 1972):

I. The *"pedagogical role"*, which encompasses the teacher-student relationship.
II. The *"colleague role"*, which defines the relationships between teachers in the institution and shapes the school atmosphere, which has a significant impact on the students' sense of comfort.
III. The *"bureaucratic role"*, in which teachers are seen as public servants with administrative responsibilities and a dependent relationship with the employing institutions.
IV. The *"parent-parent role"*, which defines the common goals of parent-teacher and lays the foundations for a collaboration that offers a more positive and unique opportunity for the fulfilment and intellectual development of each student.
V. The *'public role'*, which Lange-Garritsen (1972) takes for granted in a democratic society.

Figure 6: Five roles according to Lange-Garritsen (1972) (own editing)



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Clustered around these roles, a trend has emerged over several decades and has become almost universal in public education, in which the teacher's role as educator is first and foremost to transmit values and develop personality. In Hungary, for example, this is achieved by the method of frontal teaching. Although the last thirty years of the regime change have seen many changes and transformations in education and the role of the teacher, this type of organisation has nevertheless remained in most institutions. As a result, the teacher is the protagonist of the classroom and education is organised in a uniform way.

Nevertheless, the transformation of roles has started. Thanks to the change of regime, new, more or less neglected role concepts have started to come to the fore in this country too. As early as 1989, Ottó Mihály and his colleagues stressed that *"differences in values and patterns of behaviour are not an obstacle to successful socialisation, but rather a precondition"* (Mihály, 1989). With the emergence of alternative schools, the classical roles of teachers were denied to a greater or lesser extent. The new values have partly returned to their Rousseauian roots. The focus of attention to children and young people became the *'here and now'.* The teacher is characterised by an attitude of absorption, of being with his pupils in space and time, and in this way is not merely an implementer of methodological instructions, but is forced to be constantly creative. In such a pedagogical situation, the teacher enjoys maximum freedom, and it is precisely because of this spontaneity that the presence and participation of the personality is more prominent, provided that the teacher is able to do so.

Among the analogical role conceptions, Merril Jackson's distinction of *"remedial"* roles adapted by Joseph Adelson for teachers, which divides teachers into three types, is noteworthy (Pataki, 1976):

I. The *'shaman', a* high-energy, somewhat narcissistic but highly committed educator who embodies the charismatic teacher persona.
II. The *"pastor" is a* dedicated representative of the community and a servant of science.
III. The *"mystic healer"* whose focus is not on the curriculum but on the student, as Adelson puts it, *"I can help you because you are who you are"*.

The role that a teacher takes on is wittingly present in their everyday practice. It determines, either explicitly or implicitly, the interaction with students. The question is how the role that the teacher assumes relates to his or her personality.

Lothar Krappmann distinguishes five main variants in the relationship between *"individual and role"* (Krappmann, 1980):

I. Full identification with the role;

II. Formal fulfilment of role requirements without identification;

III. Acting the part;

IV. Compliance with the role by minimising formal expectations; and

V. The subjective reinterpretation and falsification of the role.

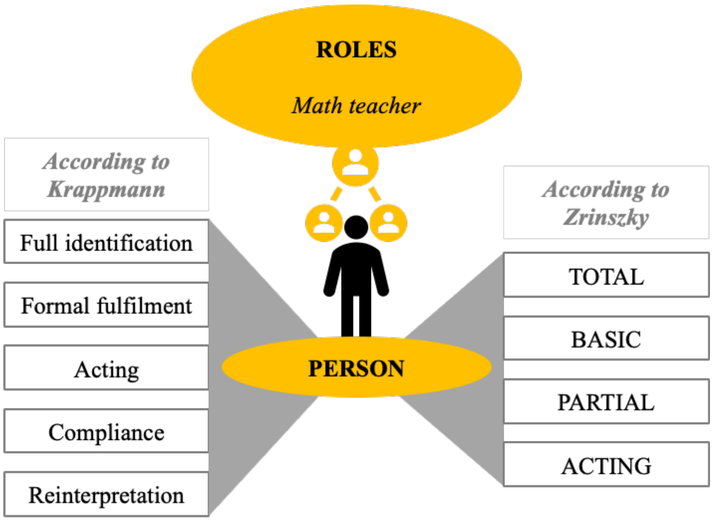
The relational framework defined by Krappmann (1980) approaches the conscious or unconscious relationships with role perceptions from the perspective of the individual. The individual's attitudes significantly determine the relationship he or she can form with the role dynamics in a situation. This relationship, in turn, determines the quality of the instructor's presence in a given situation - in our case, in interactions with students - and the impact of this presence on his or her audience.

As described by Krappmann (1980), it highlights what the individual does with the assigned role. However, the role also has an impact on the individual. In this respect, different roles function in different ways, which Zrinszky's work classifies into the following four categories (Zrinszky, 1994):

1. Total roles: they fill our lives so completely and place such strong expectations on us that we simply can't move away from them, or if we try to, we feel guilty;
2. Basic roles: also have strong expectations of the individual, but they are not as exclusive as total roles (almost any role can become a basic role.) For example, they can be professional roles such as tutors;
3. Partial roles: situational roles that are less framing of the personality. They are more specific to particular situations, in which the role "invites" part of the personality. To stay with education, a good example is when a student gives a small lecture. In this case, he or she is placed in a partial teaching role for the duration of the lecture;
4. Acting roles: typically these roles are not identified with our personality, but are imitated or symbolically represented (for example: acting).

Role interpretations can therefore be approached from several directions (Figure 7), but there is no doubt that the roles we have to assume in our lives affect us fundamentally. They thus directly or indirectly, but to a significant extent, determine our self-image and bring to the surface the problem of self-actualisation and self-abnegation.

Figure 7: Role interpretations according to Krappmann (1980) and Zrinszky (1994) (own editing)



The conflict between self-image and roles in the teaching profession frames a fundamental dilemma: *"how do I appear to my students"*. The former begins to blend inexorably with the self-image: *'what I want to be like'* and the self-image: *'I want to be seen as such'*. As a result, my internal perception and my external perception are constantly confronting each other. And the conflict that emerges is defined by the dilemma of the roles that modulate and enter into the continuous basic role on the role perception side: there can be a contrast between the *"natural"* and *"contrived"* behaviour of the instructor. This opposition has an impact on the quality of his teaching and thus on the motivation of his students.

2. Students and learning from a teacher's perspective

In the learning process, the stimuli a student receives determine his or her motivation to learn a particular subject. For example, one student reported that he wanted to get a driving licence and therefore enrolled in a driving course. He successfully passed the "driving test", but every time he went to class his teacher was so humiliating and shouted at him that he decided to stop learning and not drive.

Reward or punishment, positive reinforcement or withdrawal, determine a student's attitude and openness towards a subject. In the context of the subjects taught, it is essential that students' self-esteem is not damaged, because contrary to the *'palm growing under pressure'* hypothesis, Claude Steel reported the opposite phenomenon in 1992 (Smith & Meckie, 2004).

The black students who were in classes where their teachers failed them because they were prejudiced against them for being black and stupid, there was a very interesting psychological defence mechanism that developed in the students. Students changed their self-image to the extent that they excluded academic achievement from their self-evaluation. In this way, they lost interest in school and their desire for recognition from academic achievement, thus protecting themselves from negative feedback that was harmful to them. However, in this personal response, a self-fulfilling vicious circle was created whereby these students did not actually achieve, so their failure was inevitable.

It was also in this experiment that Steele (1992) showed that students' perceptions of the importance of school performance are closely related to the quality of their grades. This again underlines how strongly the defence of self-esteem is embedded in one's personality. If a student gets good grades, the subject becomes important to him or her and has a defining value. If his grades are poor, his interest will wane and it will be irrelevant how many marks he gets, just survive.

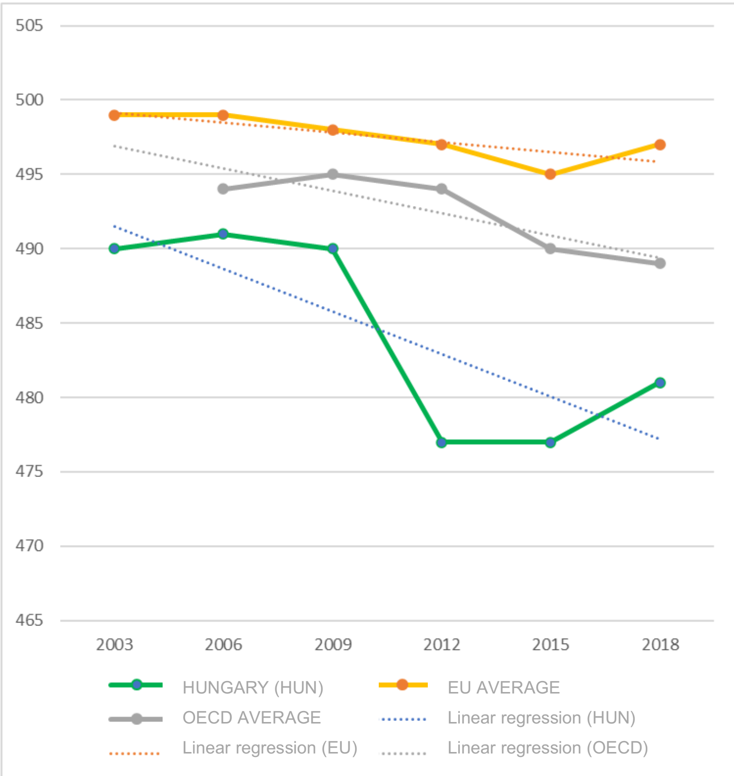
Maslow's theory (1970) presents the hierarchical relationship between different human needs in a pyramid model, with the need for self-esteem and self-actualisation at the top of the pyramid. Although Maslow believed that the higher one moves up the pyramid, the weaker the need, he also argued that the more humanised the need. The essence of a healthy human existence is to achieve self-esteem and self-actualization.

Learning, and in particular learning mathematics and its success and experience, affect these two levels of need: self-esteem and self-actualisation. The role definition and thus the behaviour of the teacher can have a very significant impact on this.

If we try to approach the question from the students' point of view, what are the qualities that are associated with the role of a teacher, we get the answer that students want *"their teachers to be knowledgeable, cheerful, friendly and fair, to whom they can turn with confidence with their problems"* (N. Tóth, 2015).

One of the key challenges of our time is to improve the effectiveness of school education. This is perhaps because a significant proportion of pupils do not reach the level of competence they require. This is particularly true in the field of mathematics education. Students are unmotivated and perceive learning as a burden. However, it is interesting to note that despite this, a 2002 survey shows that neither parents nor students dispute the need to learn mathematics (Somfai, 2010). However, the effectiveness of the teaching does not meet the expectations one would have of the performance of an *'important'* subject. A good measure of this is the trend in the performance of the PISA surveys, which have been conducted regularly for decades (Figure 8).

Figure 8: Trends in PISA scores in mathematics,
(own editing, data source: https://data.oecd.org/pisa/mathematics-performance-pisa.htm)



While the figures show that there are serious questions about the effectiveness of education, the question is how to reverse this trend. Instructors in the field often complain that even the methods that have worked in the past to keep students motivated are not working. In addition, there is a growing criticism of teachers and teaching methods, which has an impact on role perceptions and thus on students.

If you look at the younger generation, you can see that they are living in an age when the need for stimuli is very high. And this has a major impact on their behaviour. As early as 1972, Konrad Lorenz formulated the dangers of this: 'The *ability to experience pleasure is increasingly lost because of the habituation to strong and ever-increasing stimuli, and it is therefore no wonder that jaded people are always seeking new and new stimuli'* (Lorenz, 2001).

In fact, it is difficult to do anything about it. Stimulus needs vary at different ages, but the dynamics of stimuli are always similar. When the individual's activation level drops in a given situation, a feeling of boredom sets in and stimulus seeking begins. This is where a trained instructor can step in and harness this need and put it at the service of learning. When the student is looking at the mobile phone or laptop in class, it is because the stimuli coming from there are more likely to satisfy his needs.

The instructor's role definition, the behavioural pattern that comes with the role, determines whether or not he or she succeeds in engaging students, directing their attention and motivating them to learn and love a subject. We hear countless times that the subjects we love and the career paths we choose are often those where our teachers and tutors have influenced us, have been able to appeal to us.

Numerous studies have shown that the effectiveness of schools and education systems depends on teachers. Personal experience has shown that it is not the school, but the teacher, that determines how well children perform. So the question arises: what makes a good teacher?

Research on this issue clearly shows that there are three factors that significantly determine teacher quality: personality, role model and cognitive elements.

Beatrix Fűzi summarises in her work the research and directions that have been taken in the past 80 years (own translation from Fűzi (2015)):

"In the middle of the last century, it was hypothesized that refining the skills used in the classroom, such as the number of conjunctions in explanations or the order in which rules and examples are presented, could significantly improve teacher effectiveness (Allen and Ryan, 1969; Gage, 1972). As a critique of the above-mentioned research on the importance of elementary skills, some have stressed the importance of personality (Rogers, 1959, 1961; Asch, 1973). Rogers considered congruence, acceptance of oneself and others, to be the most important. Others began to focus on the teacher's thinking and decisions (Shavelson, 1976), assuming that these were in fact the drivers of action. Later, the focus shifted to pedagogical knowledge, and as part of this, great importance was

attached to routines of practice, differences between novice and experienced teachers, content and methodological knowledge (Falus, 1986, 2001). Other research has sought to identify the specific skills that are essential to the teaching profession, such as communication and conflict management (Sallay, 1996; Hegyi, 1996). Then attention has been focused on reflective thinking, which has been identified as the key to self-development in teaching (Schön, 1983; Kimmel, 2002, 2006)."

So you can see that the search for a way forward is almost continuous.

The role model that the teacher consciously or unconsciously chooses is nothing more than his or her professional self-definition. The elements of his or her personality that are mobilised in order to perform the tasks of a teacher, his or her experience and knowledge of teaching and learning, play a role in its development. Last but not least, it is his or her relationship with the students, the teacher-student bond, which also has an influence.

The psychological approach to attachment is nothing more than the search for and maintenance of closeness with another person. It is a strong and enduring emotional bond between the parties in which the bonding parties feel supported and secure, which cannot be replaced by anything else. And if this relationship is not available, the attachment partner experiences anxiety.

This is also reflected in pedagogy, where trust, affection and love are the main motives for emotional connection. Its functions are to help the other party and to socialise, in addition to support and protection in the learning process. For this very reason, the way in which teachers relate to their students and define the quality of the attachment they feel towards them is very important in their choice of role.

The three main orientations underlying the social-motivational model (belonging, competence and autonomy) are also reflected at the level of education for teachers:

- are present in the lives of students,
- strive to maintain the learning environment as a transparent and consistent system, and
- support student autonomy.

A number of studies in recent decades have shown that the teacher-student relationship is the most predictive of later behaviour in the long term (Baker, 2006): the strength of the teacher-student bond during the primary school years is negatively related to students' externalising (e.g. aggression, hyperactivity) and internalising behavioural problems (e.g. anxiety, depression, etc.).

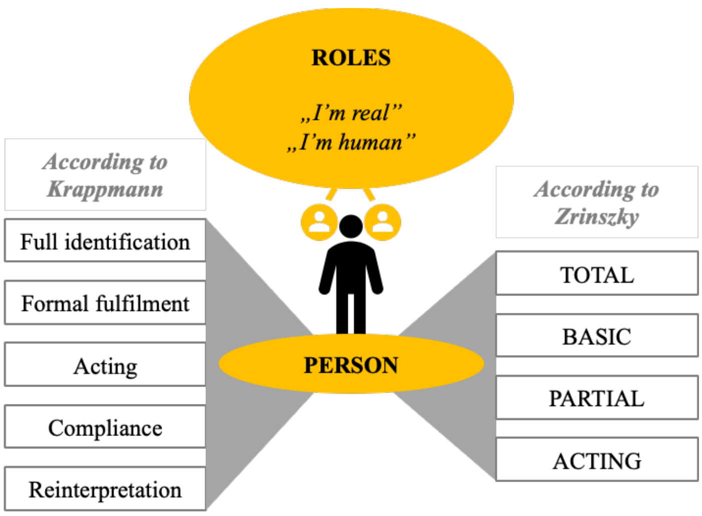
Among the domestic studies, I would like to highlight one that finds two findings in relation to the studies discussed above (own translation from Szabó et al. (2019)):

(1) *"the closeness of the teacher-student relationship in the adolescent sample is below the threshold*";

(2) *"In grade 10, the relationship between teacher and student is not coupled with a higher emotional quality - neither in terms of closeness nor conflict - and is assumed to lose its personal character, with teacher and student distancing themselves from each other"*.

In higher education, this attachment - as we are talking about adults - has even more interesting effects. However, in general, the effects described above in the field of mathematics learning and later in the field of quantification and decisions in the use of numbers have a significant impact on decision making attitudes. Just as teachers take on and communicate certain roles, students also take on roles and their self-definition is influenced throughout their lives (Figure 9). One of the most prominent of these is described as "I don't have a maths brain".

Figure 9: Role interpretations according to Krappmann (1980) and Zrinszky (1994) (own editing)



3. Education, motivation, trap

You acquire knowledge by learning. Learning can be a pleasure or a burden, but the process of learning determines the relationship we develop towards a subject or a body of knowledge. The way in which learning takes place is therefore not negligible. What kind of environment is created for learning, who are the people who guide us in it and, last but not least, what is our own attitude.

Learning can take many different forms. One of the simplest forms is called habituation or habituation (Zimbardo et al., 2022). This is typically the way in which, in the context of one or more stimuli, we learn not to respond to that stimulus. A good example is when a city dweller goes to the countryside and notices how quiet it is. This ability enables him to distinguish what is happening on a noisy roundabout from important, relevant stimuli, such as an ambulance coming and not stepping off the pavement.

The other - also a simpler form of learning - is the preference of what is learned over a stimulus that is already familiar, also known as the familiarity effect. This is when we prefer a stimulus we have already encountered to one we are not yet familiar with. Typically, this effect occurs regardless of the memory (good or bad) to which the stimulus is associated, and we are often not even aware of it (Zajonc, 1968).

Further forms of learning are much more complex, because at the higher levels our brain also takes into account the relationship between stimuli. When we see the solver of the second-degree equation, we remember the school desk, the atmosphere of the classroom, and we see our old teacher. When we learn that there is a relationship between two stimuli, for example through reward or punishment, learning takes on a new dimension. The learning process, which is called behavioural learning, is based on the stimulus and the response to it. The two main forms are classical conditioning and operant conditioning.

Last but not least, complex problem solving, learning in school further deepens the learning levels of how our minds work. Cognitive learning is a form of learning that goes beyond the previous forms of learning and is also related to internal mental processes.

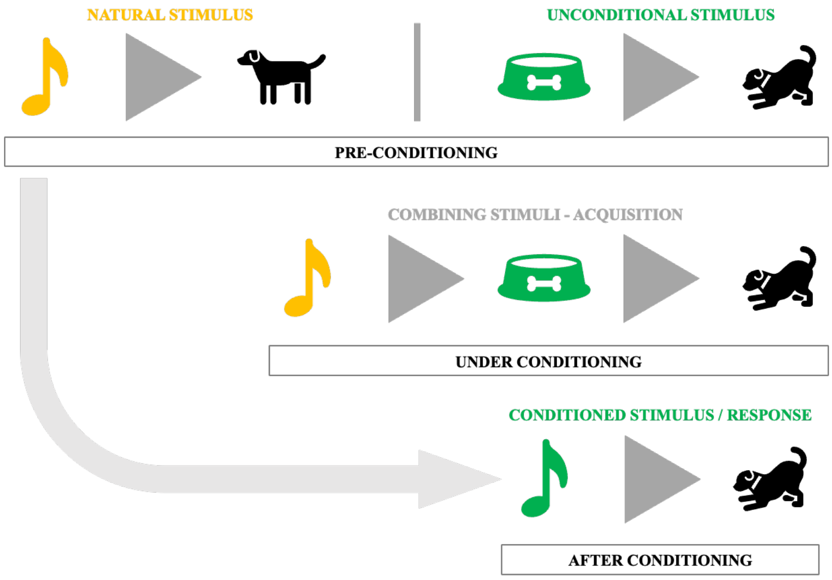
Understanding the forms of learning and how they affect people's personal experiences is very important to understand how the quality of mathematics education influences choices.

3.1. The Classical Conditioning

Ivan Pavlov was awarded the Nobel Prize in 1904 for his discovery of how to replace a stimulus that triggers an innate reflex with a stimulus that was previously considered neutral, and thus to trigger essentially the same response. He worked with dogs in his experiments and made a breakthrough with his research on the digestive system (Pavlov, 1928).

He based the experiment on an unconditional response that is instinctive in the animal: when the animal sees food, it automatically starts salivating because it will eat. However, if, for example, the animal hears a sound, such as a bell being rung, nothing happens as far as salivation is concerned, apart from, of course, turning in the direction of the stimulus with interest. Pavlov used this neutral stimulus in his experiments by repeating the stimuli in a particular sequence inx each case. First the bell rang, then came the food and the dog salivated. When this sequence occurred repeatedly, so-called acquisition occurred. The dog started to salivate as soon as it heard the bell. In other words, a neutral stimulus, the sound of the bell, became a conditioned stimulus that triggered the same response (salivation) that the unconditioned stimulus activated (salivation). The process is illustrated in Figure 10.

Figure 10: The classical conditioning process (own editing)



This learning process, in which an innate stimulus is replaced by a neutral stimulus, i.e. a conditional stimulus is created to produce a conditional response - which, like the unconditional response, differs only in the stimulus that triggers it - is called classical conditioning (Zimbardo et al., 2022).

In classical conditioning, countless reactions can occur throughout our lives. However, the Pavlovs also observed that if the conditioned stimulus is not followed by an event associated with the conditioned response, such as a reward - i.e. the animal does not get the food after the bell - then the physiological response is lost after a while. This phenomenon of the conditional response weakening and then disappearing has been termed extinction. However, this extinction is deceptive, because if the circumstances are right, we are able to recall a conditioned stimulus suddenly, even though a longer time elapses before the conditioned stimulus is perceived again. This phenomenon is called spontaneous recall.

In the context of classical conditioning, it is important to mention an additional phenomenon: stimulus generalisation. This is the phenomenon where a conditioned response extends to conditional stimuli that are similar to the original stimulus. Typically, this occurs in cases where very strong emotions are associated with an event, the consequence of a traumatic effect. For example, when someone is stung by a wasp - which can be very painful - they may fear and panic about all insects from then on, not just when they see a wasp.

This innate learning process, which can be significantly influenced by environmental influences, has an impact on us, especially when we are in a targeted situation. During our years of education, we experience countless situations in which the conditions for classical conditioning exist. It is in these situations, often unnoticed, that this learning process takes place. The teaching roles and the different communication and problem-solving situations that are repeated in them trigger similar feelings in the students again and again. In this way, they develop the conditional stimuli to which they will later respond in the non-school environment with exactly the same feelings and responses as they did in the classroom.

3.2. The Operant Conditioning

In the classical conditioning process, the stimulus is first aroused and the response is generated. However, there is another way to learn behaviourally. This occurs when the probability of a response occurring is based on its consequence, i.e. it is the events following a decision in a given situation that stimulate a change in behaviour. This learning process, which is shaped by the consequences of behaviour, typically reward or punishment, is called operant conditioning and explains the nature of the emergence of new and involuntary behaviours.

As Thorndike described in his law of effect, we prefer behaviours that lead to a more pleasant state (Thorndike, 1898). Skinner took this research further and devoted his entire career to showing that these state changes and their consequences have the greatest power to influence our behaviour. These consequences, which subsequently affect us, are what he called reinforcers (Skinner, 1956). In his definition, these reinforcers are therefore stimuli that follow and reinforce a behaviour, a response to a situation (which involves the removal or retention of the stimulus). In the case of reinforcers, Skinner distinguishes two types, which are often used colloquially: positive and negative reinforcement. Typically, more people are familiar with the former than the latter.

During positive reinforcement, the stimuli that occur as a consequence of the behaviour increase the likelihood that the response will occur again, that the behaviour that triggered it will be repeated. A good mark, praise, attention fulfil this role in exactly the same way as dessert, money or sex. If a child clears the table after dinner at home and gets a cube of chocolate for doing so, he will do it again the next time, hoping to be rewarded again. Typically, then, we are talking about stimuli that make us feel good.

Another form of reinforcement - often misinterpreted in the wrong context - is so-called negative reinforcement. Contrary to popular perception, this is not the opposite of the "reward" context associated with positive reinforcement, i.e. it is not "punishment". Rather, the term negative corresponds to an operation of mathematical *subtraction*: the removal of an unpleasant stimulus is the result of the behaviour in question. Any case in which a given behavioural response leads to the removal of a stimulus that triggered a behaviour is called negative reinforcement. (In this sense, positive reinforcement is the operational equivalent of *addition.*) There are many phenomena in everyday life that rely on this form of learning. Warning systems are a prime example. The warning sound lasts until the correct behaviour is performed: closing the door, fastening the seat belt and the warning sound stops, or even removing all metal objects from your pockets at the airport so that the security gate does not signal. When we behave correctly, the unpleasant beeping will stop in both cases.

The timing and frequency of the reinforcement that influences voluntary behaviour can be the most decisive in bringing about real change. Rewarding each correct response at the beginning of the learning process provides clear feedback on the desired behaviour. This is called continuous reinforcement. But when the desired behaviour emerges, the learner no longer needs to be rewarded for distinguishing between correct and incorrect responses. When we teach a dog a new, complex behaviour, we give a reward bite for each successful response to capture the response to the command. However, once the animal has done the correct thing every time, it is enough to reward it every third time, and it continues to do the task correctly. This is called partial reinforcement. Partial reinforcement is the method by which a learned behaviour becomes sustainable (Robbins, 1971; Terry, 2000). Its main advantage is that, unlike classical conditioning, extinction has no effect unless the reward is completely absent. In all other cases, the behaviour occurs with the same intensity in the situation.

In everyday situations, reinforcements can go both ways. For example, when a young child cries out in the night because he or she had a bad dream and the parents respond by putting him or her between them, this can have a double reinforcing effect when used regularly. On the one hand, the child receives positive reinforcement for his or her behaviour: if I cry, I can sleep between mummy and daddy. On the other hand, parents learn through negative reinforcement: if we invest him between us, he will stop crying, so we can sleep peacefully. These effects are also reflected in the interactions that students have in maths lessons, while understanding and solving problems.

3.3. Cognitive Learning

In contrast to the operant learning methods, a group of psychologists began to investigate another form of learning. They believed that many elements of the learning process were to be found in mental processes, even if many of the issues involved were subjective. Köhler demonstrated that not only responses through conditioned learning, but also those generated through insight, through perceptions of the problem, are possible in more advanced primates. This ability to mentally rearrange and reinterpret perceived situations is what he called insight learning (Köhler, 1925).

Tolman (Tolman, 1949) also set out to demonstrate the forms of learning that occur alongside behavioural learning. In his experiments, he let rats roam free in labyrinths for long hours. During these experiments, the animals received no reinforcement and simply came and went. The experience led to the conclusion that the animals had learned, had figured out the maze. In fact, when food was later placed in a remote part of the maze, animals that had mapped their environment typically reached the food faster than those that had not. Tolman called this phenomenon latent learning. This degree of cognitive mapping of the environment is of great evolutionary importance, as it is a key adaptive behaviour in organisms that have had to actively explore their food (Kamil et al., 1987).

Budura went further in his research, and while the reinforcement seen in behavioural learning previously only affected those who performed the behaviour, he showed that it could also affect those who observed it (Bandura et al., 1963). So, if we see the behaviour of an observed person (model) as leading to a result, we may also tend to choose that behaviour. This form of learning is called observational learning (also known as social learning).

In the context of the phenomenon of observational learning, the media have a very important role to play in the 21st century. By consuming a lot of video content, we see patterns that influence our behaviour. Murray and Kippax, as early as 1979, described the phenomenon of emotional numbing (Murray & Kippmax, 1979). In their experiment, they demonstrated that people who watch violent scenes are less upset when they see them. Huesmann and Moise went further (1996) and demonstrated that viewing violent behaviour increases the likelihood of violent behaviour occurring. This in turn can be traced back to the patterns of behaviour that observers see in a given situation.

3.4. The penalty

One more important behaviour to mention in relation to learning is punishment. Punishment is an aversive consequence of a behaviour that, in contrast to negative reinforcement, weakens the strength of the behaviour, i.e. negative reinforcement and punishment have the opposite effect on the behaviour (Baum, 1994).

Personality is made up of the learned behavioural tendencies that an individual acquires through personal life experience. The patterns he sees around him determine and shape his attitudes. From childhood to young adulthood, most people are obliged to participate in public education. As such, we are with our teachers for a significant part of the time. The teacher participates with his or her whole being and personality in these teaching and educational situations, given the time horizon of being together with the students. This influence is particularly important in the field of mathematics learning, and is present throughout our studies until the end of our higher education, and sometimes even beyond.

This phenomenon can also be seen in the general approach of having a 'maths brain' or 'humane brain' when learning mathematics. If the student, the soon-to-be, senior decision-maker, thinks of themselves in this way, the result will be similar: they will exclude the importance of mathematical performance in their own judgements.

4. Determinants of performance in mathematics learning

4.1. Motivation to learn and self-confidence

The interest of students in a subject is an important factor in cognitive development and learning (Hidi, 1990). Whether or not a student is interested in a particular subject is a determining factor. This is especially true in mathematics, where interest has been shown to correlate with mathematical achievement over time (Schiefele et al., 1992; Kim et al., 2015). Some studies have also shown that interest in mathematics can be a predictor of students' later academic performance (Schiefele & Csikszentmihalyi, 1995; Singh et al., 2002; Viljaranta et al., 2009). Interest in mathematics can support students' learning of mathematics in a number of ways:

(1) by increasing effort (Deci et al., 2001);
(2) by increasing the duration and frequency of mathematics learning (Ericsson et al., 1993; Schiefele, 2001);
(3) using specific learning strategies (Schiefele, 2001).

Depending on motivation, these behaviours can be implemented even when teachers' attitudes are not adaptive in their mathematics teaching practice (Bong et al., 2015). Motivation can help students to explore mathematical problems in more depth and extend their knowledge, leading to sustained mathematics learning and good outcomes (Reeve et al., 2015; Schiefele & Csikszentmihalyi, 1994).

Motivation is needed in the learning process (Irhamna et al., 2020). It plays an important role in providing passion or enthusiasm for learning, so that highly motivated students have plenty of energy to complete learning activities. If the student is motivated in learning, it will lead to good academic results. The intensity of motivation is a major determinant of the level of academic achievement. Motivation is a change in personality that is characterised by the emergence of affective factors, feelings and reactions that are directed towards the achievement of goals. Strong motivation in learners increases interest, willingness and enthusiasm for the subject matter. Research by Irhamna et al (2020) also shows that motivation to learn is closely related to learning achievement. They observed that the higher the motivation to learn, the better the academic performance. People who are more motivated in a particular area are more successful in that area.

In addition to motivation, self-confidence is an important factor that influences students' academic performance. Self-confidence is very important for students to be successful in learning mathematics. Indeed, students with high self-confidence will be more motivated and more willing to learn mathematics. This enables learners to take appropriate and effective action in different situations, even when they face challenges from themselves or others (Burton & Platts, 2006). Learners' self-confidence is also useful in creating a learning climate that supports learners to do the best possible job and to believe in the effectiveness of their own hard work without having to cheat to succeed. Often, students are unable to demonstrate their full potential because they feel unsure of their ability to complete the tasks assigned to them. This is particularly evident in mathematics, where understanding is reinforcing, but being wrong is often associated with a sense of shame, which can undermine self-confidence.

The question is how long this motivation can be sustained in mathematics. In mathematics, the focus is on problem solving, not on the acquisition of lexical knowledge. This poses a constant challenge to students, because maintaining interest and motivation becomes increasingly difficult as the complexity of the problems to be solved increases. The ability of students to take the initiative, which depends on their daring to ask questions, to think and to be wrong, is crucial to success.

4.2. Mathematical anxiety

Students' ability to solve mathematical problems is influenced by a number of external and internal factors. In the 20th century, much research focused only on external factors, learning and teaching methods or strategies, when in fact internal factors also play a fairly large role in the ability to solve mathematical problems (Sukarti, 2018). Mathematical problem solving is inherently more than a routine application of what is learned and therefore requires a higher level of understanding. This in turn can lead to internal conflicts in learners.

When we talk about learning, it takes place in a constructed situation, in a targeted way, through pedagogical effects, embedded in the educational system and through predefined means. Learners' performance in this context is primarily achieved through motivation, through the experience of personal abilities, but it is also influenced by a number of other psychological factors. Among these, one should be highlighted that affects students' low academic performance (Kumar & Karimi, 2010) and is specifically related to numeracy tasks: mathematical anxiety. Mathematical anxiety is related to what researchers have been noticing for fifty years, namely that students feel anxious when working with numbers or solving mathematical problems (Richardson & Suinn, 1972).

The more widely accepted definition of anxiety is the fear of actual or expected communication with another person or persons (Opt & Loffredo, 2000). This is a consequence of low self-esteem in the mathematics learning context (Daane et al, 1986). It is also a cause of fear and negative reactions to interactions with others (Beatty & Beatty, 1976; Smith et al, 1994; McGuire et al., 1995). Xu (2004) defines mathematical anxiety as a complex of feelings of tension, aversion, frustration and fear. It is the phenomenon that makes it virtually impossible for students to learn mathematics. Experiences in the classroom lead to a state of discomfort in students, which causes them to avoid mathematics as much as possible.

Many students have low self-confidence in solving mathematical problems, which is a result of mathematical anxiety (Scarpello, 2005). Unpleasant experiences in mathematics class, experiences mediated by parents, or even teachers' attitudes are factors that can influence students' mathematical anxiety (Barnes, 2006). According to Ashcraft and Krause (2007), if a student has difficulties with his working memory, i.e. he calculates more slowly, he also becomes a mathematics anxious person. By examining these results, it can be concluded that stress, which affects the use of working memory, undoubtedly has an impact on mathematical performance. However, this process also has a reverse effect: mathematical performance is also associated with an increase in mathematical anxiety (Furner & Duffy, 2002; Hopko et.al. 2003). The higher the level of mathematical anxiety, the lower the students' achievement in mathematics, the lower their rate of mastery and the lower their motivation towards new knowledge. Regression analyses have shown that math anxiety is a significant predictor of achievement, with a negative correlation between math anxiety and math achievement (Devine et al., 2012). Olaniyan and Salman (2015) reported that students with math anxiety tend to claim that math is a difficult subject, dislike math, do not practice, and even skip class. This can certainly affect their success in mathematics exams.

One solution to overcome anxiety may be for the instructor to use strategies methodologically that reduce the load on the students. This will make students feel comfortable (Oxford & Vordick, 2006) and reduce their resistance. It is important for teachers to set goals for students that are achievable, and through this to focus on increasing students' confidence in mathematics. If one is constantly confronted with statements such as 'maths is not important in the world' or 'you are not a good maths student', or even 'let's look again for the sake of the understanding', these statements can eventually translate into negative beliefs in students about their own mathematical competence (Chinn, 2008): 'I don't have a maths brain', 'I am stupid at maths'.

Another way to find a solution is through teamwork. Alsup (2004) concluded that mathematical anxiety was significantly reduced when students worked together to solve a mathematical problem. By using interactive and collaborative teaching strategies, he also found that students became less anxious about mathematics, more confident in their mathematical problem-solving abilities, and more empowered in their own learning to initiate problem solving. According to Iossi (2007), other strategies to minimise anxiety include individual learning pathways, distance learning, peer classes and targeted mathematics anxiety courses.

But maths anxiety does not only affect those who are learning. It also affects teachers. Mathematics anxiety leads maths teachers and teacher candidates to passively develop their competences and avoid situations where they need to rely on their mathematical expertise. A good example of this is when a student does not solve a problem correctly according to the method they have learnt. In such cases, the teacher has two options. Either he or she reconsiders the solution and praises the student, or he or she does not accept the solution because it does not follow the learned/taught pattern. Choppin (2011) has highlighted that teachers with mathematics anxiety rely mainly on textbook-based teaching, which emphasises basic skills, mechanical application of learned formulas and minimal discussion activities. Teachers with mathematical anxiety are less able to apply a variety of mathematics teaching-learning strategies in the classroom (Swars et al., 2007). Thus, real mathematical problem solving does not occur in the classroom and teachers are less effective in identifying unique situations and responding to student needs and solutions.

Mathematics anxiety makes it difficult for students to learn and apply mathematical concepts (Gleason, 2008). Students who grow up not liking mathematics at all, seeing it as a subject that is not fun, difficult to understand, requiring them to solve different problems or problems that not everyone can do. Maths anxiety is a feeling of fear when faced with a numeracy challenge, the possibility of dealing with a maths problem. Mathematical anxiety adversely affects the process and outcomes of learning mathematics. It affects student performance. Mathematical anxiety is nowadays becoming a global problem for which new strategies need to be developed to effectively address it (Luttenberger et al., 2018).

4.3. Mathematical self-efficiency

In the context of mathematical anxiety, a number of studies mention the concept of so-called mathematical self-efficacy. This is defined as an individual's beliefs or perceptions about his or her mathematical abilities and how he or she likes to learn. In other approaches, self-efficacy can also be defined as an individual's perception of his or her abilities in the context of successfully completing a given task (Zimmerman, 2000). In the context of motivation, mathematical self-efficacy is defined as the individual's uniqueness and ability to perform organized, detailed, and specific work (Pastorino & Doyle-Portillo, 2013). According to Margolis and McCabe (2006), self-efficacy is defined as an individual's belief about his or her ability to achieve his or her goals. Therefore, learners with self-awareness are able to perform difficult tasks and are motivated to find solutions. They have also pointed out how learners achieve self-efficacy through experience, verbal persuasion and emotional state. Self-efficacy beliefs influence individual decision-making processes (May & Glynn, 2008).

Research has shown that the learning environment and teaching method can improve self-efficacy in the classroom (Bandura, 1991). Fencl and Scheel (2005) showed that teachers need to use different strategies in teaching in order to improve students' self-efficacy. One way to measure teaching methods is through the classroom climate. In their study, they describe that collaborative learning and inquiry-based activities contribute greatly to the development of learners' self-efficacy. Bandura (1991) also concludes in his work that collaborative learning strategies can improve learners' self-efficacy and academic achievement. Schunk and Pajares (2002) also identified other pedagogical methods that can help to improve learners' self-efficacy, such as personal diaries and reflective essay writing. A solution may also be to compare students' performance against the target set by the teacher rather than against the rest of the class.

There are studies showing that self-efficacy and mathematical performance are related. Pajares and Miller (1994) followed by Bong (1998) have shown that self-efficacy can be a good predictor of overall academic performance. According to Higbee and Thomas (1999), mathematical self-efficacy influences student performance. And Hall and Ponton (2002) suggested that this finding supports Bandura's view that mathematical achievement is the greatest source of self-efficacy. If students have mathematical self-efficacy, there is a possibility that they are more competent in successfully solving mathematical problems (Kabiri & Kiamanesh, 2004; Liu & Koirala, 2009).

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Research suggests that interest in a number of subject areas, particularly mathematics, is closely related to self-efficacy (Lent et al., 1991; Bong et al., 2015;). Rottinghaus et al. (2003) conducted an empirical meta-analysis on 60 independent samples. Results showed that the correlation between interest and self-efficacy was noticeably stronger in mathematics than in other subjects. One reason for this may be that feedback from others or on one's own activities can help individuals to strengthen their self-efficacy, thus making them feel competent and not giving up on learning (Bandura, 1986). In the case of mathematics, such feedback may carry much stronger messages both in terms of one's own experience: 'I understand, I have found the solution', and in terms of the reactions of others: 'great that you have figured out the relation between the variables'.

Research over the past decades has clearly shown that self-efficacy is a very strong predictor of mathematical performance (Pajares & Miller, 1994; Pajares & Kranzler, 1995; Pajares & Graham, 1999; Kitsantas et al., 2011; Cleary & Kitsantas, 2017). Students with high self-efficacy tend to believe that they are capable of achieving the goals and solving the problems they face. Thus, they tend to exert more effort (Sakiz et al., 2012), persist better than other students (Bandura, 1986; Schunk & Zimmerman, 2006; Hoffman & Schraw, 2009), use more effective strategies (Butler & Winne, 1995), and achieve better mathematical performance (Fast et al., 2010). Self-efficacy is a determinant of students' mathematical performance (Skaalvik et al., 2015; Schöber et al., 2018).

4.4. Mathematical anxiety and self-efficacy

Students who have low self-efficacy and feel that they have poor concentration tend to experience mathematical anxiety (Fennema & Peterson, 1983). According to May, mathematical anxiety and low self-efficacy have implications for mathematics learning. These factors affect students' mathematical performance, so their relationship is very important. May's (2009) study found that students who passed exams had higher self-efficacy and lower anxiety than those who failed.

Di and Chan (2020) investigated the mediating role of self-efficacy and math anxiety and the effect of math interest on math performance. A total of 158161 eighth grade students from 4 provinces in China participated in their study. The results showed that

(1) interest in mathematics had a direct and positive impact on students' mathematical performance;
(2) the positive relationship between interest in mathematics and mathematical performance is influenced by self-efficacy;
(3) self-efficacy and mathematical anxiety determine mathematical performance.

Students who are anxious about learning mathematics tend to be concerned about their performance on mathematics tests (essays, exams, tests) (Ashcraft & Ridley, 2005). The resulting stressful situation depletes their working memory resources, which are essential for problem solving and are key to using mathematical skills to solve mathematical problems (Richardson & Suinn, 1972; Ashcraft & Kirk, 2001; Beilock et al, 2004; Mammarella et al., 2015). Mathematical anxiety is often accompanied by physiological reactions such as pain (Lyons & Beilock, 2012; Pletzer et al., 2015) or negative emotions (Ma, 1999; Ashcraft, 2002; Young et al., 2012; OECD, 2013; Ramirez et al., 2018), and therefore students avoid mathematics or mathematics-related situations that trigger this anxiety (Beasley et al., 2001; Ashcraft & Ridley, 2005; Ashcraft & Moore, 2009; Chang & Beilock, 2016).

Several studies have demonstrated a significant negative correlation between math anxiety and math achievement (Ma, 1999; Chang & Beilock, 2016; Foley et al., 2017; Gunderson et al., 2018; Ramirez et al., 2018; Wang, 2019). Students with high math anxiety tend to perform worse than their peers with low anxiety (Ashcraft & Moore, 2009; OECD, 2013), especially when solving complex math problems (Ashcraft & Kirk, 2001; Ashcraft & Moore, 2009). Among OECD (Organisation for Economic Co-operation and Development) countries, 14% of the variation in student math performance is explained by math anxiety (OECD, 2013).

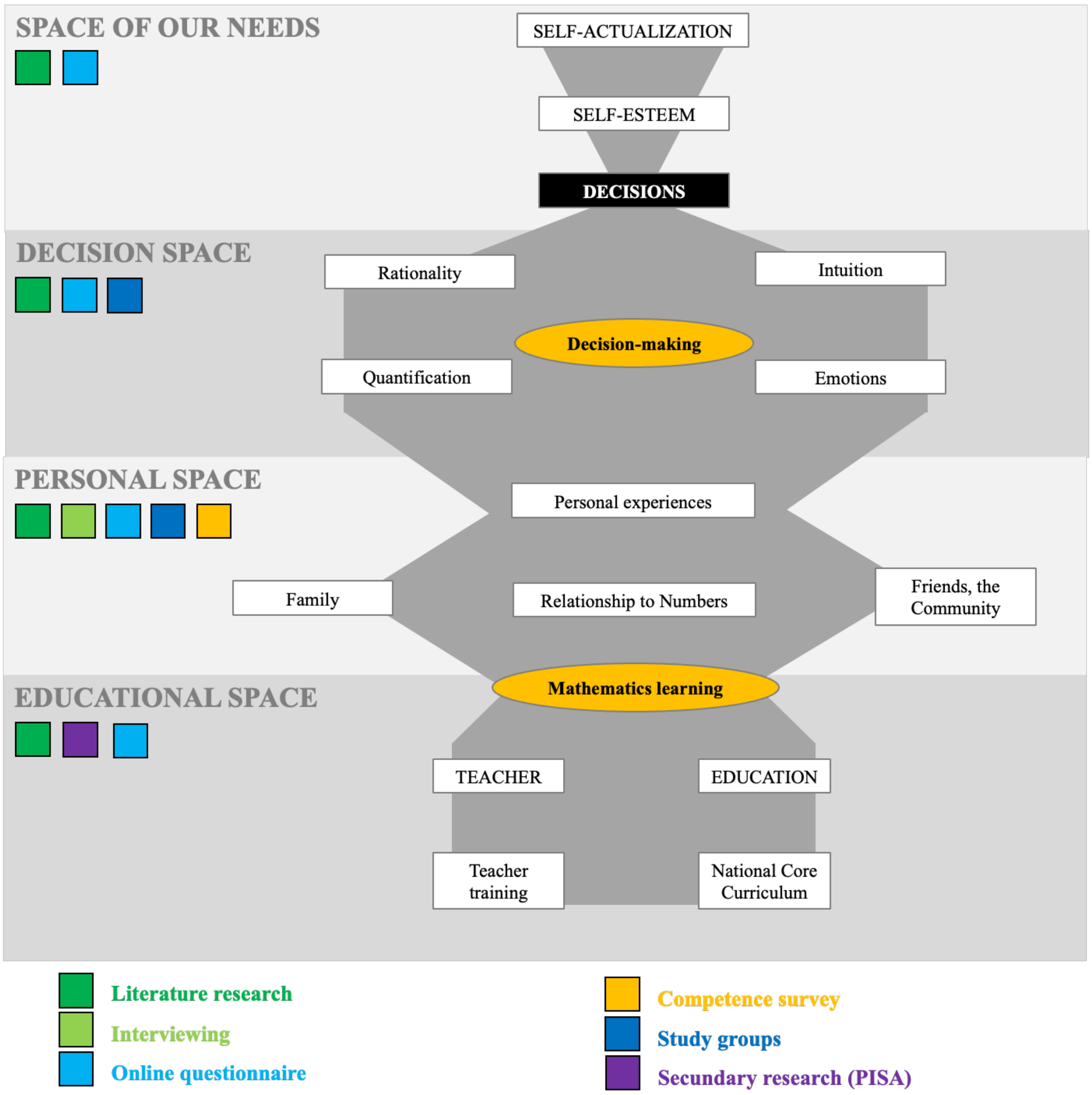
Research has shown that self-efficacy is significantly related to mathematical anxiety (Pajares & Miller, 1994; Pajares & Kranzler, 1995; Jameson, 2014). Bandura's (1986, 1997) social cognitive theory also addresses the relationship between self-efficacy and anxiety. According to the theory, the learner's perceived self-efficacy plays a crucial role in the validation of anxiety. Those who are perceived to be able to manage "adverse events" that cause anxiety tend to be more successful in converting and reframing anxiety into positive outcomes. While students with poor self-efficacy in mathematics tend to feel anxious about meeting academic requirements. In other words, social cognitive theory suggests that students' mathematics self-efficacy is responsible for mathematics anxiety

(Bandura, 1997). In addition, empirical research has shown that mathematics self-efficacy significantly predicted mathematics anxiety (Akin & Kurbanoglu, 2011). Students with high math self-efficacy have lower math anxiety and better performance in solving difficult problems than their peers with lower self-efficacy (Lee, 2009; Hoffman, 2010).

V. PATHWAYS - THE RESEARCH I PLAN AND ANALYSIS METHODOLOGY

The research is methodologically mixed because it approaches the research questions from several angles. As such, both quantitative and qualitative research methodologies are used. However, it should be stressed that the research axis is based on an online questionnaire survey, the results of which are complemented and reinforced by other related research and data collection methods. The relationship between the research field and the elements of the research design is shown in Figure 11.

Figure 11: Relationship between the research field and the elements of the research plan (own editing)



1. Primary research

The primary research aims at understanding the existing trends in the teaching of mathematical competences and the personal experience and perception of number concepts and quantification in relation to the usefulness of mathematics learning, as well as the decision-theoretical context. Given that the primary research includes qualitative methodological research, in places the author provides a first-person, singular researcher's self-reflection.

1.1. Literature research

To analyse the publications of the past decades related to the present research, in order to explore the results and views previously reached on the research topic.

The researcher' role definition: the investigator.

Positioning in the research field: the main area of investigation is research on the topics covered by the entire research field.

Research Methodology: The research is based on a comprehensive literature review. It is theoretically based on the choice of a targeted topic area based on and supported by personal experience and, in this case, on the exploration of the interrelationships between these areas. In the light of this, and taking into account the methodological considerations, the following steps were followed:

- Defining the research focus: the aim was to interpret the information available in the fields of economics, decision theory and mathematics education in a comprehensive context.
- Conceptual overview of the topics: the key concepts through which the thesis will be structured and interpreted have been identified. The keywords along which the literature review was conducted were also identified.
- Literature review: three online search/databases were available for the literature search, which were (1) Scopus, (2) Web of Science and (3) Google Scholar. In addition to these, a significant primary source was provided by printed literature sources that were derived from previous self-collection following personal

professional interest. Typically, references to literature found in these provided as a secondary source.

- Literature analysis and synthesis: the selection of the literature sources included in the research was based on their relation to the theoretical synthesis, in addition to the best-known concepts of the discipline.
- Finalisation: bringing together the relevant ideas from each source along the historical path built up.

1.2. Interviewing

The research involved interviews with individuals from several target groups in order to explore different aspects of mathematics and mathematical knowledge as outlined earlier:
-Adult workers/managers: the interviews were designed to find out to what extent mathematics is present in their lives and work;
-Students (primary, secondary and higher education): the interviews aimed to explore young people's attitudes towards mathematics and their perceptions of their experiences of mathematics education;
-Mathematics teachers: the interviews aimed to find out what and why they would like to teach mathematics to different age groups.

The researcher's role definition: the curious.

Placement in the research field: the main field of study is the Educational Space and the Personal Space.

Research methodology: the interviews were conducted online (due to the epidemiological measures during the epidemic) or face-to-face in a semi-structured format. In all cases, audio material was recorded and a transcript of the interview was prepared. The output of the recording was anonymous and in all cases the recording of the audio material was based on personal consent.

The sample was selected by expert selection, because the aim was to get people from different social, educational, age and backgrounds to report on their own mathematical narratives.

The pre-defined interview questions were:

-If you had to think of mathematics as a colour, what would come to mind?

-If you had to think of mathematics as a piece of cloth, what would come to mind?

-When did you first encounter mathematics?

-What are your memories from primary and secondary school and higher education?

-What has learning maths given you?

-What were your maths teachers like?

-What was a moment in your life when you felt you really needed maths?

-How do your children feel about maths? Do they need motivation?

-If you had the resources, what would you change about teaching maths?

The data was collected in 2020.

An interesting feature of the interviews is that one of his fellow researchers also interviewed the author of this doctoral thesis in the same manner that he interviewed others for the research. In this way, self-reflection on the author's position will also be part of the analysis.

1.3. Online questionnaire

The online questionnaire explored the typical attitudes towards mathematics, how these are influenced by experiences of learning mathematics and how the role of mathematics is perceived. The questionnaire consisted of 5 blocks. In the first, the demographic background of the respondents was assessed. The second focused on free associations with mathematics. The third was to recall experiences of learning mathematics. The fourth block asked about the relationship between career choices, work and mathematics. Finally, personal opinions were explored, the extent to which respondents thought mathematics was needed in the context of each occupational area.

In evaluating the results of the questionnaire, I had to be particularly aware that the questions I asked would be influenced by my preconceptions. Therefore, I had to be able to deviate from my research objectives and not to steer my thinking in one direction through the questions.

Based on the questions and data collection methods used, I analysed the responses using descriptive and multivariate statistical methods. During these analyses my involvement was minimal, helped by the way the data was collected.

The researcher role definition: the accountant.

Positioning in the research field: the main field of study is the Education, Personal and Decision Space.

Research methodology: the questionnaire was designed, shared and completed in the digital space. The data collection tool was Qualtrics. The data collection was self-completed, so it was important that the line of questioning was clear, avoiding as much misinterpretation as possible, in order to exclude any resulting bias.

Given the lack of knowledge of the population in relation to the characterisation according to the study criteria defined in the research context, sampling was carried out using the snowball method. The link to the questionnaire was shared in online social spaces (Facebook, LinkedIn and direct email using the easy access method). Furthermore, a related communication campaign accompanied the distribution of the questionnaire.

After demographic evaluation based on the responses received, the representativeness of the sample was checked, but as expected, the conditions for this could not be met. Considering the exploratory nature of the research, this does not hinder the evaluation and interpretation of the results.

In terms of analysis, both qualitative and quantitative analysis of the data had to be carried out based on the answers to the questions. In the qualitative domains, personal narratives were analysed after coding, and free association responses were analysed from both symbolic and psychological archetypes. Questions asked for quantitative analysis were typically dominated by multiple-choice or 5-point Likert scale questions. For these, in addition to descriptive statistical analysis, multivariate statistical analysis could also be performed. Finally, it should be underlined that, given the large sample size, the analyses could also be carried out along clusters based on the main demographic data, with a separate analysis of the differences in results between the clusters that were formed.

1.4. Study groups

The Co-operative Doctoral Programme provided an opportunity to set up learning groups (two in total) with a business partner to implement a mathematical competence development programme according to a planned methodology. Group activities were audio and, where possible, video recorded.

The researcher role definition: the mentalist.

Positioning in the research field: the main research areas are Personal Space and Decision Space.

Research methodology: the study group was designed to consist of two groups of employees from the research partner's company, with representation from all levels of the organisation, from the managing director to the receptionist. In this way, the sampling was carried out using the easily accessible subjects method. The same programme was developed for each group, along similar themes.

1.5. Competence survey

A pre-test and a post-test were completed and analysed in order to implement the planned methodology for developing mathematical skills in the selected groups of students. The purpose of this was to measure the impact of the planned mathematics skill development on the personality and competence level.

The researcher role definition: the soul diver.

Placement in the research field: the main study area is the Decision Space and the Space of Our Needs.

Methodology of the research: the competence survey was carried out for the participants in the study groups. As such, the sampling was done using the easily accessible subjects method with expert selection. The data collection was planned to be conducted in two sessions: (1) before the start of the work with the learning groups; (2) after the development activities with the learning groups.

The ProfileXT (PXT) competency assessment test was used for the survey. This test assesses the whole personality according to the following criteria:

- cognitive skills and thinking style;
- behavioural characteristics;
- interests.

The questionnaire took on average 70-90 minutes to complete, but there is no time limit. The survey itself was conducted online, so the data could be collected conveniently without the need to be present in person.

2. **Secondary research.**

The "PISA research"

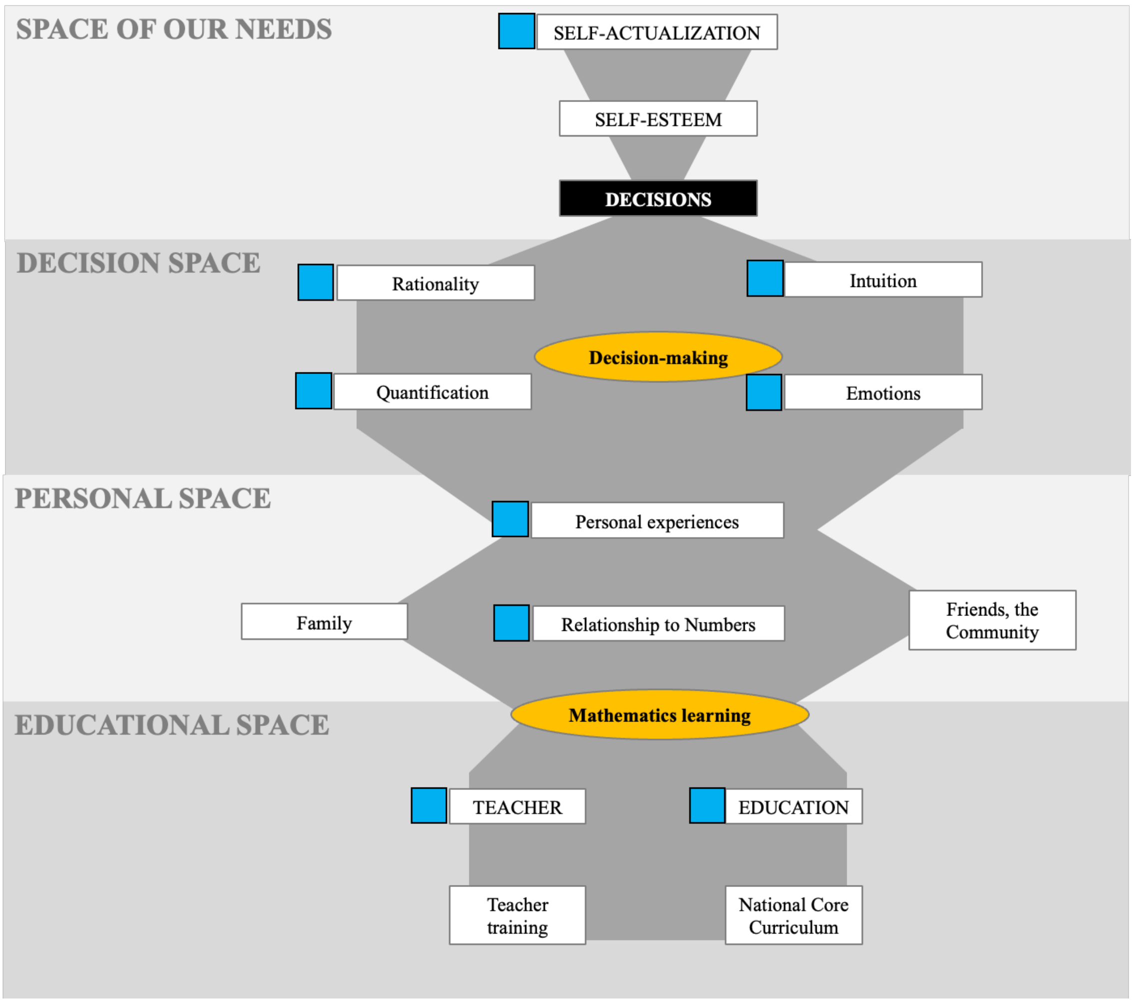
The results of the so-called Programme for International Student Assessment (PISA) studies have been analysed to determine student competence levels and to put them in an international context. The aim of this research was to assess the knowledge background of 15-year-old students in secondary education.

The number of countries taking part in PISA is growing (PISA, 2019), with the seventh assessment taking place in 2018, when 79 countries and education systems were represented. There are nearly 32 million 15-year-old students in each country, with around 600 000 students represented in the survey. The survey considers as 15-year-olds those students aged between 15 years 3 months and 16 years 2 months at the time of data collection. It represented 5132 pupils from 245 schools in the country. 324 pupils were in year 7 or 8, 3,800 in year 9, 1,006 in year 10 and 2 in year 11 at the time of the survey.

VI. FOCUS - PRESENTING THE RESULTS OF THE RESEARCH

In view of the complexity and scope of the research and the limitations of the form and content of the thesis, as well as the recommendations of the reviewers of the draft thesis, the main quantitative results of the primary research of the on-line questionnaire survey are presented and analysed in this thesis (Figure 12). The results of the further research detailed in the research plan will be published thematically in national and international professional journals and conferences in the future, building on the results of the present doctoral thesis.

Figure 12: Relationship between the research field and the questions in the research questionnaire. The blue square indicates the elements of the research field to which the questions relate (own editing).



1. Detailed presentation of the online questionnaire

1.1. The sampling

The research used a non-random sampling procedure. This is because the sampling was explicitly designed to select subjects who are more likely to have spent more time in the education system learning mathematics, and thus have direct experience of its short-, medium- and long-term effects. Hence, the technique used was snowball sampling (Sajtos and Mitev, 2007), which is a non-probability sampling procedure. The explicit aim was to select research subjects belonging to specific groups.

The application and adoption of the snowball method is discussed in a number of scientific publications, such as Goodman (1961), who examined the technique in one of his early studies, or Biernacki and Waldorf (1981), who analysed in detail the potential and limitations of the method. These works help to understand the theoretical basis and practical application of snowball sampling, as well as its effectiveness and limitations in different research contexts.

One of the main disadvantages of the sampling procedure in this study is that it does not provide random sampling, so the results obtained may not be generalizable to the larger population. In addition, there is a risk of sample bias, as the sample is often limited to the social networks of the initial participants, which may affect the objectivity of the results. The sampling procedure sought to compensate for these limitations in two ways:

1) On the one hand, the starting point for the sampling was not only personal social platforms (Facebook and LinkedIn), where the researcher first reached out to subjects in his own personal and professional network, but also to online articles related to the research, which provided access to the questionnaire to independent subjects;
2) Given the large number of respondents (sample of over 500), sampling error is reduced and reliability is increased (Alreck-Settle, 1995).

It is important to stress, however, that the research is exploratory in nature, and thus its short-term aim is to test the research questions and their primary hypotheses and to prepare the ground for follow-up research that the present findings justify. For this reason, the fact that the questionnaire was completed by those who felt that they were in some way related to its subject matter, given the way in which the sample was selected, is of particular importance. At this stage, it was not intended to survey those who were not related to mathematics. Data collection was carried out during 2021.

1.2. Detailed structure of the questionnaire

The questionnaire consisted of 5 blocks. The data collection tool was Qualtrics, in which the respondents answered the questions step by step, taking into account the protection of their personal data and anonymity. The data collection was self-completed. The questionnaire was shared in online social spaces (Facebook, LinkedIn), targeting easily accessible subjects, using the snowball method presented earlier. Given the exploratory nature of the research, representativeness of the sample was not expected. For a number of questions, which will be discussed later, the categorisation defined by the Central Statistical Office (KSH) was taken into account in determining the possible answers.

(1) Block 1: Understanding the demographic background

In the case of demographic background, the characteristics that are most likely to determine the personal characteristics of the respondents were surveyed. For the purposes of this research, these are:

- Respondent gender (Q1): male, female, other; variable type: nominal;
- Year of birth of respondent (Q2); variable type: ordinal;
- Respondent's type of residence (Q3, based on KSH): capital city, small and medium-sized city, county seat / city with county rights, municipality, big city, other; type of variable: nominal;
- Respondent's highest level of education (Q4): primary education, secondary education, tertiary education, academic degree; variable type: nominal;
- Respondent's personal preference (Q16): more 'realistic', more 'humanistic', more 'artistic', mixed; variable type: nominal;
- Respondent's field of work (Q5): Physical, Service, Intellectual; variable type: nominal;
- Respondent's job type (Q8): freelance, subordinate, team leader, middle manager, senior manager, student, retired; variable type: nominal;
- Respondent's type of job (Q6): self-employed, small and medium enterprise, large domestic enterprise, large multinational enterprise; student, not working, working in education; variable type: nominal;

(2) The second block: about mathematics, free

The purpose of the survey of personal free associations was to get a qualitative picture of the respondents' feelings about mathematics curriculum and mathematics education. In all cases, responses were free-form. The evaluation of these responses is not the purpose of this analysis, but for the sake of completeness the questions are presented here:

- According to respondents, if mathematics were a colour, what colour would it be (Q37);
- According to respondents, if maths were a garment, what kind of garment would it be (Q40);
- When respondents think about mathematics, what feelings, thoughts and images come to mind (Q39);
- When respondents think of their maths teachers, what feelings, thoughts and images come to mind (Q41);
- When respondents think of mathematics lessons, what feelings, thoughts and images come to mind (Q42).

(3) The third block: our experiences of learning mathematics

The third block was a five-point Likert scale to assess the learning of mathematics and the experiences related to it. In each case, the answers were to be given according to the level of education, i.e. primary, secondary, tertiary and academic. In the case of mathematical success, the five-point scale corresponded to the average results obtained in the Hungarian assessment system. The questions covered the following areas:

- How successful respondents remembered being in mathematics during their studies (Q9_1: Primary Mathematics; Q9_2: Secondary Mathematics; Q9_3: Higher Mathematics; Q9_4: Degree; Together: Q9); selecting five stars indicated an A and selecting one star indicated an unsatisfactory academic performance. Variable type: metric.
- How much respondents remembered how much they enjoyed studying mathematics during their studies (Q10_1: Undergraduate Mathematics; Q10_2: Secondary Mathematics; Q10_3: Postgraduate Mathematics; Q10_4: Academic Degree. Together: Q10); the selection of five stars indicated "I liked it very much", the selection of one star indicated "I did not like it at all". Variable type: metric.

- How much respondents remembered liking their mathematics teachers during their studies (Q11_1: Primary Mathematics, Q11_2: Secondary Mathematics, Q11_3: Higher Mathematics; Q11_4: Degree; Together: Q11); the five star selection indicated "liked very much" and the one star selection indicated "did not like at all". Variable type: metric.
- How useful did the respondents feel the mathematics they studied was, according to their recollection (Q12_1: Primary Mathematics, Q12_2: Secondary Mathematics, Q12_3: Higher Mathematics; Q12_4: Degree, Together: Q12); the five stars selected were for "very useful" and the one star selected were for "I don't understand why at all". Type of variable: metric.
- Respondents were asked how much they remembered being engaged in mathematics classes during their studies (Q18_1: Primary Mathematics, Q18_2: Secondary Mathematics, Q18_3: Higher Mathematics; Q18_4: Degree; Together Q18); the five stars selected were for "completely engaged" and the one star selected were for "not at all engaged". Variable type: metric.
- Did respondents ever need a paid tutor and if so, during which period of their studies (Q13): during primary education; during secondary education; during higher education; when they obtained a degree; never. Type of variable: nominal;
- For respondents who had a private tutor, what was the experience of the private lesson (Q21): it was good and they finally understood the maths; they could practise individually but it was not different from the lesson; the private tutor did not help them understand better. Type of variable: nominal.
- Have respondents attended a mathematics course (Q22): yes, no. variable type: nominal;
- Respondents who have obtained a school leaving certificate, at what level (Q20): intermediate (used to be school leaving certificate); advanced (used to be university entrance examination); no mathematics. Type of variable: nominal.
- What role did mathematics play in respondents' career choice (Q15): chose a career that did not require mathematics; chose a career that required mathematics; chose a career that required mathematics; chose a career that required mathematics; about to choose a career. Type of variable: nominal.
- Respondents were also asked to indicate their favourite mathematical topic (Q14). The aim here was to ensure that at least one area appeared before each respondent,

so it was not possible to give an answer that avoided it. Mathematical logic, Set theory, Number theory, Algebra, Plane geometry, Spatial geometry, Vectors, Trigonometry, Coordinate geometry, Combinatorics, Graphs, Functions, Sequences, Elements of the analysis of univariate functions, Statistics, Probability calculus. Type of variable: nominal.

(4) The fourth block: work and mathematics

The fourth block examined the relationship with mathematics in the triangle of career choices, work and decisions:

- Respondents were asked to indicate when they use a mathematical approach in decisions about their everyday tasks (Q26): always use mathematics; depends on the decision; not usually, definitely not consciously. Type of variable: nominal.
- Respondents were asked whether they are influenced by numbers in their decisions about their daily tasks (Q28): yes; depends on the decision; no, rarely encounter numbers. Type of variable: nominal.
- Do respondents think that a mathematical approach would help them in making their own decisions (Q29): yes, definitely; it depends on the decision; they don't think they would make a different decision. Type of variable: nominal.
- Respondents think that people who are good at maths can make better decisions (Q30): yes, definitely; it depends on the decision; you wouldn't think they would make a different decision. Type of variable: nominal.
- What has helped respondents the most in learning maths (Q31): learnt to count well; learnt to think logically; sees minimal benefit. Type of variable: nominal.
- How much of a decision-maker does the respondent consider him/herself to be, where the respondent was asked to rate this on a 10-point Likert scale, with a value of 1 if he/she is a fully intuitive thinker and a value of 10 on the right if he/she is a maximally rational thinker (Q32). Type of variable: metric.
- Respondents were asked to list instances in their lives when they have had to use their knowledge of mathematics (Q33). Type of variable: nominal.

(5) The fifth block: occupations and mathematics

In this block, the aim was to assess respondents' perceptions of the need for mathematical skills in the context of each field of employment (Q23). Responses were given on a 5-

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point Likert scale, where a score of 5 represented that studying mathematics at school was very useful in that field and a score of 1 represented that studying mathematics at school was completely unnecessary in that field. The employment areas covered in the survey are based on the KSH and are detailed in Appendix 1.

An extracted description of the questions of the research questionnaire is given in Annex 1.

2. About the sample - results from the demographic background

The research sample was reduced to 505 people after data cleaning. Respondents who only started but did not complete the questionnaire or dropped out at some point (did not complete the questionnaire) were excluded from the analysis. The reduced sample size of 505 respondents is informative for the research: for a mathematics questionnaire of this depth, it is challenging to relate to mathematics and to do a focused piece of work in this context. The reduced sample of 505 respondents answered almost all the questions, for their age group. However, there were a few incomplete fields, which means that when analyses are presented on the data for the full sample, in some cases the total number of respondents is a few values below the 505.

2.1 Respondents' sex (Q1) and year of birth (Q2)

69.1% of respondents are female, 30.5% male and 0.4% have other identities. That means that more than twice as many women completed the questionnaire as men (Table 3).

Table 3: Gender distribution of respondents (own editing)

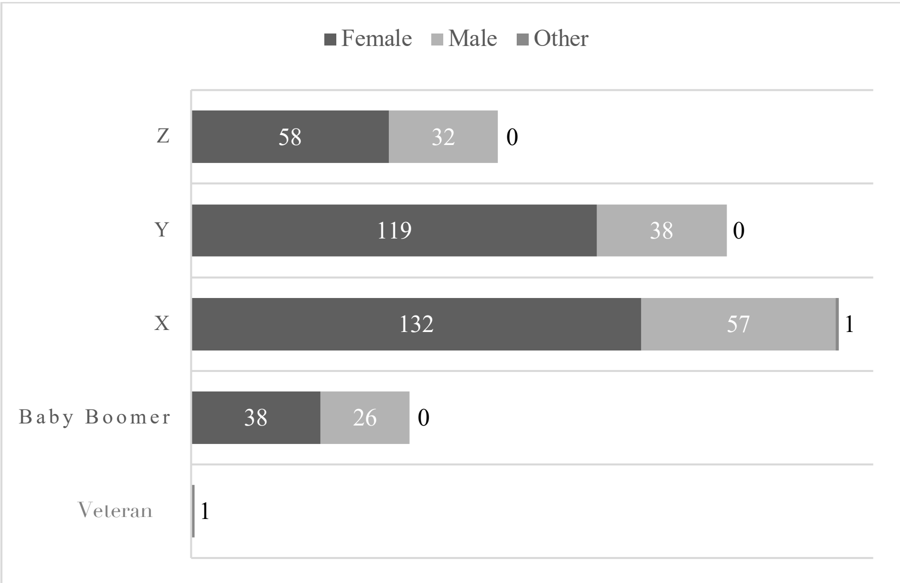
	No	
	Main (pcs)	Share (%)
Other	2	0.4%
Male	154	30.5%
Female	349	69.1%

In terms of the age of the respondents, five generational clusters were defined for the research based on Steigervald (2020): (1) the veteran generation includes respondents born in 1945 or before, i.e., those aged 79 or older; (2) the baby boomer generation includes respondents born between 1946 and 1964, i.e., those aged 60 to 78; (3) Generation X includes respondents born between 1965 and 1979, i.e. aged 45 to 59; (4)

Generation Y includes respondents born between 1980 and 1994, i.e. aged 30 to 44; and finally (5) Generation Z includes respondents born between 1995 and 2009, i.e. aged 15 to 29.

The breakdown of respondents by generation (Figure 13) shows that the largest proportion of respondents to the questionnaire was from generations X and Y, with 347 respondents, representing nearly 70% of the total. Not surprisingly, given the composition of the sample, more women than men responded to the questionnaire for each generation, and, excluding Baby Boomers, nearly twice as many women as men completed the questionnaire per generation. However, the sample sizes by generation allow them to be examined separately in terms of the research questions as part of the exploratory research.

Figure 13: Gender distribution of respondents by generation. The horizontal axis shows the number of respondents (pieces), the vertical axis shows the generations (own editing).



2.2 Respondent's place of residence (Q3) and highest level of education (Q4)

Respondents were typically from the capital and the county capital/cities with county status, with nearly 68%. However, when looking at the proportion of respondents from within and outside the capital, nearly 58% of respondents were from the capital and nearly 42% were from outside the capital. Also in terms of educational attainment, nearly 68% of respondents have a tertiary education and a further nearly 10% have an academic degree. The breakdown of responses is shown in Table 4.

Table 4: Breakdown of respondents (in number) by place of residence and highest level of education (own editing)

	CAPITAL	County seat / City with County Rights	Big city	Small and Medium City	Municipality
Primary Education	7	1	1	1	1
Secondary Education	47	16	4	18	12
Tertiary Education	200	32	8	78	21
Scientific Degree	33	5	0	9	1

The high proportion of tertiary graduates in the sample is particularly important for the research because, given the exploratory nature of the research and the impact of mathematical literacy, it was specifically aimed to include subjects who had spent a long time in the education system and therefore had more experience.

2.3 Personal attitudes of respondents (Q16)

One of the defining characteristics of respondents for this research is the kind of thinker/decision-maker they consider themselves to be. Drawing on classical stereotypes and mainstream research, three main types were defined: (1) the "realistic" thinker, (2) the "humanistic" thinker and (3) the artist. Alternatively, a so-called "mixed" category has been defined for those who do not feel committed in either direction. The results are presented in Table 5.

Table 5: Breakdown of respondents by personal attitudes (own editing)

Personal attitudes	Main (pcs)	Share (%)
no reply	*1*	*0.2%*
I'm more "humanist"	122	24.2%
I am more of an "artist"	21	4.2%
I am more "realistic"	184	36.4%
Mixed	177	35.0%

The majority of respondents, 36.4%, identified themselves as realistic, just over 24.2% as humanistic, 4.2% as artistic and the remaining 35% as mixed (0.2% of respondents did not answer this question).

2.4 Respondents' field of work (Q5)

Analysing the composition of the sample, 85.5% of the respondents work in the field of "Intellectual work", 9.1% in the field of "Service work" and only 3% declared their field as "Physical work" and 15 did not answer (Table 6).

Table 6: Distribution of respondents by field of work (own editing)

Workspace	Main (pcs)	Share (%)
no reply	15	3.0%
Physical work	12	2.4%
Intellectual work	432	85.5%
Service area	46	9.1%

2.5 Respondents' job seniority (Q8)

The job title of the respondents is of particular importance for the research, as it is the job title that is a good representation of the decision-making competences. The more senior the job (or the more jobs the respondent has), the more professional competences the respondent has and the more decisions the respondent has to make.

The three categories of managers that are relevant for decision-making accounted for a total of 24.6% of respondents, representing almost a quarter of them. Of these, 21 were "Team Leaders" (4.2% of the sample), 59 "Middle Managers" (11.7% of the sample) and

44 "Senior Managers" (8.7% of the sample). A further 37.4% of respondents were "Subordinate" (189) and 17.4% "Freelance". The remaining respondents were "Retired" (5%, 25 people) and "Student" (14.5%, 73 people). Their distribution is shown in detail in Table 7.

Table 7: Breakdown of respondents' jobs (own editing)

Job title	Main (pcs)	Share (%)
no reply	6	1.2%
Posted on	189	37.4%
Team leader	21	4.2%
Student	73	14.5%
Top management	44	8.7%
Middle manager	59	11.7%
Retired	25	5.0%
Freelance	88	17.4%

2.6 Type of respondents' workplace (Q6)

The type of workplace is also representative of problem-solving and decision-making needs and skills. Half of the respondents (50.1%, 253 respondents) work in the corporate sector, 22.6% in "SMEs", 11.7% in "Large domestic companies" and 15.8% in "Large multinational companies". The second highest proportion of respondents (19%) are in "Education" (96). Of the remaining respondents, 62 respondents indicated "Student" (12.3%) and 26 "Not working" (5.1%). Their distribution is presented in detail in Table 8

Table 8: Distribution of respondents by type of job (own editing)

Type of job	Main (pcs)	Share (%)
no reply	16	3.2%
Sole trader	52	10.3%
Large domestic company	59	11.7%
Small and medium-sized enterprises	114	22.6%
Large multinational company	80	15.8%
I do not work	26	5.1%
I work in education	96	19.0%
I am a student	62	12.3%

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3. Correlations observed based on the results of the demographic background

In order to understand the basic characteristics of the sample, it is essential to identify possible correlations between the individual criteria being tested. At this stage of the research, the correlations were first determined using frame tables, followed by the calculation of correlation coefficients to show the existence of relationships and their closeness.

3.1 Relationship between generations (Q2) and personal attitudes (Q16)

The distribution of responses by personal attitudes for each generation is shown in detail in Table 9.

Table 9: Generational breakdown of respondents by personal attitudes. Values represent the number of respondents (in persons) (own editing)

	Veteran	Baby Boomer	X	Y	Z
Real	0	26	74	48	36
Human	0	13	48	43	18
Artist	0	4	3	11	3
Mixed	1	21	65	55	34
Total	1	64	190	157	91

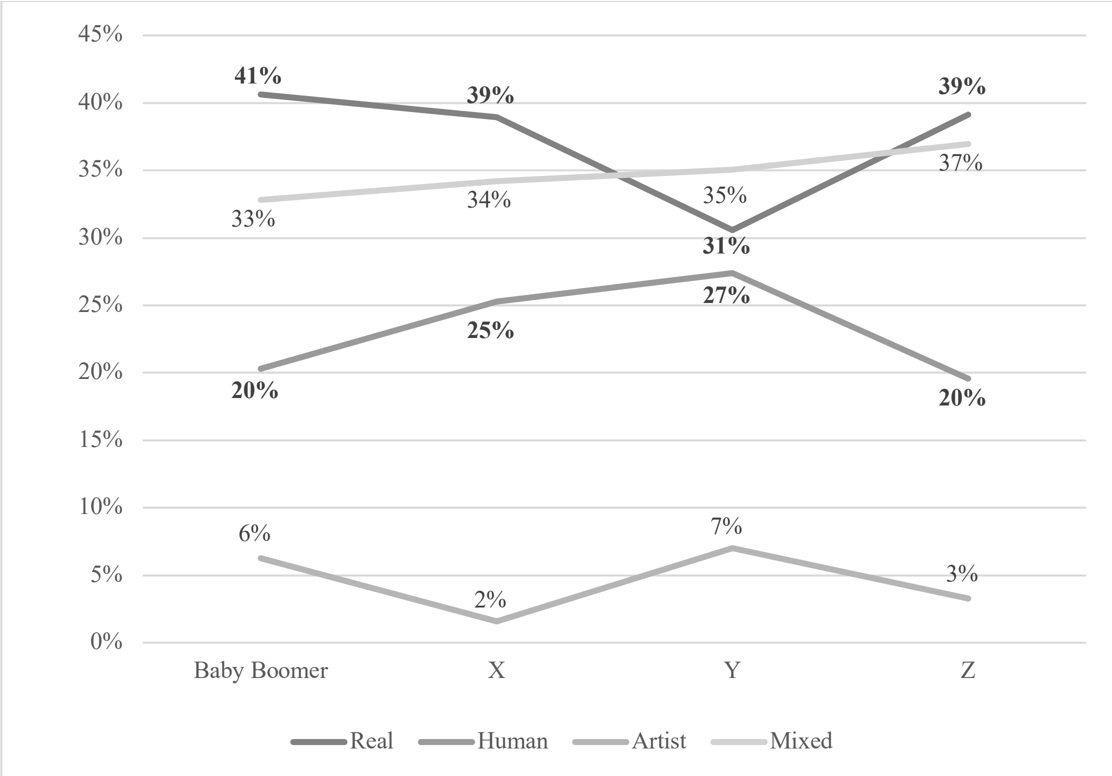
The data presented in Table 9 shows the proportion of personal attitudes in each generation, which is illustrated in Figure 14. Three characteristics can be observed in relation to the sample:

(1) The proportion of "human" and "realistic" respondents varies from generation to generation in opposite directions for Baby Boomers, Generations X, Y and Z. The Pearson correlation coefficient between the two groups is -0.79.

(2) The number of "mixed" respondents has been increasing from generation to generation, rising slowly but in a strictly monotonous fashion from 33% of the Baby Boomer generation to 37% of Generation Z.

(3) And the proportion of respondents with an "artist" attitude varies from generation to generation, with higher and lower proportions.

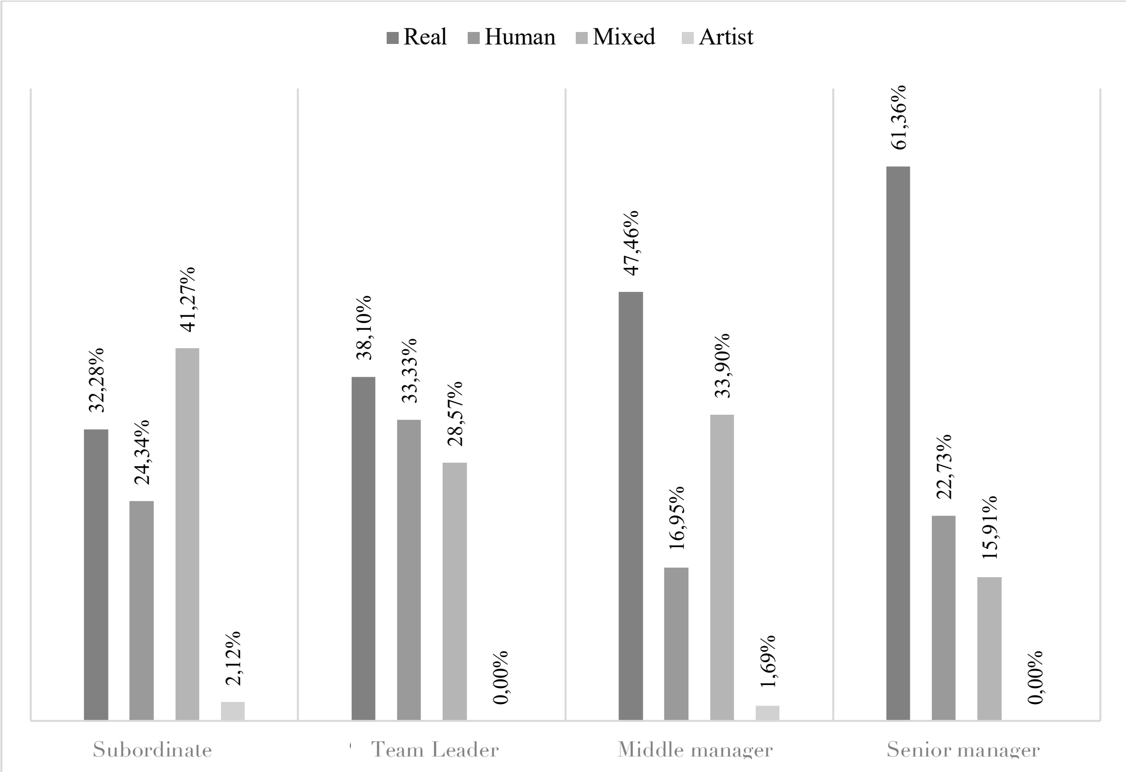
Figure 14: Percentage distribution of personal attitudes by generation (own editing)



3.2 Relationship between job (Q8) and personal attitude (Q16)

As regards respondents, it is worth observing the differences between job and personal attitudes. While in the case of 'Subordinate' and 'Team Leader', the proportion of those with a 'Real' and 'Human' attitude is almost equal, 2.8 per cent of 'Middle Managers' and 2.7 per cent of 'Senior Managers' say they have a 'Real' attitude compared to a 'Human' attitude. The proportions are illustrated in Figure 15.

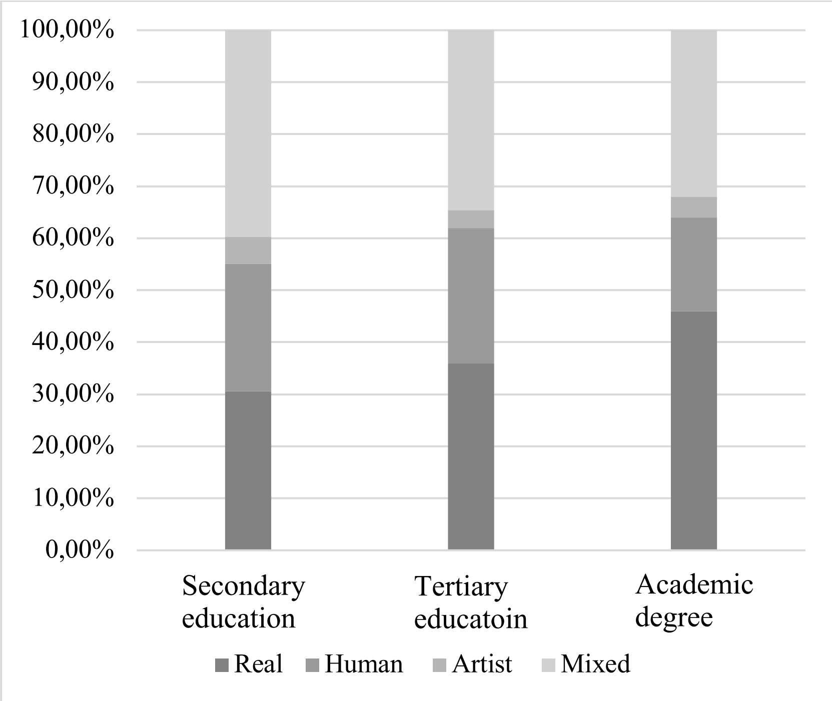
Figure 15: Percentage distribution of respondents' personal attitudes by job (own editing)



3.3 Relationship between educational attainment (Q4) and personal attitude (Q16)

In the case of the relationship between educational attainment and personal attitudes, the higher the level of education, the higher the proportion of respondents who consider themselves to be realistic. While this is 30.61% for secondary education, it is 36.05% for tertiary education and 46% for those with an academic degree. At the same time, the proportion of respondents who say they are mixed declines, from 39.8% for those with upper secondary education to 34.59% for those with tertiary education and 32% for those with an academic degree. Humanities attitudes are almost the same for those with secondary and tertiary education at 24.49% and 25.87% respectively, while the proportion is lower for those with an academic degree at only 18%. The results are illustrated in Figure 16.

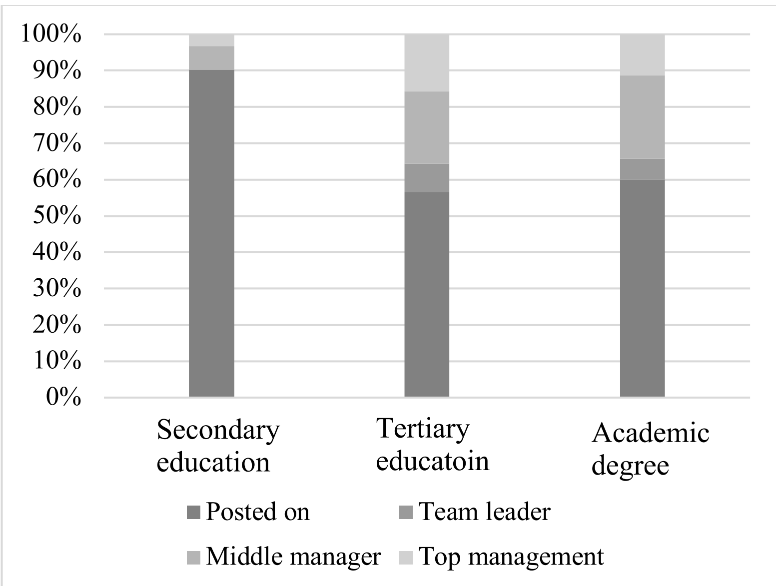
Figure 16: Relationship between educational attainment and personal attitudes (own editing)



3.4 Relationship between educational attainment (Q4) and job (Q8)

The largest difference between education and job is between those with secondary and tertiary education (Figure 17). 90% of those with secondary education are in a subordinate job, i.e. not in a managerial position. In contrast, 43.43% of the tertiary graduates in the sample are in managerial positions and 15% are senior managers.

Figure 17: Distribution of respondents' jobs by educational attainment (own editing)



4. **Mathematical success (Q9)**

Of particular relevance to this research is the analysis of respondents' experiences of mathematical success (Q9_1, Q9_2, Q9_3, Q9_4). Mathematical success is a measure of the level of knowledge that a given respondent claims to have based on the conditions (marks) expected by the education system. Success rates by educational level are presented in Tables 10-13. The values *'NA'* indicate respondents who did not answer because they were no longer studying at the level of education. In the latter case, it can be said from the responses that almost all of the respondents who had studied at primary school went on to study at secondary school. However, 23% of the respondents did not continue their studies after secondary education.

Table 10: Respondents' success in mathematics in primary school (own editing)

	Primary school	
	Main (pcs)	Share (%)
1	1	0.2%
2	6	1.2%
3	38	7.5%
4	97	19.2%
5	359	71.1%
NA	4	0.8%

Table 11: Respondents' success in mathematics in secondary school (own editing)

	High school	
	Main (pcs)	Share (%)
1	5	1.0%
2	33	6.5%
3	80	15.8%
4	127	25.1%
5	251	49.7%
NA	*9*	*1.8%*

Table 12: Respondents' success in mathematics in higher education (own editing)

Higher education	Main (pcs)	Share (%)
0	*4*	*0.8%*
1	20	4.0%
2	55	10.9%
3	101	20.0%
4	125	24.8%
5	84	16.6%
NA	*116*	*23.0%*

Table 13: Respondents' mathematical success in obtaining an academic degree (own editing)

Obtaining a scientific degree	Main (pcs)	Share (%)
0	*17*	*3.4%*
1	28	5.5%
2	3	0.6%
3	16	3.2%
4	13	2.6%
5	17	3.4%
NA	*411*	*81.4%*

Descriptive statistical analysis of the responses for the whole sample (Table 14) shows that mathematical success scores worsen as educational levels progress. While at primary school the average grade point average (success rate) is 4.61 with a very low standard deviation of 0.697, at the academic degree level the average is only 2.33 and the standard deviation increases significantly to 1.816.

Table 14: Some descriptive statistical indicators of mathematical success by level of education (own editing)

	Average	Source	Slant	Simplicity
Maths success in primary school	4.61	0.697	-1.857	3.201
Maths success in secondary school	4.18	0.997	-1.022	0.149
Maths success in higher education	3.48	1.183	-.544	-.261
Success in maths for science degrees	2.33	1.816	0.198	-1.458

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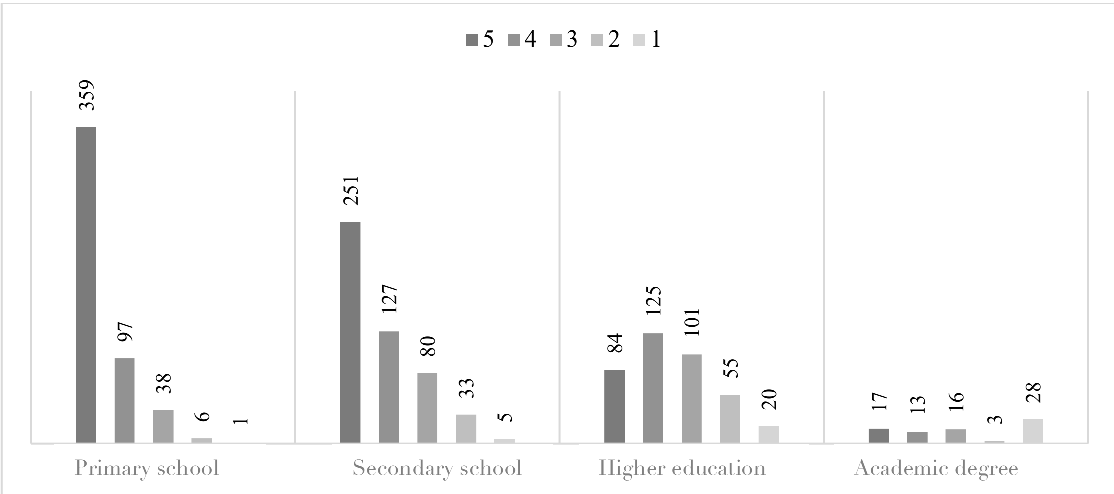
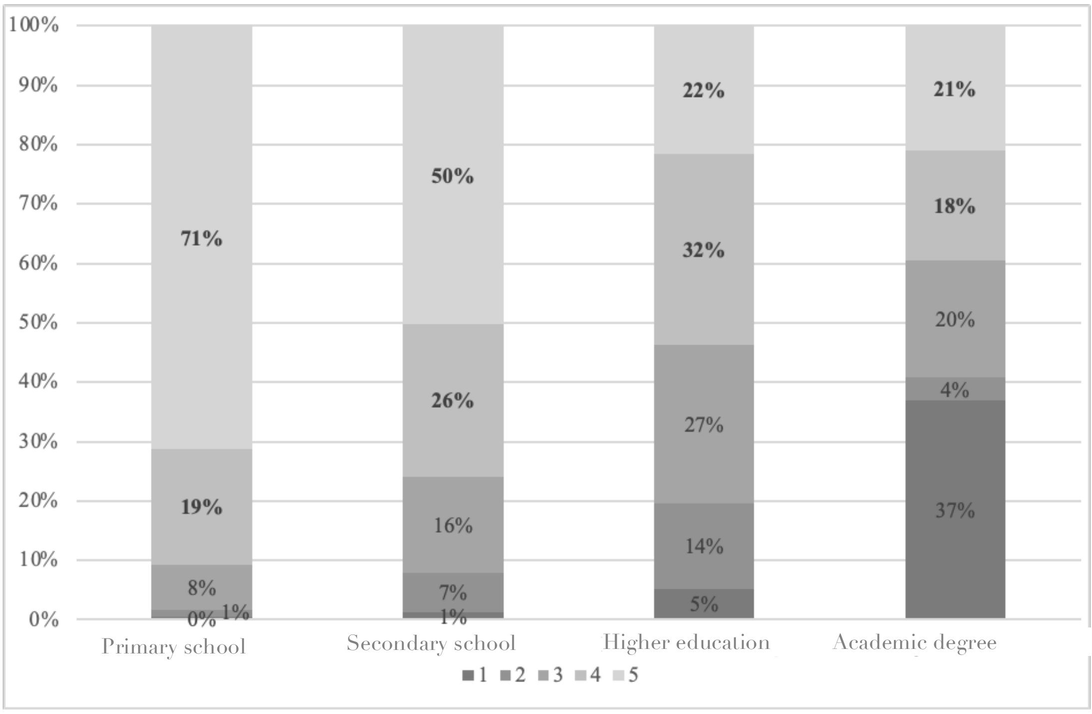
The skewness indicators show that while the data are still heavily concentrated in the direction of an A in primary school with a value of -1.857, the value is only -1.022 in secondary school, -0.544 in higher education and 0.198 in academic degree, i.e. a shift towards unsatisfactory performance. There is also a downward trend in the peak performance, from a value of 3.2 to almost -1.46, which means that while there is a significant peak at the beginning (almost all A's in primary school), this dominance disappears as the studies progress and the distribution of performance among respondents flattens out. The results are illustrated in Figure 18.

Figure 18: Distribution of respondents' success rates by educational level (own ed)



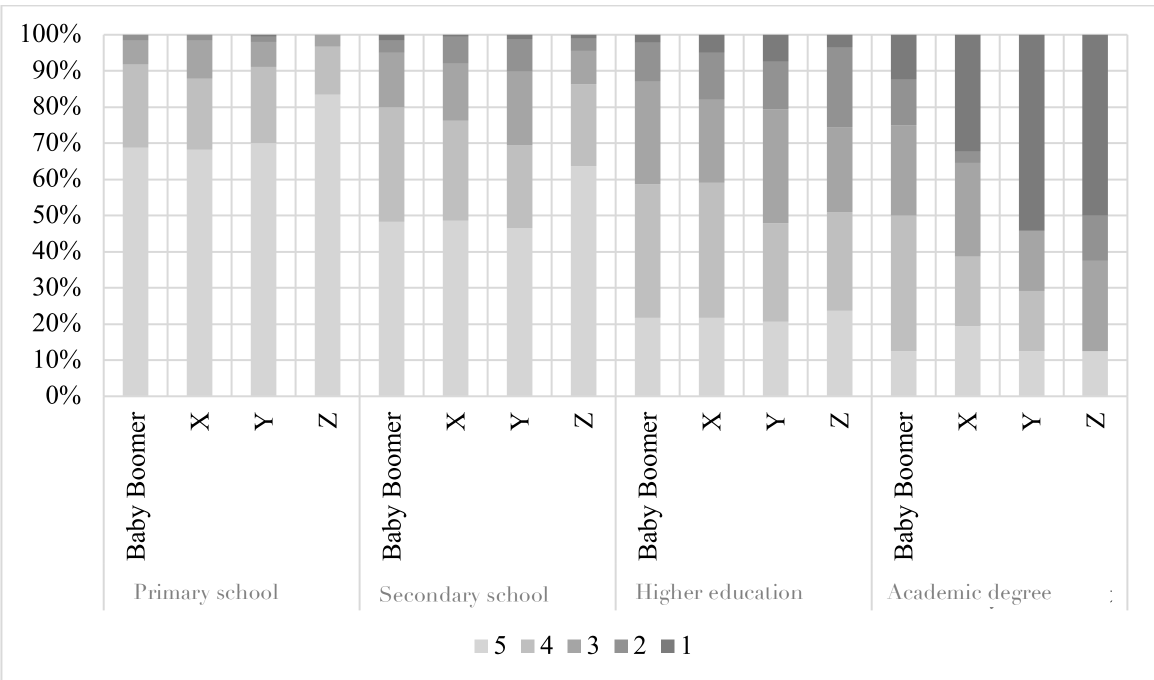
It is striking that the performance of respondents in primary education is outstanding (Figure 19). Nearly 90 per cent of them recall receiving a grade of 4 or better, and 71 per cent achieved an excellent result. However, this positive performance changes as the years progress. Already during secondary education, the proportion of those with the best performance (4 or 5) drops from 91 per cent to nearly 76 per cent, which is only 54 per cent in higher education and less than 40 per cent for academic degrees. In contrast, poor performance shows a significant spike and there are signs of a bipolar pattern of mathematical performance. Some know mathematics and some do not, and a gap appears between the two: the 2-score performance is close to 4 percent and is on either side of 37 and 59 percent of respondents, respectively.

Figure 19: Success rate of respondents by educational level (own ed)



In the context of mathematical success, the question arises as to whether generations have performed differently. The large number of respondents allowed the sample to be analysed by generation, the results of which are shown in Figure 20.

Figure 20: Success rate of respondents by educational level compared by generation (own editing)



At each level of study, the generational distribution of mathematical success was very similar, with very similar levels of success. Therefore, a statistical analysis of the distribution of data between each data set was determined by calculating a correlation coefficient (Figure 21). Rounded to two decimal places, the value of the correlation coefficient between any two generations is less than or equal to 0.96. This means that all generations of respondents had a very similar performance, so that in essence, the trend in success rates is the same across generations.

Figure 21: Performance of generations compared by pair (own editing)

	Baby Boomer	X	Y	Z
Baby Boomer	1,00	0,99	0,98	0,96
X		1,00	0,98	0,97
Y			1,00	0,97
Z				1,00

5. Research model and results related to the hypotheses of H1

The research hypothesis H1 is the following:

H1 Cognitive mathematical skills are related to problem-solving and decision-making skills:

H1a The more educated someone is in relevant mathematical skills related to their profession, the more likely they are to be a problem solver and decision maker.

H1b The mathematical competences acquired have an impact on thinking skills and behaviour, thus indirectly leading to more successful decisions.

5.1 Research model related to hypothesis H1

The phenomena, qualities and abilities represented in the hypothesis are discussed in the research questionnaire by the following questions:

(1) Cognitive mathematical skills: variables based on mathematical knowledge acquired in the education system during primary, secondary, tertiary and post-secondary studies, measured through mathematical success. Measured variables and their notation: Mathematical success in primary school: Q9_1; Mathematical success in secondary school: Q9_2, Mathematical success in higher education: Q9_3, Mathematical success in science degree: Q9_4.

(2) Problem-solving and decision-making skills: problem-solving and decision-making skills are measured indirectly, based on personal attitudes and respondents' decision-making style, as well as on respondents' highest level of education and job title. The latter cases are justified by the fact that, on the one hand, higher educational attainment implies higher levels of problem-solving skills in the relevant field of expertise; on the other hand, job titles in all cases define a level of responsibility, with more decision-making powers typically being assigned to managerial jobs. The variables measured and their labels are.

(3) Vocationally relevant mathematical skills: in this research, vocationally relevant mathematical skills are defined as the skills acquired in higher education and in the mathematics taught at the time of obtaining a scientific degree, and whether mathematics was a criterion for the respondent's choice of career in further education. Given the exploratory nature of the research, the focus here is on respondents who have a tertiary education and hold corporate positions. The variables measured and their labels are.

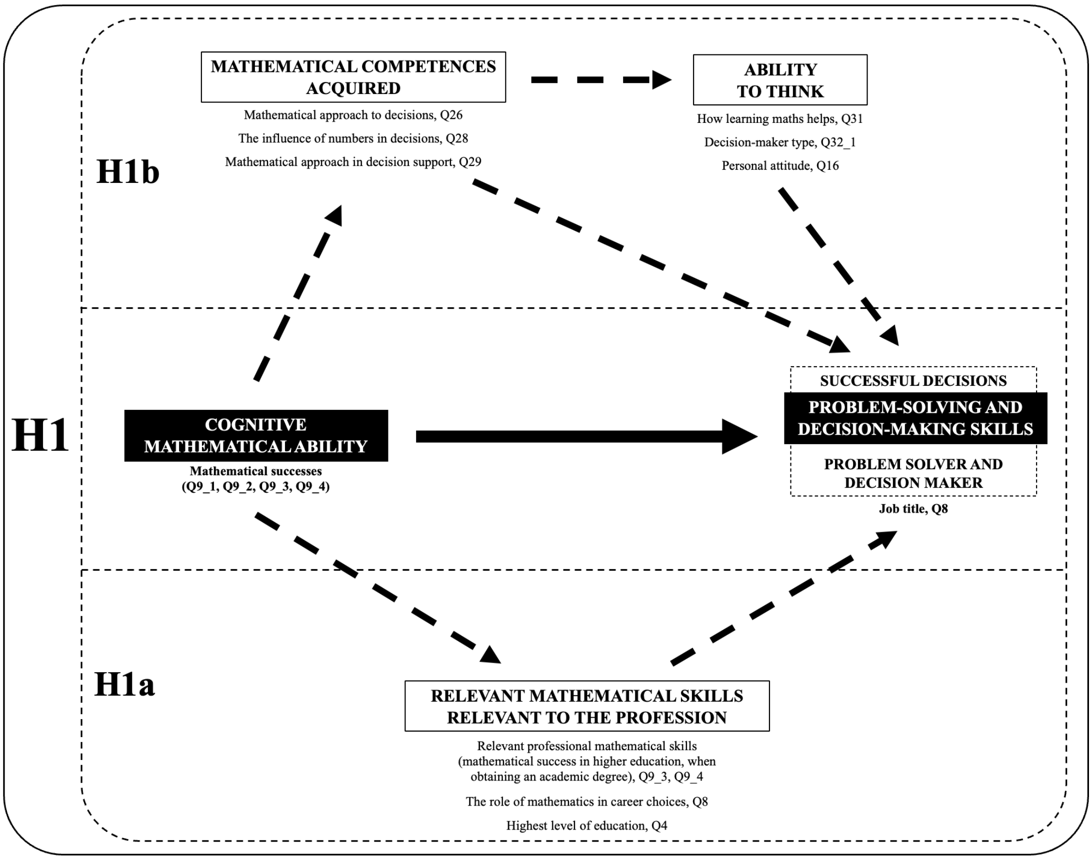
(4) Acquired mathematical competences: for the purposes of the research, 'acquired mathematical competences' are defined as all actions that are related to mathematics and are used by respondents in problem-solving and decision-making situations. These include mathematical approaches such as the use of and attitudes towards quantification. Measured variables and their notation: mathematical approach in decision making: Q26; Influence of numbers in decision making: Q28; Aiding effect of mathematical approach in decision making: Q29.

(5) Thinking skills: thinking skills are measured indirectly in two ways. First, respondents' perceptions of how their mathematics studies have helped them. On the other hand, what respondents think about themselves, what their personal attitudes are and what kind of decision-maker they consider themselves to be. Variables measured and labelled.

(6) Successful decisions: successful decisions are also measured by the level of professionalism achieved in the occupations, indirectly, assuming that the successful professional decisions made are those that have led the respondent to become more recognised in his/her career, thus his/her professional work has been followed by promotions. Therefore, the corresponding variable is also Job title: Q8.

The research model related to hypothesis H1 is shown in Figure 22.

Figure 22: Distribution of respondents according to what kind of decision-maker they consider themselves to be (own editing)

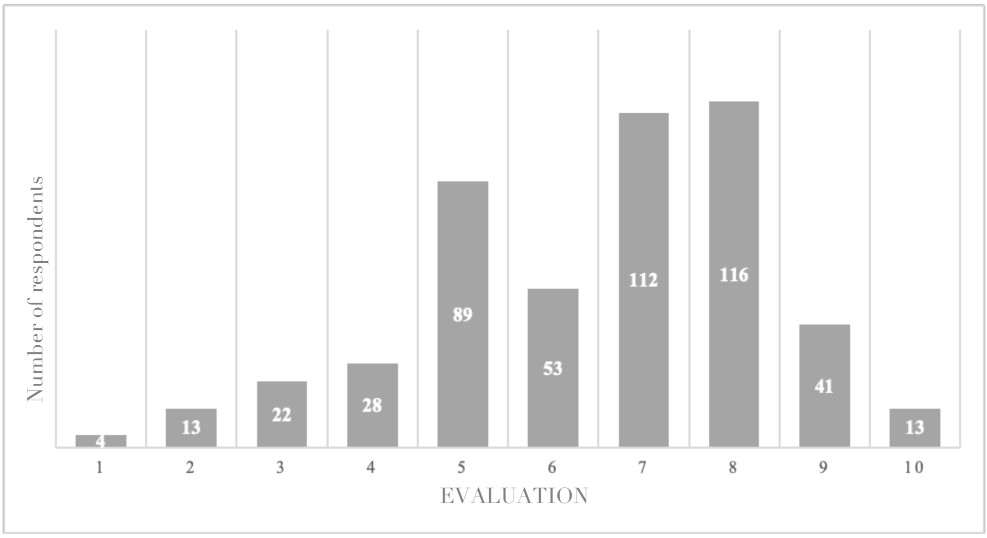


5.2 Research findings related to hypothesis H1

5.2.1 What kind of decision-maker is the respondent: Q32_1

Respondents were also asked to say what kind of decision-maker they consider themselves to be. They were asked to rate their responses on a Likert scale of 1 to 10, with a score of 1 indicating that they make decisions in a fully intuitive way and a score of 10 indicating that they make a maximum effort to make rational decisions. The results are shown in Figure 23.

Figure 23: Distribution of respondents according to what kind of decision-maker they consider themselves to be (own editing)



The choice of graph representation also shows that respondents tend to shift towards rationality when the question is about how they decide. The skewness calculated for the data is 7.4, i.e. the sample is asymmetric towards rationality.

5.2.2 The role of mathematics in career choices: Q15

Almost 70% of respondents chose (or had to choose) a career that required mathematics and almost 20% of them went into a career that specifically required mathematics. This shows that a significant proportion of the sample of career choices cannot exclude mathematics from their further education requirements. Results are detailed in Table 15.

Table 15: Distribution of respondents according to how much mathematics played a role in their career choice (own editing)

Mathematics and career choices

	Respondents (persons)	%
NA	*5*	*1.0%*
I chose a career that requires mathematics.	246	48.7%
I chose a career that is specifically mathematics-oriented.	95	18.8%
I chose a career that didn't require mathematics.	146	28.9%
I am about to choose a career.	13	2.6%

5.2.3 Mathematical approach in decision-making: Q26

The research question asked respondents to what extent they think they rely on mathematical approaches for their everyday decisions. Almost three quarters of respondents avoided taking a clear position on the question, indicating that it depends on the decision. The remainder, however, were clear on this question and while 6.3% indicated that they always take a mathematical approach to their decisions, 17.6% did not. That means that 2.8 times more of the confident respondents make a non-mathematical decision compared to those who do. The results are summarised in Table 16.

Table 16: Distribution of respondents according to the extent to which they use a mathematical approach in their decisions (own editing)

Mathematical approach to decisions

	Respondents (persons)	%
NA	*10*	*2.0%*
It depends on the decision.	374	74.1 %
Yes, I always base my decisions on mathematics.	32	6.3%
I don't, certainly not consciously.	89	17.6 %

5.2.4 The influence of numbers in the decision: Q28

Since mathematics involves the use of numbers, the question of the extent to which respondents are influenced by numbers (i.e. quantified information) when making decisions was also raised. Here only 40% of respondents were undecided. More than half of respondents, nearly 52%, said that they were influenced by numbers and only nearly 5% thought that they were not influenced by numbers in their decisions. This means that among confident respondents, 10 times as many people think that numbers influence their decisions. Table 17 shows the results in more detail.

Table 17: Distribution of respondents according to how much numbers influence their decisions (own editing)

Do the numbers influence the decision

	Respondents (persons)	%
NA	*10*	*2.0%*
It depends on the decision.	209	41.4%
Yes.	262	51.9%
No, I rarely see numbers.	24	4.8%

5.2.5 How the mathematical approach helps in decision-making: Q29

Of the personal perceptions of mathematics, it is particularly important to understand how respondents view mathematical competences. Therefore, they had to answer the question on their perception of whether a mathematical approach can help in decision making. Nearly 60% of the sample evaded this question, in the sense that it is determined by the decision they make. However, only 11.5% of confident respondents think it can't and the remaining nearly 26% think it can help. This means that among them, more than twice as many think that maths helps than do not. The results are shown in more detail in Table 18.

Table 18: Distribution of respondents according to the extent to which the mathematical approach helps them in their decisions (own editing)

Would a mathematical approach help you decide

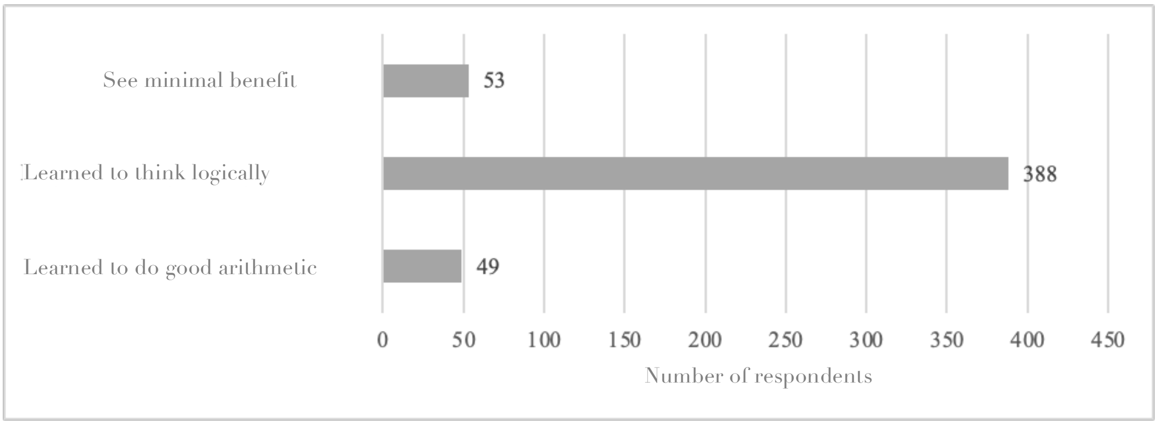
	Respondents (persons)	%
NA	*10*	*2.0%*
It depends on the decision.	306	60.6%
Yes, definitely.	131	25.9%
I don't think I would have chosen otherwise.	58	11.5%

5.2.6 How has learning maths helped, Q31

Another key question was how respondents think learning maths has helped them. There were three easy-choice options: (1) they see minimal benefit from studying mathematics,

(2) they have learned to think logically, and (3) they have learned to do good arithmetic. The results show that the vast majority of respondents (almost 80%) said that they had learned to think logically, and only almost 11% said that they saw minimal benefit from studying mathematics. However, the latter also assumes that one in 10 respondents in the sample think this. The results are shown graphically in Figure 24.

Figure 24: Distribution of respondents according to how learning mathematics has helped them (own editing)

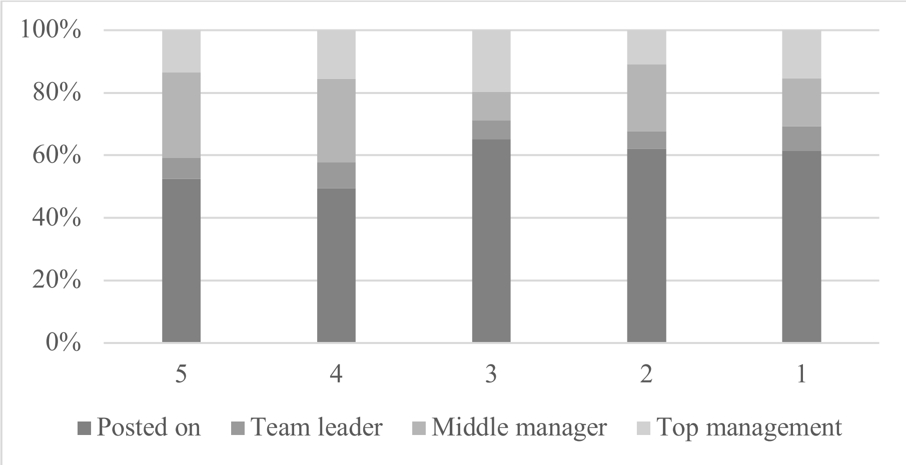


5.3 Relationships between research findings related to hypothesis H1

5.3.1 The relationship between mathematical success and job

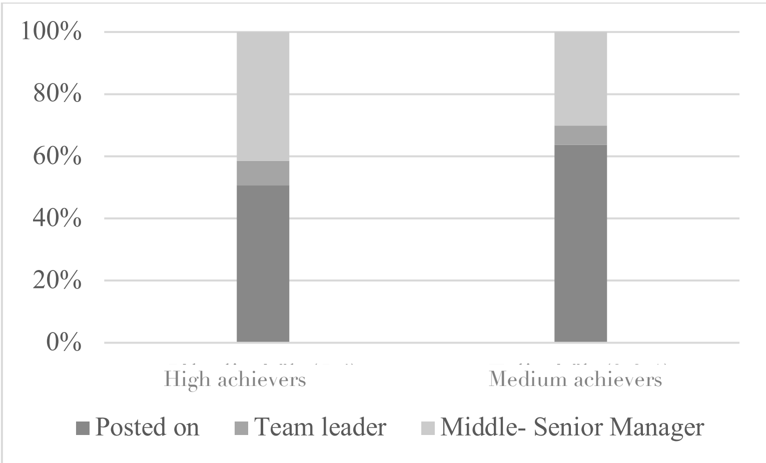
57% of respondents with tertiary education work as subordinates, 7% as team leaders, 21% as middle managers and 16% as senior managers. When looking at this proportion in the context of mathematical performance, different proportions hold managerial positions depending on the level of performance (excellent: 5, good: 4, medium: 3, fair: 2 or unsatisfactory: 1) at which they are declared to have performed (Figure 25).

Figure 25: Distribution of respondents according to the proportion of management positions held in terms of mathematical success in higher education (own editing)



If the data are analysed according to two clusters, one of which is composed of respondents who are at least high achievers, i.e. those who score 4 or 5 in terms of success in higher education (Q9_3) (high achievers), and the other of respondents who are at best medium achievers, who score 3, 2 or 1 (high achievers), the results between the two groups are significantly different (Figure 26).

Figure 26: Breakdown of respondents by the proportion of high performers and low performers in managerial jobs (own editing)



Only 50.7% of those who do well are employed as subordinates. In other words, nearly half of the respondents who were successful in mathematics became managers. Of these, 7.7% are team leaders, 26.8% are middle managers and 14.8% are senior managers.

The proportion of leaders is lower for those who pass maths. In their case, 63.8% of respondents, or 1.25 times more in proportion, hold a managerial position. And at middle and senior management level, only 30.3% are employed, compared with 41.6% of high achievers.

5.3.2 Relationship between mathematical success (Q9_3) and respondents' decision type (Q32_1)

When comparing the two criteria, the responses to the decision maker type were grouped into 3 categories. The first group, the more rational decision makers, gave themselves a score of 10, 9 or 8. The second, those in the middle, gave a score of 7, 6, 5 or 4. And finally the intuitive decision makers, who gave a score of 3, 2 or 1. The relationship between the mathematical success of tertiary graduates and their decision maker type led to the finding, in terms of sample characteristics, that respondents who are more successful in their mathematical studies consider themselves to be more rational decision makers than those who do not perform as well (Figure 27).

Figure 27: Distribution of respondents according to how mathematical success in higher education shapes the distribution of respondents by type of decision-maker (own editing)

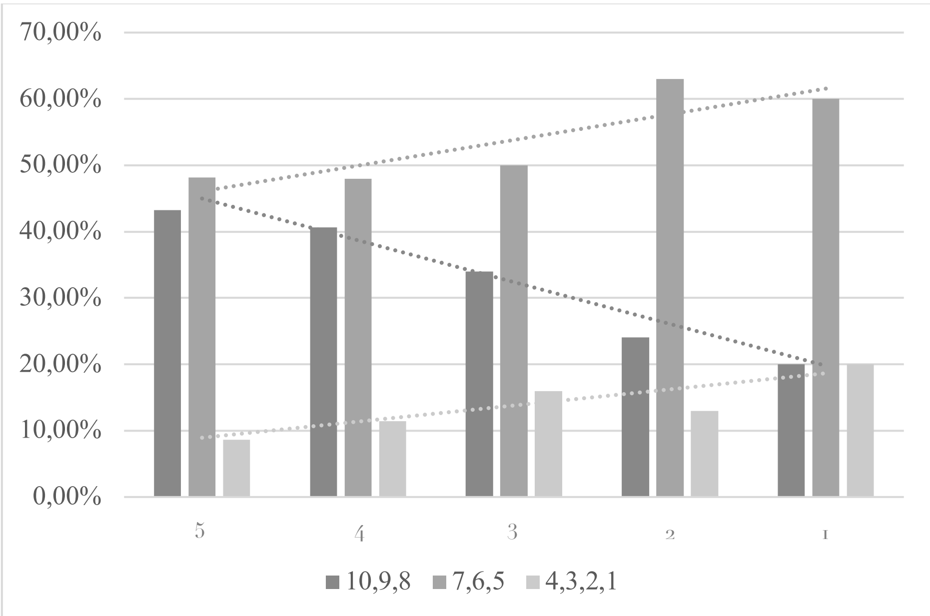
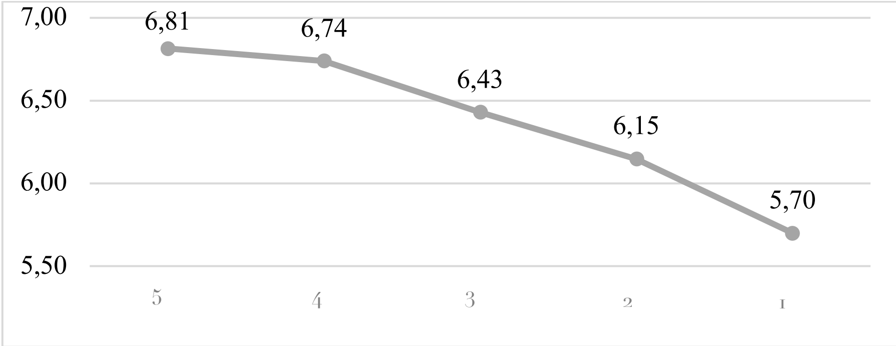


Figure 27 clearly shows that the proportion of students who consider themselves rational thinkers decreases significantly as they move from being A students to F students. There are now less than half as many low achievers as high achievers.

This decrease is illustrated by looking at the average of the values given by respondents for each level of mathematical performance. Figure 28 shows that its value decreases strictly monotonically, by nearly 1.1 units per achievement. Thus, it can definitely be said that those with higher mathematical achievement consider themselves more rational.

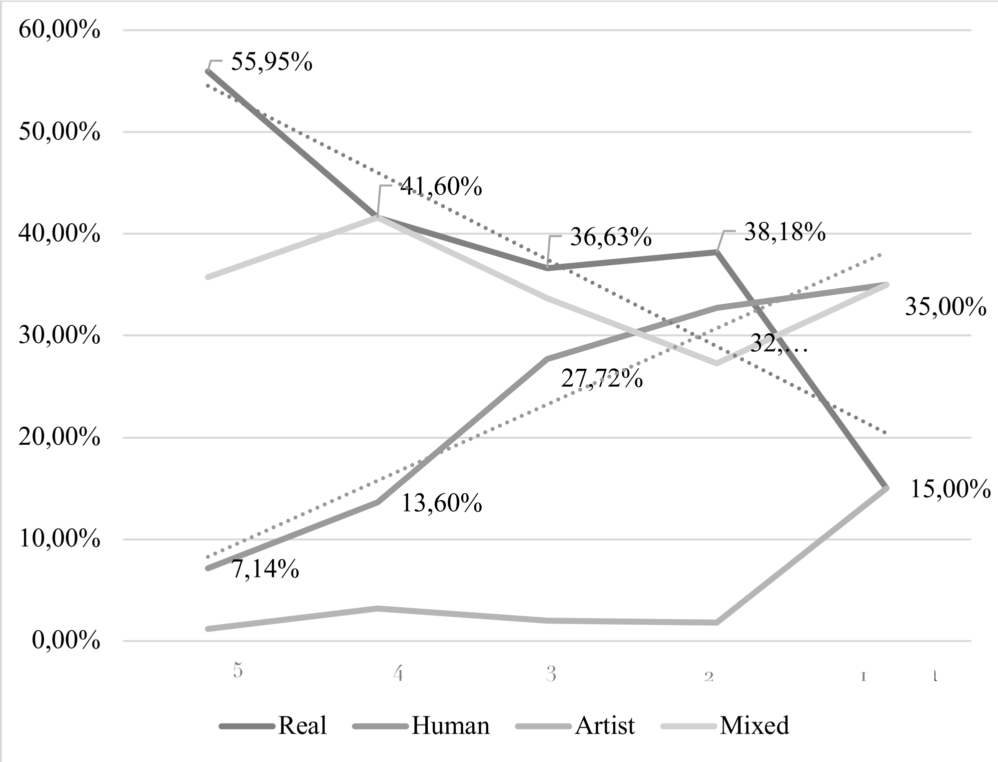
Figure 28: Average of the values given by respondents for each level of mathematical performance by type of decision-maker (own editing)



5.3.3 The relationship between mathematical success and personal attitudes

When looking at the relationship between mathematical success in higher education and personal orientation, it is striking how high the proportion of realistic achievers is: 55.95%, meaning that one in two respondents consider themselves to be realistic. This proportion also decreases as the success rate falls, down to 15%, or less than a third. However, it is worth noting that the proportion of mixed respondents at all five levels is in the range of 30-40%. In other words, as the mathematical success rate decreases, the human bias towards realism increases. The humanistic type is only 7.14% for the marked respondents. However, this value increases in a strictly monotonic manner as success rate decreases. For respondents with unsatisfactory marks, it is as high as 35%, which is proportionally five times as high as for the excellent respondents. The artist's attitude hovers around 2% almost all the way through, but shows a significant spike among unsatisfactory performers, where it rises to 15%. The results are illustrated in Figure 29.

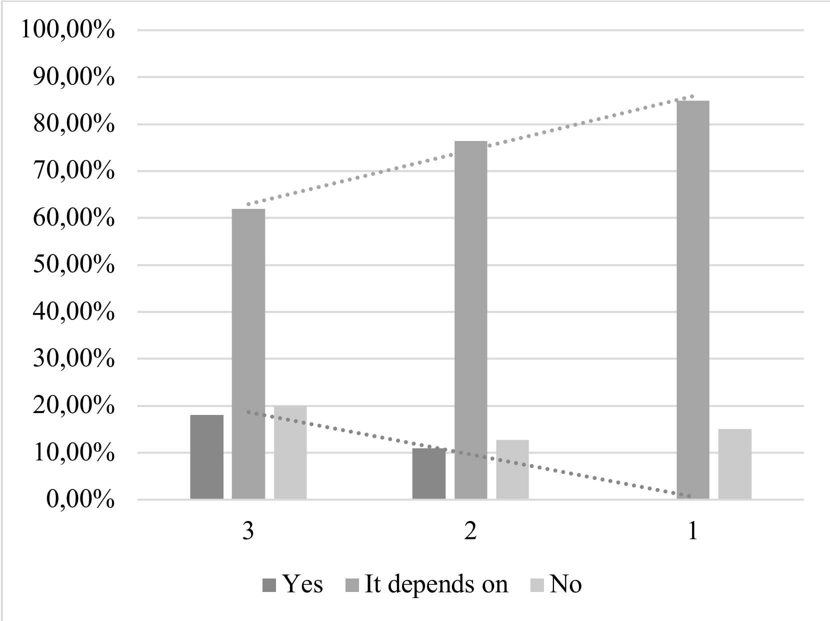
Figure 29: The relationship between mathematical success in higher education and personal attitudes (own editing)



5.3.4 The relationship between mathematical success and mathematically based decisions

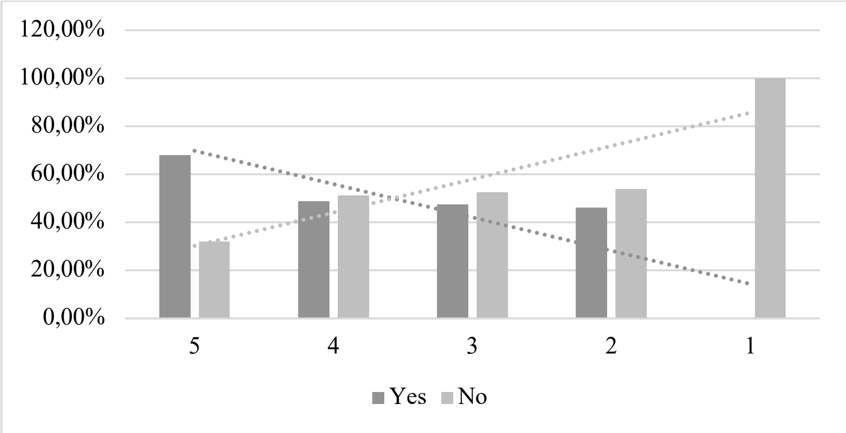
Those with tertiary education have studied mathematics in some form for the longest time in their lives. Therefore, it is particularly interesting to look at their perceptions of the role of mathematics in their decisions: if people are successful in mathematics, do they make better decisions. For this question, respondents had three options to choose from: yes; no; depends on the question. The experience with the responses to this question was that, from moderate to unsatisfactory, the majority of respondents (the higher the proportion, the lower the mathematical performance) moved towards a neutral, third answer (Figure 30). At the same time, it can be observed that the proportion of yes answers is also decreasing.

Figure 30: Distribution of views among those with tertiary education on, whether good mathematical skills lead to better decisions (own editing)



However, if the sample is narrowed down to respondents who have a firm idea on the question, it is worth looking at how the 'yes' and 'no' answers vary as a function of the success rate in mathematics during higher education (Figure 30). Those who have achieved an unsatisfactory (1) result in mathematics completely reject the idea that mathematics influences their decisions. It is true that we are talking about only 3 people in the sample, so this proportion is by no means representative. When the performance is fair (2), medium (3) or good (4), respondents are almost equally likely (between 46.15% and 48.78%) and almost equally likely to think that the better mathematician makes a better decision. Conversely, this nearly 47% to 53% ratio in favour of "no" is heavily skewed in the case of marked (5) performance: 68% of respondents think that the better mathematician makes a better decision, as shown in Figure 31.

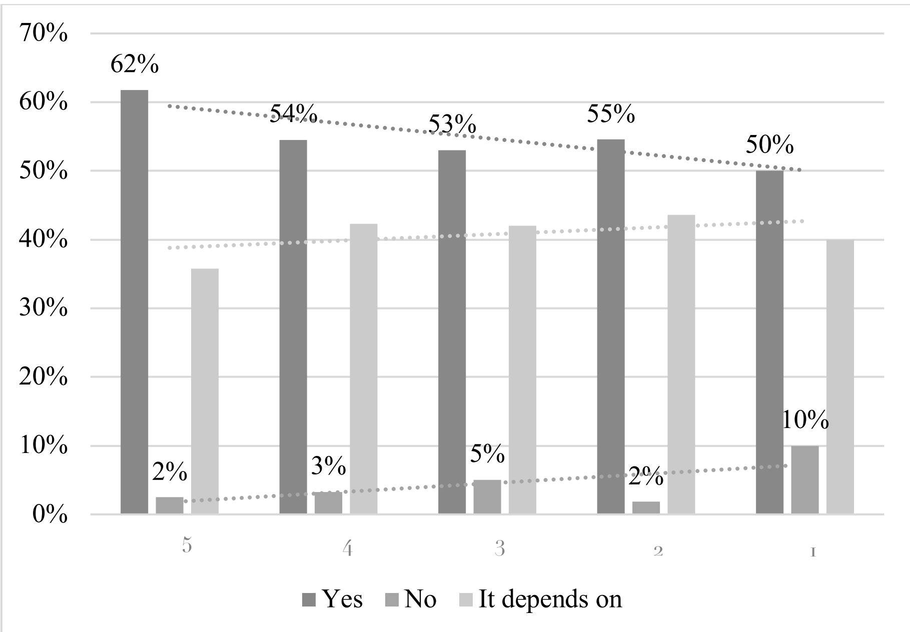
Figure 31: Distribution of opinions among tertiary graduates with a definite answer on whether good mathematical skills lead to better decisions (own survey)



5.3.5 The relationship between mathematical success and the decision influence of numbers

Regardless of mathematical success, it can be said that numbers influence at least one in two respondents in their decisions. The rate is 62% for A+ respondents and 50% for B+ respondents. In other words, although the rate is falling, it is not falling at the same rate as the success rate. Conversely, as success rates fall, the number of those who say they are not influenced by numbers increases. Their share is 2% for "good" and 10% for "unsatisfactory". So five times more people think that numbers do not influence their decisions if they are unsuccessful in mathematics (Figure 32).

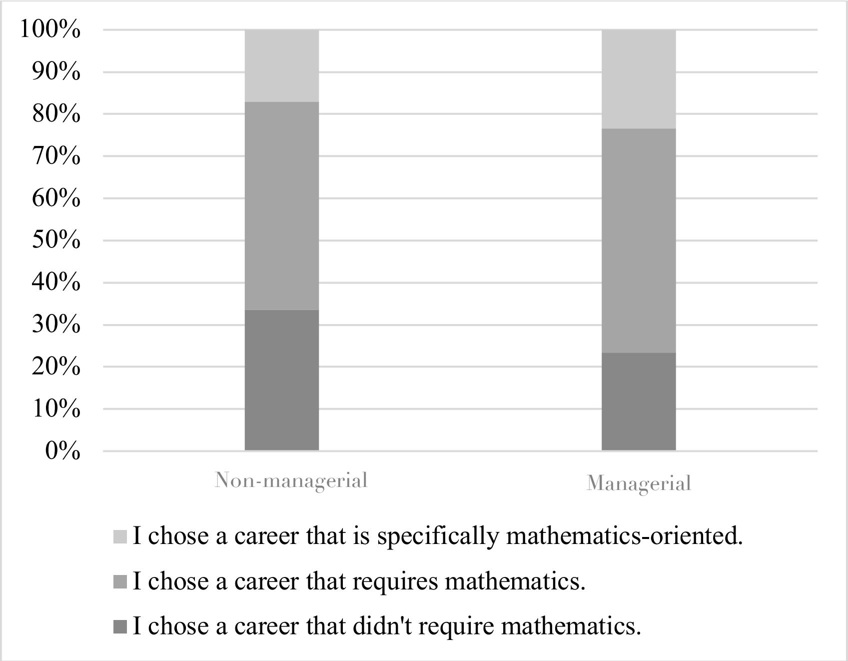
Figure 32:
Relationship between mathematical success in higher education and the influence of numbers (own editing)



5.3.6 The relationship between job and career choice in mathematics

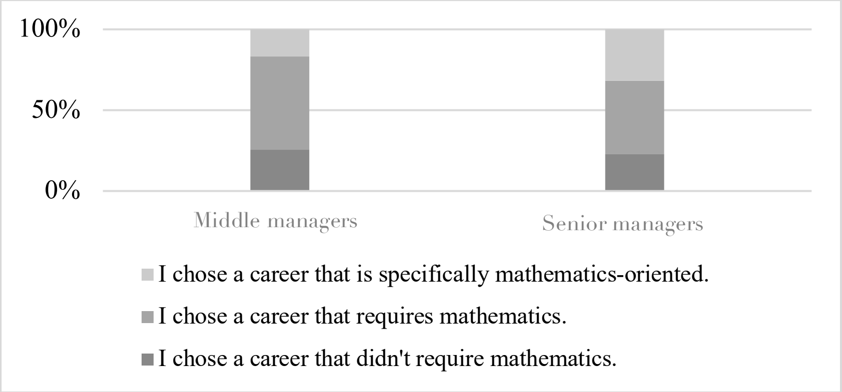
An important question is the extent to which mathematics is needed for schooling and related career choices. This was measured by the questionnaire on the relationship between mathematics and career choice. Almost 10% more people in a managerial position chose a career that required mathematics or specifically required mathematics (Figure 33).

Figure 33: Distribution of career choice and respondents' job type between managerial and non-managerial jobs (own editing)



When comparing senior management positions, it is clear to see how much higher the proportion of senior managers in the sample who have chosen a career that is specifically mathematics-intensive. It is almost twice as high: 31.82% compared to 16.95% for middle managers (Figure 34).

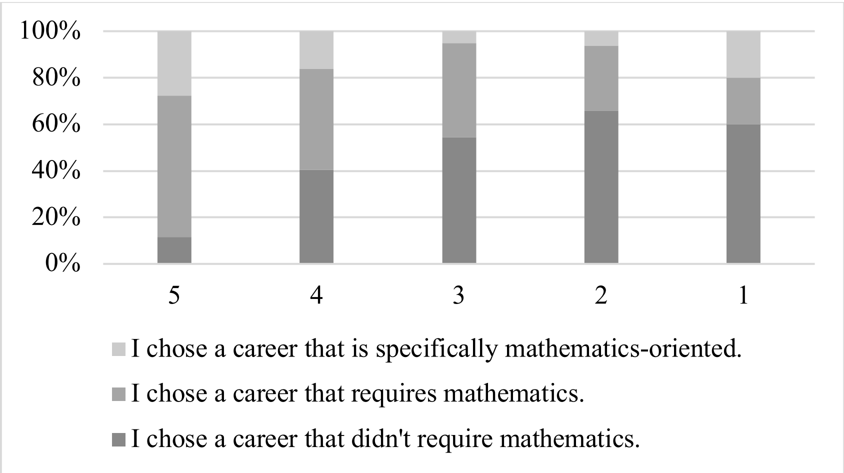
Figure 34: Distribution of career choice and respondents' job type between middle and senior managers (own editing)



5.3.7 The relationship between mathematical success and career choices in mathematics

If we look at it from the perspective of mathematical success rather than job title, the sample shows that those who have already achieved success in mathematics are overwhelmingly those who choose a career that requires mathematics. Based on the high school results, it is our experience that while 65.63% of the respondents who are proficient (2) students choose a career that does not require math, this proportion is only 11.52% for those who are honor (5) students. What is particularly interesting is that there is a relatively large jump between good (4) and excellent (5) students. While 40.32% of good (4) students choose a career that does not require maths, the figure for excellent (5) students is 11.52% (Figure 35).

Figure 35: Distribution between career choice and respondents' success in secondary school mathematics (own editing)



5.4 Conclusions and proof of hypothesis H1

When analysing the indicators related to the relationship between cognitive mathematical skills and problem-solving and decision-making skills (Q9 and Q8, Figure 25), it appears that among those who perform well in mathematics (excellent or good, i.e. 5 and 4), there is a higher proportion of middle and senior managers. While this is 41.55% for the high achievers, it is only 30.17% for the (lower) achievers. In other words, it can be said that respondents in the sample who are more successful in mathematics are more likely to be middle or senior managers. In fact, if we look only at the managerial classification, almost half (49.3%) of the high performers hold a managerial position, i.e. one in two respondents are managers. And if we look at the results of the high performers, the figure is 36.2%, which means that only nearly one in three respondents hold a managerial position. It follows that mathematical success shows a positive relationship with job function, but to nuance this picture we need to look at the results of other correlations between variables (H1).

For those who have a prior need to take maths because of their profession (either because it is necessary for further education or because it is the direction they want to go in), the impact of being successful in maths is even more pronounced. When viewed in the light of success in mathematics at secondary school (Q9_2), those who are more successful in mathematics are much more likely to choose a career that is mathematics demanding. Also, Figure 34 shows that nearly 90% (88.47%) of high school students who achieve an A (5) in math choose a career that requires math. This percentage drops towards the fair (2) achievers, where it is only 34.37%. So, while only one in ten of the high achievers in maths do not choose a maths track, only one in three of the low achievers go into a track that requires maths.

It is very important to see that the above results show that the more successful maths students are more likely to choose a maths-related further education pathway, while at the same time almost two thirds (67.5%) of all respondents said that they needed maths in their career choice, i.e. maths is not bypassable.

And when comparing senior and middle managers (Figure 33), the sample shows that there is almost twice the proportion of senior managers who choose a career specifically requiring mathematics (31.92%) compared to middle managers (16.95%). In other words, senior managers are indeed more dominated by those who are more successful in mathematics skills specific to their profession (H1a).

In terms of acquired mathematical competences, further exciting results emerge for the sample, which raises questions for additional research.

In terms of competences acquired, a significant proportion of respondents (74.1%) said that it was situationally dependent on whether they used a mathematical approach to decision-making. The majority of confident respondents, 17.6%, do not use mathematics and only 6.3% think they do.

Another measure of acquired competences was the extent to which respondents were influenced by numbers in their decisions. Interestingly, the proportion of confident respondents is much higher in this respect: 51.9% of them are influenced by numbers in their decisions.

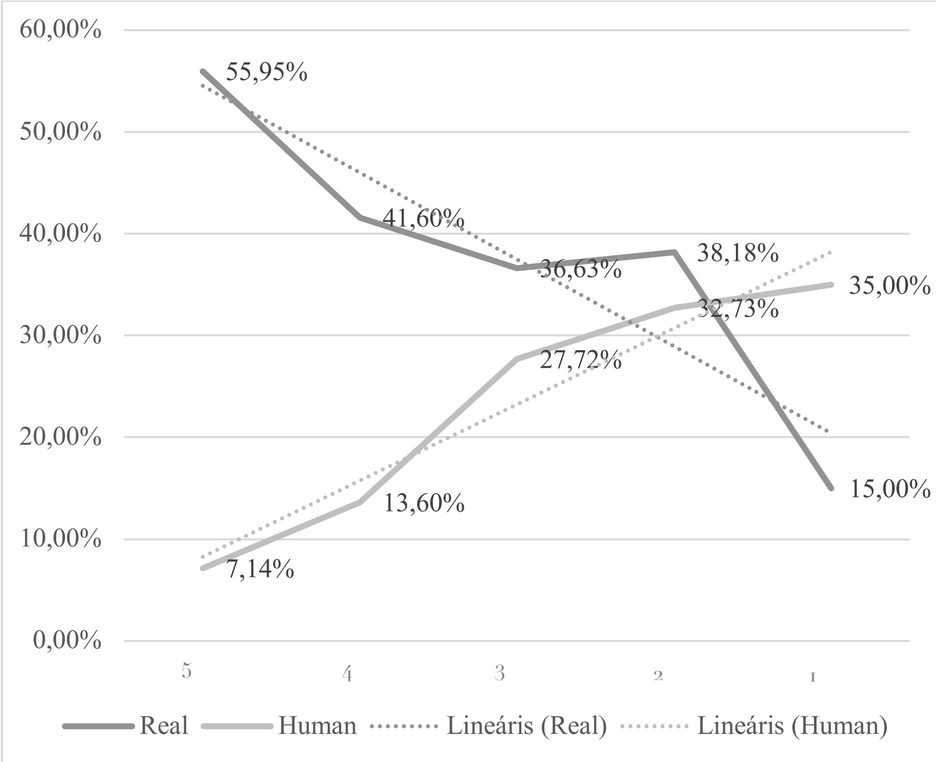
If we look at the cross-tabulation of these two questions, we can see that those who always make decisions based on mathematics are also influenced by numbers. Among those who said that it depends on the decision whether or not they use a mathematical approach (374 respondents), a significant proportion (56.95%) are those who are influenced by numbers in their decisions. And even among those who do not use a mathematical approach, one in five respondents (21.34%) said that they are nonetheless influenced by numbers.

25.9% of respondents agree that a mathematical approach helps them to make decisions and only 11.5% think they would not make a different decision if they used a mathematical approach.

In terms of acquired competences, taking into account the success in higher education, the percentage of those who consider themselves rational decision-makers is the highest for those who are A (5): 43.21%. Typically, this percentage decreases in a strictly monotonic way as the success rate in mathematics related to the profession decreases, up to 20%. Thus, those who are more successful in mathematics consider themselves more rational decision-makers than those who are not, and twice as many unsuccessful than successful people consider themselves intuitive decision-makers. Plotting the average of respondents, the trend is clearly shown in Figure 27.

This dichotomy is even more apparent when looking at the intersection of personal attitudes and mathematical success. If we focus only on the real and human responses (Figure 36), it is very striking to see the extent to which successful mathematical outcomes dominate real attitudes for the marked (5) performance, and as mathematical performance deteriorates, the human approach takes over (H1b).

Figure 36: The relationship between mathematical success in higher education and realistic or humanistic personal attitudes (own editing)



The results of the exploratory research show that

1. In the sample, respondents' cognitive mathematical abilities, measured through mathematical success, are related to their problem-solving and decision-making abilities, i.e. their job function.
2. The more educated a person is in relevant mathematical skills related to their profession, i.e. more successful in mathematics in higher education, the more likely they are to be a problem solver and decision maker, because the proportion of people with higher mathematical success is higher among those in managerial jobs.
3. The mathematical competences acquired have an impact on thinking skills, because those who are more successful in mathematics are more likely to be rational decision-makers and realistic than those who are not. Furthermore, they are more likely to take a mathematical approach and take numbers into account when making decisions, so this has an impact on their behaviour, thus indirectly leading to more successful decisions because they hold higher job titles.

Hypothesis H1 is confirmed in the research sample.

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6. Research model and results related to H2 hypotheses

The research hypothesis H2 is:

H2 The quality of mathematics teaching plays a decisive role in the perception of the usefulness of the subject:

H2a Traditional mathematics teaching methodologies are not suited to prepare students in a sustainable way to meet the cognitive challenges of the 21st century.

H2b Innovation in mathematics education can lead to sustainable thinking if it increases the self-esteem of future generations, who will then be able to empower themselves as decision-makers.

6.1 Research model related to hypothesis H2

The phenomena, qualities and abilities represented in the hypothesis are represented in the research questionnaire by the following questions:

(1) The quality of mathematics education: the quality of education can be measured by a number of factors. In the present research, quality is measured by the extent to which students enjoy learning the subject, how engaged they are in the class work and whether what they are learning in the school system is enough or whether they need additional lessons to keep up. The research also looked at whether, in the case of someone who uses a tutor to achieve better results, whether their perception of learning mathematics changes when they take lessons with a tutor. The variables measured and their labels are: liking of learning mathematics at each level of education (primary school, secondary school, higher education, academic degree): Q10_1, Q10_2, Q10_3, Q10_4; How engaging are math lessons for students at each educational level (primary school, secondary school, higher education, academic degree): Q18_1, Q18_2, Q18_3, Q18_4; Whether it was necessary to involve a special teacher: Q13, What was the experience. How much time did you need to study with a tutor: Q21.

Of course, quality can also be looked at along other dimensions, such as mathematical success or the perceived usefulness of the subject, but the measurement results of these variables in the sample have been described in detail earlier, so their results will not be presented again in Chapter 6.

(2) Perceptions of the usefulness of mathematics: the usefulness of learning mathematics was deliberately measured in relation to each level of education. This is an attempt to measure the extent to which learners feel that what they are learning makes sense for their future lives and careers. The variables measured and their notation.

(3) Traditional mathematics teaching methodology: the methodology of mathematics teaching was not directly measured in the questionnaire. Its indirect assessment is a function of the results of the measured variables, looking across generations and analysing changes between them. That is, methodological changes are judged by comparing the intergenerational distribution of outcomes measured in relation to mathematics learning. The variables measured and their notation are: Q18_1, Q18_2, Q18_3, Q18_4; Mathematics success at each level of education (primary, secondary, higher, science): Q9_1, Q9_2, Q9_3, Q9_4.

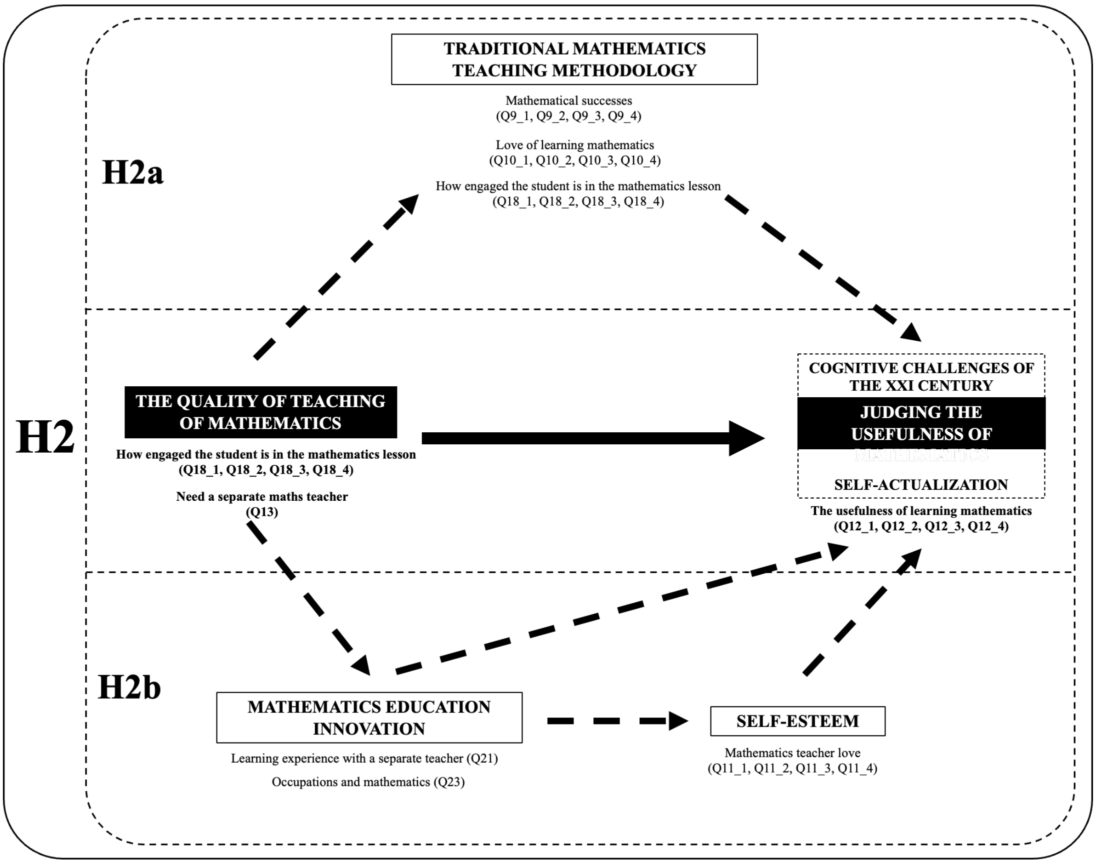
There are two variables that are important to highlight and whose measurement is crucial for the evaluation of the results:

(4) Love of the mathematics teacher: since the perception of the subject is related to the love of the teacher, this was also measured in the research and has an impact on the perception of the quality and methodology of teaching. The variables measured and their notation: the liking of the mathematics teacher at each level of education (primary school, secondary school, higher education, academic degree).

(5) An examination of the relationship between occupations and mathematics, what respondents think is the extent to which mathematics is associated with each occupation as an expected skill. The variable measured is: How much mathematics is required for the occupation (Q23).

The research model related to hypothesis H2 is summarised in Figure 37.

Figure 37: Research model related to hypothesis H2 (own editing)



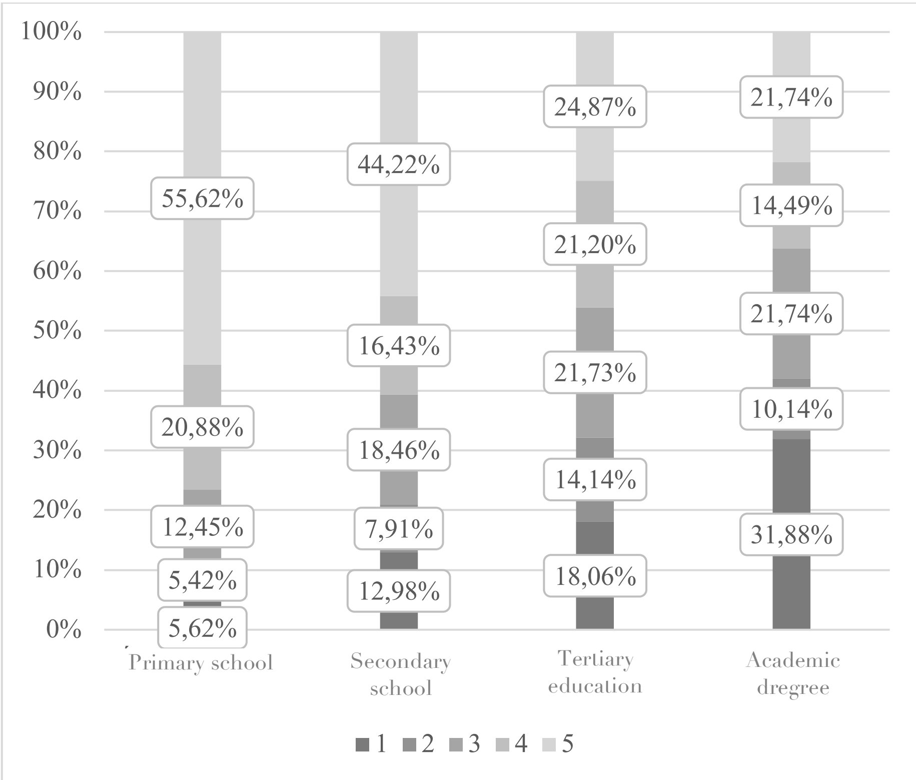
6.2 Research findings related to hypothesis H2

6.2.1 Love learning mathematics (Q10_1, Q10_2, Q10_3, Q10_4)

The number of respondents' who liked learning mathematics was measured according to educational level, i.e. they were asked to indicate how much they liked learning mathematics at that level. The results show that 76.5% of respondents initially rated their liking of learning mathematics as 4 or higher. Importantly, 55.62% gave a score of 5, which means that one in two respondents liked learning mathematics as a child. As we move forward in time, i.e. at the level of study, it is striking that the love of mathematics decreases and respondents' love of learning mathematics decreases. The initial 55.62% drops to 44.22% in secondary school and 24.87% in higher education. This means that at the start of their studies, one in two students still explicitly like to study mathematics, but by the time they reach higher education, this is true for one in five respondents.

The other camp, those who do not like to learn mathematics (respondents with values of 1 and 2), is growing. While 11.4% do not like to study mathematics in primary school, this figure rises to 20.89% in secondary school and 32.21% in tertiary education. The number of people who do not like learning mathematics almost triples. The data are presented in more detail in Figure 38.

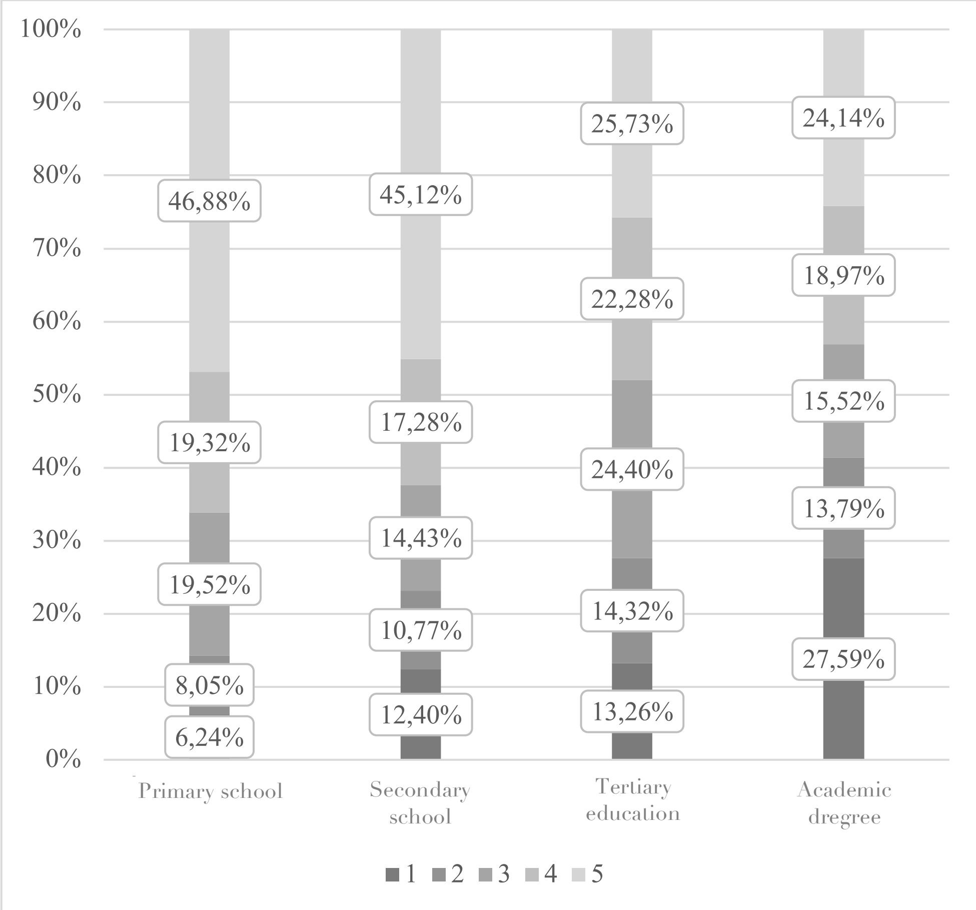
Figure 38: Preference for learning mathematics at each level of education (own editing)



6.2.2 Love your maths teacher (Q11_1, Q11_2, Q11_3, Q11_4)

The likeness of mathematics teachers was measured in relation to each level of education. In terms of results, the highest percentage of those who like their teachers is in primary school: 46.88% of them gave a score of 5, while 19.32% gave a score of 4. These values decrease slightly for secondary school, however, a big jump can be seen for those giving a 5 when entering higher education. Moving into higher education, the previous 45.12% drops back to 25.73% and essentially stays at 24.14% for science degrees. This seems to indicate a double counting. Before graduation, 46% of students in general, or nearly half, one in two, like their maths teacher. Then, after graduation, this proportion drops to almost 25% for those who continue their studies, i.e. only one in four students are enthusiastic about their maths teacher. The partial distribution is shown in Figure 39.

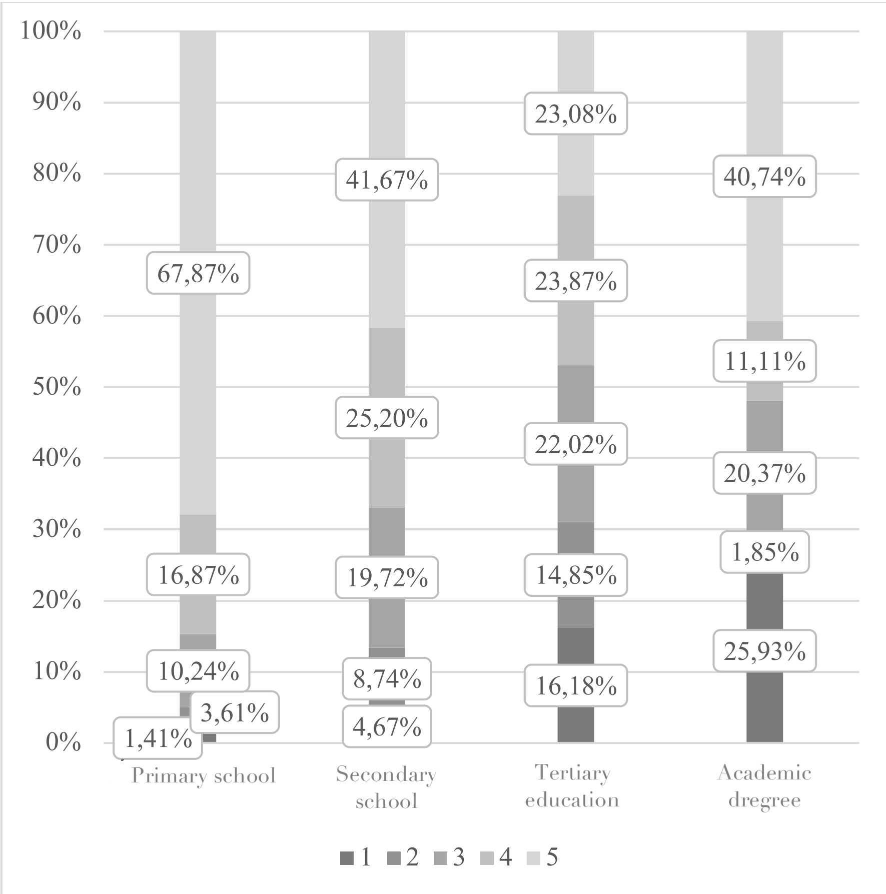
Figure 39: Mathematics teacher preference at each level of education (own editing)



6.2.3 The usefulness of learning mathematics (Q12_1, Q12_2, Q12_3, Q12_4)

Respondents clearly perceived the usefulness of studying mathematics as being the most useful in primary school. 67.87% of them gave a rating of 5 and 16.87% a rating of 4, i.e. only 15.44% of those who did not consider studying mathematics at this level to be useful or of little use. However, notice how strongly the perception of the usefulness of the subject falls as we progress in our studies. The initial 67.87% is reduced to 41.67% in secondary school and 23.08% in higher education. All this while relevant knowledge of mathematics relevant to the future profession should be acquired at increasingly higher levels of study, yet this is seen by respondents as increasingly useless. What is striking is that, although the lowest level of judgement of the usefulness of learning mathematics is at level 1 when obtaining a degree, those giving a rating of 5 almost double that at higher education, from 23.08% to 40.74%, as mentioned earlier. The reasons for this are worth investigating, but presumably it could be due to the usefulness of statistical and probabilistic mathematical models, which are essential for academic work and should be used in an indispensable way. The detailed results are shown in Figure 39.

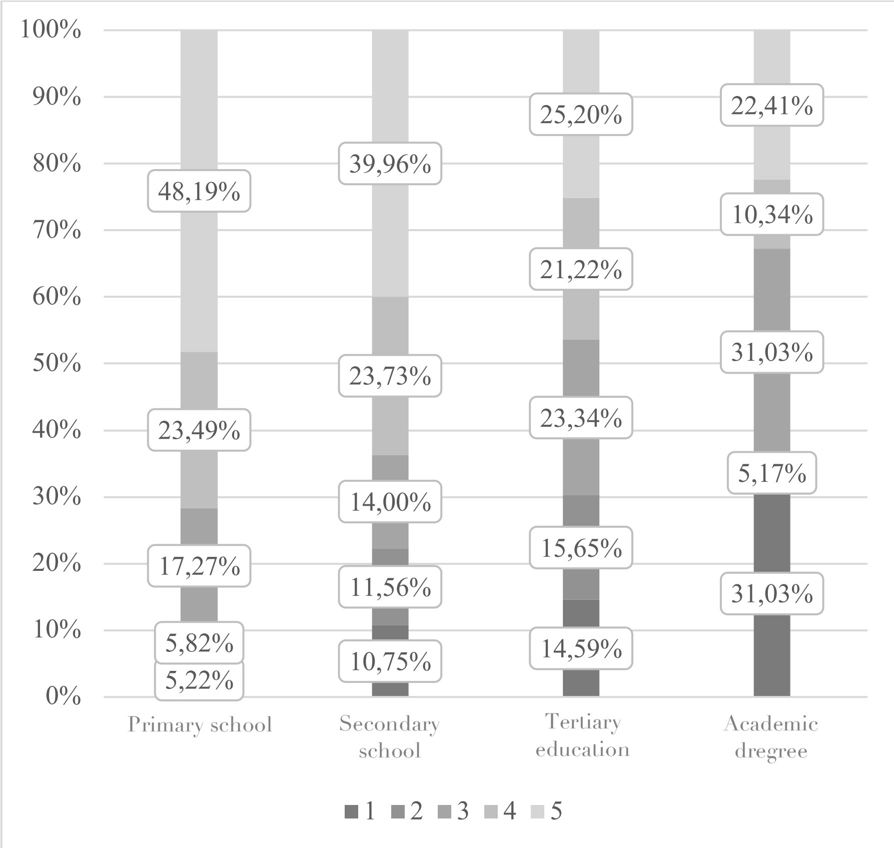
Figure 40: Perceptions of the usefulness of learning mathematics at different levels of education (own editing)



6.2.4 Attention in mathematics lessons (Q18_1, Q18_2, Q18_3, Q18_4)

Respondents were also asked to provide attention levels during classes linked to educational levels. The same trends as before can be seen here: primary education performs best, and as you move up the levels of education, the level of attention decreases. Looking at the 4 and 5 scores together, attention levels at primary school are still relatively high at 71.68%. It drops to 63.69% in secondary school, then to 46.42% in higher education and to only 32.75% in science. In other words, the proportion of those who can/was able to pay attention in class drops by almost half in higher education. The results are shown in Figure 41.

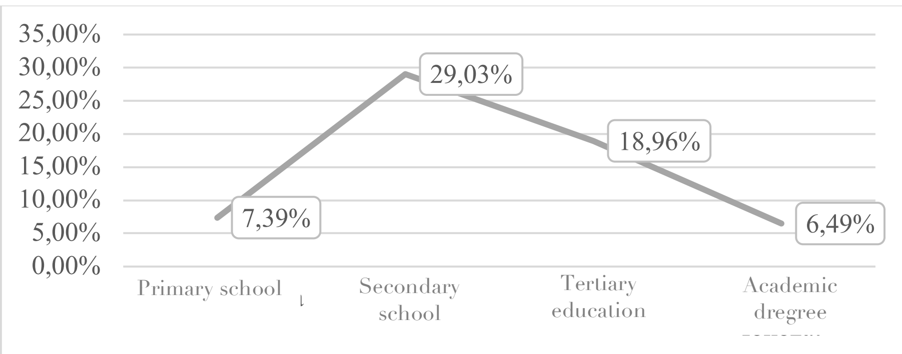
Figure 41: Attention levels in mathematics lessons (own editing)



6.2.5 Use of a separate teacher (Q13)

The use of a private teacher is an important factor during the studies. The results show that the use of a private tutor during secondary school is the most intensive (Figure 42). Almost 30% of respondents used a private tutor to ensure success in mathematics and to enhance their learning. A relatively high number of respondents, one in five, also used extra-curricular tutoring in higher education.

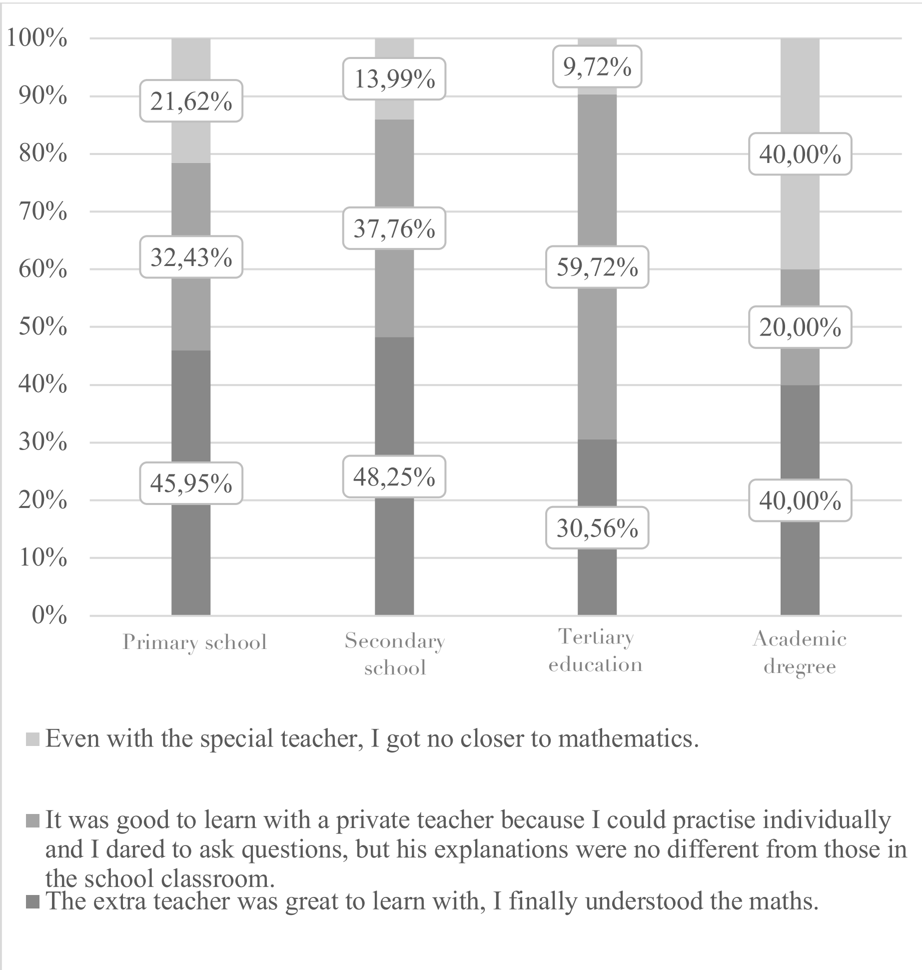
Figure 42: Percentage of students using a separate teacher at each level of education (own editing)



6.2.6 Learning experience with a special teacher (Q21)

The purpose of using a special teacher cannot be circumvented from a research point of view. The results show that, both during primary and secondary school, the main experience of having a private tutor was that, unlike compulsory lessons, it was finally fun to learn mathematics. In primary school, 45.95% of respondents and in secondary school 48.25% of respondents said that they were able to understand mathematics during the extra lesons. This proportion drops to 30% in higher education, but is also present in science degree courses, with a 40% share. Further details of the results are shown in Figure 43.

Figure 43: Experiences of having a private teacher by level of education (own editing)



6.3 Relationships between research findings related to hypothesis H2

Looking at the research results, very similar trends emerge in terms of how (1) success in mathematics: Q9, (2) liking mathematics teacher: Q11, (3) benefits of learning mathematics: Q12, (4) liking learning mathematics: Q10 and (5) attention in mathematics lessons: Q18, change during the studies. A relationship between these variables can be presumed, so a correlation matrix was constructed for the above five variables, broken down by level of study, and is presented in Table 19.

Table 19: Correlations between Q9, Q10, Q11, Q12 and Q18.
Black highlights indicate a strong positive relationship,
light grey indicates a medium positive relationship (own editing)

	Primary school	*High school*	*Higher education*	*Obtaining a scientific degree*
Q9-Q10	0.67	**0.78**	0.65	**0.74**
Q9-Q11	0.49	0.58	0.44	**0.71**
Q9-Q12	0.52	0.62	0.41	0.68
Q9-Q18	0.54	0.66	0.48	**0.72**
Q10-Q11	0.61	0.69	0.56	0.67
Q10-Q12	0.57	0.68	0.53	0.68
Q10-Q18	**0.72**	**0.78**	**0.71**	**0.77**
Q11-Q12	0.38	0.54	0.35	**0.73**
Q11-Q18	0.62	**0.71**	0.59	**0.75**
Q12-Q18	0.56	0.67	0.56	**0.80**

The Pearson correlation coefficient values indicate that the hypothesised relationship between the criteria is indeed present. Typically, there is a positive relationship of medium strength per pair. It can also be said that these relationships are markedly stronger during secondary education and when obtaining an academic degree.

6.4 Conclusions and proof of hypothesis H2

The quality of mathematics teaching plays a crucial role in determining the usefulness of the subject:

(1) The extent to which respondents are engaged in the class is strongly positively correlated with their liking of the subject at all levels of education. The magnitude of this correlation is 0.72 at primary school, 0.78 at secondary school, 0.71 at tertiary level and 0.77 at degree level.

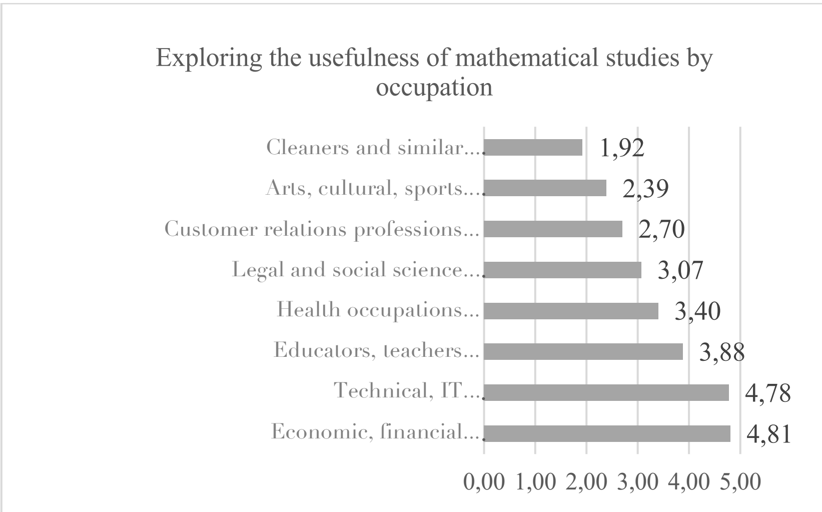
(2) If respondents are successful in mathematics, they also like learning mathematics. This correlation is strongest in secondary school, i.e. before choosing a career, when it is 0.78, i.e. a strong positive correlation.

(3) An analysis of success in mathematics across generations shows that the last four generations have similar results in mathematics success, with no increase in the proportion of successful students, despite a number of methodological innovations.
(4) In primary and secondary school, nearly one in two of those who had a separate teacher managed to understand mathematics. The other half were not given any methodological support other than what they received in class, and this did not help them to succeed in mathematics.

It can therefore be seen that, when taught according to an appropriate methodology and focused on the needs of the individual, students are more successful in acquiring the skills related to mathematics.

In the 21st century, the question of which profession/career to choose is particularly challenging. But it is important to see how this choice is linked to mathematics. As expected, respondents to questions on how much mathematics is needed for their profession (Q23) clearly put professions that deal with numbers in the category where the study of mathematics can be very useful. In parallel, it can also be seen that jobs that are typically humanities or simple skilled workers and administrators were in turn downgraded in the direction of the usefulness of mathematics, according to the stereotypes learned (Figure 44).

Figure 44: Perceptions of the usefulness of studying mathematics, grouped by occupation. Value of 5 for maximum need, value of 1 for no need (own editing)



This assessment also shows that, in relation to the performance in terms of participation in the education system, for the professions linked to a degree (exceptions are culture, arts and sport), respondents are expected to have a mathematical qualification. This is particularly true in the field of management, so when discussing how to develop management models that ensure sustainability, it is worth bearing in mind that the basic expectation is that those who make decisions about sustainability should be mathematically literate. This can only be achieved if we can change the current benchmarks and innovate the methodology of mathematics education.

As related research has shown, performance is closely linked to mathematical anxiety, which can be resolved by increasing self-esteem through success. The results of this research clearly show that it is success that strengthens our connection to mathematics and as a result we are more likely to become prepared decision makers.

Based on the results of the respondents, I consider hypothesis H2 to be confirmed for the sample.

VII. SUMMARY AND CONCLUDING THOUGHTS

As Albert Szent-Györgyi said, "The future will be like the education of today." We will think, we will make choices, like the models we have been shown, like what we have been taught, like what we have been allowed.

It can be seen that there are many approaches to understanding and judging human behaviour and decisions. However, if we look closely, what they all have in common is that numbers, mathematical thinking, are present in all of them. In the classical approach, the "rational models", this is not an issue, as they can typically be described in mathematical terms. The utility function, the optimal solution are all defined on a mathematical basis. Decision making itself is based on the weights and probabilities assigned to each alternative. However, even if we abandon this rationality, the numbers, the accounting mechanisms that can be traced back to ancient times, remain either consciously or unconsciously. We pay attention to information, picking out some and discarding others. We assign numbers to them, either because they have a specific value or because of some internal motivation. In the wake of research findings on bounded rationality, numbers and representation through numbers have come back into focus because of their significant impact on decision making through heuristics. The main reason for this is that numbers still have the potential to give us a grip, since our association with them is nothing other than the increased presence of rationality itself. Suffice it to think that when we put numbers to an explanation, what we say seems more credible.

It is no coincidence, then, that mathematics is so important in life. It is where we meet numbers, where we do most of our work. Our relationship with numbers is formed while we are learning mathematics. Hence the key to success, how we think, is also here. We need to find the cognitive mathematical knowledge that can be attributed to professions/degrees. With a properly structured and methodologically correct approach to mathematics education, progress can be made in many areas:

- in the carpentry profession, knowledge of geometric relationships, "geometrical thinking", can lead to better problem solvers, structural designers and builders;
- in hotel process management, a receptionist who is aware of linear relationships and has an overview of optimisation procedures is better able to coordinate tasks and manage resources;

- the set-theoretic and mathematical reasoning knowledge supports the thinking of legislators;

- a transport engineer can work more efficiently by using the tools of discrete mathematics through knowledge of graph theory and flows.

One could go on and on with examples of the benefits of the field of statistics, which profession/sub-discipline, which branch of mathematics can be more effective with a deeper understanding of mathematics.

Mathematics, mathematical thinking, has always had a decisive significance in the history of science as the embodiment of rationality and logical thinking. It is also a strong, positive calling word in the context of our decisions. It is the mathematical knowledge used in the preparation of management decisions, behind economic analyses, behind data analysis or artificial intelligence. However, few people have the chance to understand this knowledge, to know its secrets. We have seen that this is the area of education that we spend the most and the longest on, yet it is accompanied by many failures. Many are stuck with the "I don't have a maths brain" explanation and do not think more deeply about the reasons for failure. My research aimed to open the door to rethinking our relationship with mathematics. What happens, why does our positive attitude towards mathematics change after initial success? The results presented show that the initial relationship is strong and successful. Later on, this relationship deteriorates and this process may be closely correlated with the person of the teacher and the success achieved. This relationship, according to the analysis of the sample, is the same for generations, one could say it is hereditary. How can this be changed? How can we preserve the initial positive experiences, successes, positive perceptions and put mathematics at the service of developing problem-solving and decision-making skills? Many people understand the function and benefits of mathematics, that it teaches us to think, which is important for decision making. However, as studies become more complex, we are moving away from it, despite the fact that the importance of mathematics is not in question. So here is the challenge: how do we maintain a good relationship with mathematics?

One possible solution is to make this discipline sustainable. It is necessary to be able to change constantly and to innovate mathematics education in order to change the trends resulting from research: innovative mathematics education leads to sustainable thinking without lowering the self-esteem of future generations, so that they are able to self-actualise. This requires adherence to three principles:

(1) The first is that what we want to teach must not exceed the receptive and processing capacity of the students.

(2) The second is that what is expected as performance should not exceed the performance of the students.

(3) Third, when a problem is solved incorrectly and the students' self-esteem declines, it should not exceed the rate at which students can be brought to a level of understanding so that they can experience self-actualization.

The results suggest that the teacher is a key player in this process. The right methodological approaches and the right trainers need to be found. In the future, further research is needed to deepen our understanding, comprehension and development of our relationship with mathematics, and thus to further develop our decision-making skills. As a result of my doctoral work, in addition to proving my hypotheses, I have the following goals:

- To lay the foundations for a new, cognitive and problem-solving approach to mathematics education and competence development;
- To develop a methodology that shows that taught mathematical knowledge leads to more successful students and future employees, by easing mathematical anxiety;
- The new wave approach to maths education has resulted in the redefinition of "maths subjects" and the transformation of maths from a necessary evil to knowledge that underpins sustainable thinking and supports self-actualisation.

I can tell you that there is no such thing as hopeless mathematics, only abandonment. Perhaps the hardest thing is to bring people back from giving up. For the last 20+ years, I have been involved in mathematics education, sometimes more, sometimes less. Ninety percent of my college students and mentors gave up on math, and my job was more than just teaching math. I had to give them back what they had lost: hope. Hope that they could understand math and not be as stupid as they thought they were. And when they do, they become more competent in their own decisions, because they see the world differently.

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THE AUTHOR'S PUBLICATIONS ON THE SUBJECT

I. Scientific books, book excerpts

Zoltay Paprika, Z. & Farkas-Kis, M. (2021) The Myth of Maths in Decision Making, in Matteo, Cristofaro (Eds.) Emotion, Cognition, and Their Marvellous Interplay in Managerial Decision-Making, Newcastle, UK / England, Cambridge Scholars Publishing (2021) 268 p. pp. 142-161. , 20 p.

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II. Professional journal articles

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III. Other

a. International conference presentations

Farkas-Kis, M. (2021): Count on it? How numbers and mathematical education influence business decisions and managerial thinking. International Doctoral Seminars about Inovations in Economics and Management, The Doctoral School of Wroclaw University of Economics and Business

Farkas-Kis, M. (2022): Decision Making in The Shadow of Mathematical Education IFIP WG 8.3 Decision Support

Farkas-Kis M. (2023):Mathematics & sustainable future - Why mathematics is the key to sustainability? VI BBS International Sustainability Student Conference, Budapest Business University

Farkas-Kis M. (2023): Rationality, Mathematics & Self-Esteem - Innovation In Mathematics Education Is The Key For Sustainability EDULEARN23 (15th annual International Conference on Education and New Learning Technologies)

Farkas-Kis M., Remsei S. (2023): Obstacle for sustainable and circular economical thinking: how math anxiety influence managerial decisions? COS'23 Conference, Győr, Hungary

b. National conference presentations

Farkas-Kis M. (2022) Forget numbers, learn to think - innovation in mathematics education is the key to sustainability. Bridges and Roads Conference, Corvinus University of Budapest, ISBN: 978-963-503-934-0

Farkas-Kis, M. (2024): inherited schemas - the learned inertia of mathematics education. 15th International Scientific Conference on Teaching and Learning Online

c. Corvinus Research Week

Farkas-Kis, M. (2022). Research Week, Corvinus University of Budapest

Farkas-Kis, M. (2023). - Tension in the shadow of numbers: mathematical anxiety. Research Week, Corvinus University of Budapest

d. Researchers' night

Farkas-Kis, M. (2021). Researchers' Night, Corvinus University of Budapest

Farkas-Kis, M. (2023): Our finite realities in infinity. Researchers' Night, Corvinus University of Budapest, Budapest, Hungary

APPENDIX 1

The employment areas included in the survey according to the Hungarian Central Statistical Office:

1. Legislators, administrative, advocacy leaders;
2. Heads of economic and budgetary organisations;
3. Managers of production and service units;
4. Heads of units supporting economic activities;
5. Technical, IT and science activities;
6. Health professions (related to higher education);
7. Social service occupations (related to higher education);
8. Teachers, educators;
9. Occupations of an economic nature;
10. Legal and social science professions;
11. Cultural, sports, artistic and religious activities (related to higher education);
12. Other highly qualified administrators;
13. Technicians and similar technical occupations;
14. Professional managers, supervisors;
15. Health sessions;
16. Teaching assistants;
17. Social care and labour market service occupations;
18. Business services for administrators, public administrators, agents;
19. Art, culture, sports and religious activities;
20. Other administrators;
21. Office and administrative jobs;
22. Customer relations sessions;
23. Commercial and catering occupations;
24. Service occupations;
25. Agricultural occupations;
26. Forestry, wildlife management and fishing occupations;
27. Food industry occupations;
28. Light industry occupations;
29. Metal and electrical trades;
30. Handicraft activities;

31. Building trades;
32. Other industrial and construction occupations;
33. Operators of processing machinery;
34. Assemblers;
35. Stationary machine operators;
36. Drivers and operators of mobile machinery;
37. Cleaners and similar simple occupations;
38. Simple service, transport and similar occupations;
39. Simple industrial, construction and agricultural occupations;
40. Occupations of armed forces.

APPENDIX 2

The questionnaire was in Hungarian. This is the English translation about it.

The online questionnaire: with or without him - mathematics in my life

Q36 **With or without it - mathematics in my life** Learning mathematics is an inevitable part of life. Basically, it is a compulsory part of our life until the end of secondary education, for 12 years. It has an impact on us either directly or indirectly. Almost everyone has at least one maths-related memory, story, strong experience. In my research, I want to explore these in a little more depth, in the context of maths experiences. I ask you to support my work by filling in this questionnaire and thus supporting my PhD research to understand the mystery around learning mathematics. The questionnaire consists of 5 blocks. In the first, I measure the demographic background. The second focuses on our free associations with mathematics. In the third one, the experiences of learning mathematics are recalled. In the fourth, I ask about the relationship between career choices, work and mathematics. Finally, I am interested in your views on how much you think mathematics is needed in each field of employment. The questionnaire is anonymous, so please complete it only once. It should take no more than 15 minutes to complete and I hope you have an exciting adventure!

Q37 Prove that you are not a robot and start the questionnaire.

Q25 Understanding the demographic background

Q1 Please state your sex:

- ◯ Female (1)
- ◯ Male (2)
- ◯ Other (3)

Q2 Please enter your year of birth:

▼ 1920 (1) ... 2019 (100)

Q3 Please select the type of residence:

▼ Capital (1) ... Other (6)

Q4 Please indicate your highest level of education:

▼ Primary education (1) ... Academic degree (4)

Q16 Please tell me which one applies to you:

▼ I am more "realistic" (1) ... Mixed (4)

Q5 Please specify the field of work you do:

▼ Physical work (1) ... Intellectual work (3)

Q8 Please state what job you do:

▼ Freelance (1) ... Retired (7)

Q6 Please indicate the type of job you have:

▼ Self-employed (1) ... Not working (6)

Q43 That was a bit boring, but necessary. Thank you for your patience! Here come the more exciting blocks! Click to continue.

Q36 About mathematics, free

Q37 If **mathematics were a colour**, what colour do you think it would be?

Q40 If **mathematics were a piece of clothing**, what kind of clothing would it be?

Q39 When **you think of mathematics**, what feelings, thoughts and images come to mind? (Feel free to write a free-word list.)

Q41 When **you think of your maths teachers**, what feelings, thoughts and images come to mind? (Feel free to write a free-word list.)

Q42 When you **think of maths lessons**, what feelings, thoughts and images come to mind? (Feel free to write a free-word list.)

Q44 After free thinking, here are our experiences of learning maths, in a bit more detail. Three more blocks to go!

Q17 **Our maths learning experiences**

Q9 Please tell us **how successful you** remember **being in mathematics** during your studies. Selecting five stars indicates an A, selecting one star indicates an unsatisfactory academic performance. Please give your answer for all levels up to the highest level of education (if you have studied mathematics) and leave the rest blank.

Undergraduate Mathematics (1)	☆	☆	☆	☆	☆
Secondary Mathematics (2)	☆	☆	☆	☆	☆
Postgraduate studies in mathematics (3)	☆	☆	☆	☆	☆
Academic degree (4)	☆	☆	☆	☆	☆

Q10 Please tell us **how much you enjoyed learning mathematics** during your studies. Selecting five stars indicates "I liked it a lot" and selecting one star indicates "I did not like it at all". Please give your answer for all levels up to the highest level of education (if you have studied mathematics), leaving the rest blank.

Undergraduate Mathematics (1)	☆	☆	☆	☆	☆
Secondary Mathematics (2)	☆	☆	☆	☆	☆
Postgraduate studies in mathematics (3)	☆	☆	☆	☆	☆
Academic degree (4)	☆	☆	☆	☆	☆

Q11 Please tell us **how much you liked your mathematics teachers** during your studies. Selecting five stars indicates "I liked it a lot" and selecting one star indicates "I did not like it at all". Please give your answer for all levels up to the highest level of education (if you had a mathematics education), leaving the rest blank.

Undergraduate Mathematics (1)	☆	☆	☆	☆	☆
Secondary Mathematics (2)	☆	☆	☆	☆	☆
Postgraduate studies in mathematics (3)	☆	☆	☆	☆	☆
Academic degree (4)	☆	☆	☆	☆	☆

Q12 Please tell us **how useful you felt the maths you learned** during your studies. Choosing five stars indicates "very useful", choosing one star indicates "I don't understand why at all". Please give your answer for all levels up to the highest level of education (if you have had mathematics studies), leaving the rest blank.

Undergraduate Mathematics (1)	☆	☆	☆	☆	☆
Secondary Mathematics (2)	☆	☆	☆	☆	☆
Postgraduate studies in mathematics (3)	☆	☆	☆	☆	☆
Academic degree (4)	☆	☆	☆	☆	☆

Q18 Please tell us to **what extent you have been distracted by maths lessons** during your studies. Selecting five stars indicates "completely engaged" and selecting one star indicates "not at all engaged". Please give your answer for all levels up to the highest level of education (if you have had mathematics studies), leaving the rest blank.

Undergraduate Mathematics (1)	☆	☆	☆	☆	☆
Secondary Mathematics (2)	☆	☆	☆	☆	☆
Postgraduate studies in mathematics (3)	☆	☆	☆	☆	☆
Academic degree (4)	☆	☆	☆	☆	☆

Q13 **Did you need** a tutor or **paid private tutor to help you with your** maths? If yes, please specify during which period of study. You can select more than one option if necessary.

☐ Students tutored during primary mathematics studies (1)

☐ Tutored during secondary mathematics studies (2)

☐ Tutored during studies of mathematics at tertiary level (3)

☐ Tutored at degree level (4)

☐ I never needed a tutor (5)

Q21 If you have had a private tutor to learn mathematics, please tell us what the **experience** was like:

○ The special teacher was great to learn with, I finally understood the maths. (1)

○ The private teacher was good to learn with, because I could practice individually and I dared to ask questions, but his explanations were no different from the school class. (2)

○ I didn't get any closer to mathematics with the special teacher. (3)

Q22 Please tell us if you have attended a **maths course** or participated in a talent management programme or competition preparation?

▼ Yes I participated (1) ... I did not participate. (2)

Q20 Please indicate at **which level,** if any, you obtained **your school leaving certificate:**

▼ Intermediate level, formerly the school-leaving certificate (1) ... I did not graduate in mathematics (3)

Q15 Please tell us **what role mathematics played** in your career choice?

▼ I chose a career that did not require mathematics (1) ... I am about to choose a career (4)

Q14 Please tell us which were your **favourite maths subjects**. You can choose more than one.

- ☐ Mathematical logic (1)
- ☐ Set theory (2)
- ☐ Number theory (3)
- ☐ Algebra (4)
- ☐ Plane geometry (5)
- ☐ Geometry of space (6)
- ☐ Vectors (7)
- ☐ Trigonometry (8)
- ☐ Coordinate geometry (9)
- ☐ Combinatorics (10)
- ☐ Graphs (11)
- ☐ Functions (12)

Series (13)

Elements of the analysis of univariate functions (14)

Statistics (15)

Probability calculation (16)

Q45 Many memories have come up. But let us move on and please answer how this is present in the workplace. Two more blocks and that's it!

Q24 **Work and mathematics**

Q26 Please tell us whether you use **a mathematical approach to your** everyday decisions?

▼ Yes, I always make my decisions based on mathematics (1) ... No, not consciously. (3)

Q28 Please tell us if you are **influenced by numbers** as a parameter for your daily decisions about your tasks?

▼ Yes. (1) ... No, I rarely see numbers. (3)

Q29 Do you think **it would help you to make decisions if** you approached them mathematically?

▼ Yes, definitely. (1) ... I don't think I would choose otherwise. (3)

Q30 Do you think people who **are good at maths can make better decisions**?

▼ Yes, definitely. (1) ... I don't think they would decide otherwise. (3)

Q31 What do you feel **has helped you the** most in **learning maths**?

▼ I learned to count well. (1) ... I see a minimal advantage. (3)

Q32 **What kind of decision-maker do you** consider yourself to **be?** The value on the left is 1 if you are a fully intuitive thinker, the value on the right is 10 if you are a maximally rational thinker.

	Intuitive thinker / decision maker	Rational thinker / decision maker
	0 1 2 3 4 5 6 7 8 9 10	
1 ()		

Q33 Please try to list some of the times in your life when you have **had to use your knowledge of mathematics**. The memory could be work-related or personal, but the distinguishing feature of the thread should be that it is related to the experience of "yes, that's why I had to learn maths".

Q46 Thank you very much for your cooperation so far! There is only one block left, which is a small look at the labour market in general. Stay tuned for the end!

Q34 **Occupations and mathematics**

Q23 Please tell us **how useful you** think **studying mathematics at school is in** the following occupational areas. Five stars indicate "very useful", one star "completely unnecessary".

Legislators, administrative, interest group leaders (1)
Heads of economic, budgetary organisations (2)
Managers of production and service units (3)
Heads of units supporting economic activities (4)
Technical, IT and science activities (5)
Health occupations (related to tertiary education) (6)
Social service occupations (related to tertiary education) (7)
Teachers (8)
Administrative occupations (9)
Legal and social science professions (10)
Cultural, sports, artistic and religious activities (related to higher education) (11)
Other highly qualified administrators (12)
Technicians and related technical occupations (13)
Professional managers, supervisors (14)
Health activities (15)
Teaching Assistants (16)
Social care and labour market services (17)
Business services administrators, public administrators, agents (18)
Arts, culture, sports and religious activities (19)
Other administrators (20)
Clerical and administrative occupations (21)
Customer relations sessions (22)
Trade and accommodation (23)
Service occupations (24)
Agricultural occupations (25)
Forestry, hunting and fishing occupations (26)
Food industry occupations (27)
Light industrial occupations (28)
Metal and electrical trades (29)
Handicraft activities (30)
Building trades (31)
Other industrial and construction activities (32)
Processing machine operators (33)
Assemblers (34)
Stationary machine operators (35)

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Drivers and operators of mobile machinery (36)	☆	☆	☆	☆	☆
Cleaning and similar simple occupations (37)	☆	☆	☆	☆	☆
Elementary service, transport and similar activities (38)	☆	☆	☆	☆	☆
Elementary industrial, construction and agricultural occupations (39)	☆	☆	☆	☆	☆
Armed forces occupations (40)	☆	☆	☆	☆	☆

Q47 This is the end of the questionnaire! Just a few more clicks to close.

Q35 **Congratulations!**
You managed to complete all five levels.
Thank you very much for your cooperation and your answers!
I hope it was exciting to reminisce and reflect on the usefulness of mathematics. If you are interested in the results of my research, please enter your email address in the box below, which I will use solely to communicate the progress of the research.
If you like the questionnaire, please support my research and send it to your friends so that it reaches as many people as possible!
Thank you,Farkas-Kis Máté MBA

○ Your e-mail address is: (1) ____