

THESIS COLLECTION

András Máté Farkas-Kis

**Cognitive mathematical competences, and the impact
of their development on decision-making processes**

for his Ph.D. thesis

Supervisor:

Dr. Zoltayné Dr. Paprika Zita
Professor

Budapest, 2024

Doctoral School of Business Administration

THESIS COLLECTION

András Máté Farkas-Kis

**Cognitive mathematical competences, and the impact
of their development on decision-making processes**

for his Ph.D. thesis

Supervisor:

Dr. Zoltayné Dr. Paprika Zita
Professor

© Farkas-Kis András Máté

Table of contents

I. Research background and rationale for the topic	4
II. The methods used	7
a. Literature research	7
b. The research field	8
c. The hypotheses	9
d. Sampling and questionnaire	9
III. Results of the thesis	12
a. Featured results	12
b. Conclusion	20
IV. Main references	22
V. List of own publications on this topic	23

I. Research background and rationale for the topic

From primary school onwards, the study of mathematics is designed to help students learn to think. The Program for International Student Assessment (hereafter: PISA) assessment, of which one of the key components is the measurement of mathematical competences, defines mathematical literacy as follows (PISA, 2007):

"Applied mathematical literacy means that an individual recognises and understands the role of mathematics in the real world, makes well-informed decisions, and uses his or her mathematical knowledge to help him or her solve real-life problems and become a constructive, interested, thoughtful member of society."

By definition, applied mathematical literacy is essential for making informed decisions, solving problems correctly and playing a full role in society. This definition shows that, if we are talking about decisions and decision-making, mathematical literacy is inevitable. At the heart of decisions is always the human being, whose relationship with mathematics (whether conscious or not) is decisive.

Since the information used in the pre-decision process, the way it is used and the tools used are all human-defined, based on mathematical competences, the ultimate key to success in decision-making is to be found in this. Concepts of enterprise and approaches to decision theory have evolved and developed together since the second half of the 20th century. However, one thing is still present in both areas today. And that is that they are based on information, which in most cases is numerical or quantified data. It makes no difference whether they

are objective measurements or subjective judgements. At the level of mental abstraction, almost everything is expressed in numbers at some point of judgement and decision making. And these numbers exert an important and inescapable control and influence on us in our decisions.

At the same time, the research findings of bounded rationality make numbers and representation through numbers inescapable. In the case of anchoring heuristics, numbers have a particularly strong influence on the way a decision is made. Kahneman and Tversky's experiments have shown (Kahneman, 2012) that numbers, even when they seemingly have nothing to do with the decision, can influence our decisions. Therefore, numbers can also legitimize intuition-based decisions (Zoltayné & Farkas-Kis, 2021).

Data, however, do not speak for themselves, but instead are "embedded in narratives that give them form and meaning" (Loukissas, 2019), and are woven into a process of "telling different stories at different moments, for different purposes" (Dourish & Gómez Cruz, 2018). In this way, numbers and their mathematical interpretation, through data visualizations, both "mathematize" reality and transform data into a narrative, and are not at all neutral representations.

Mathematics is multifaceted and intertwined across disciplines, appearing in almost every discipline and in our everyday lives, sometimes unnoticed. However, numbers are always present, and the way they are quantified and the way they appear in personal, corporate and consumer decision-making is of particular interest. To make sense of these numbers, we need cognitive mathematical skills. Within this complex, interconnected system, the following research questions arise:

- How does the level of mathematical literacy influence our decision-making approaches?

- Which mathematical areas and competences can be traced back to different decision situations at individual, consumer and company level?
- How does the development of cognitive mathematical skills affect problem solving, decision-making and behaviour?
- How does understanding mathematics affect self-esteem and thus assertiveness in relation to our decisions?

The aim of the research is to understand the relationship between mathematical competences, numerical representations and decision making, with the aim of demonstrating the close relationship between the two, and furthermore to identify a new direction in mathematics education, in the development of mathematical competences, which can provide useful knowledge for everyone.

II. The methods used

a. Literature research

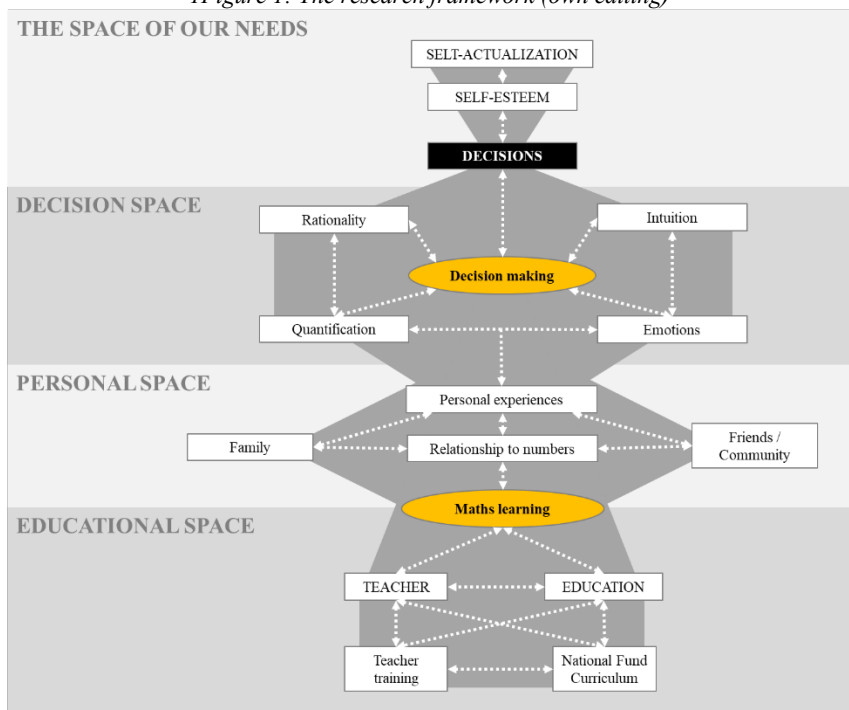
The research is based on a comprehensive literature review. It is theorised around the choice of a targeted topic area based on and underpinned by personal experience and, in this case, the exploration of the links between these areas. In the light of this, and taking into account the methodological aspects, the following steps were followed:

- Defining the research focus: the aim was to interpret the information available in the fields of economics, decision theory and mathematics education in a comprehensive context.
- Conceptual overview of the topics: the key concepts through which the thesis will be structured and interpreted have been identified. The search terms along which the literature review was conducted were also defined here.
- Literature review: three online search/databases were available for the literature search, which were (1) Scopus, (2) Web of Science and (3) Google Scholar. In addition to these, a significant primary source was provided by print literature sources that were derived from previous self-collection following personal professional interest. Typically, references to literature found in these provided a secondary source.
- Literature analysis and synthesis: the selection of the literature sources included in the research was based on their relation to the theoretical synthesis, in addition to the best-known concepts of the discipline.
- Finalisation: bringing together the relevant ideas from each source along the historical path you have built up.

b. The research field

The complexity of the research field and the interconnectedness of the elements within it can be assessed through several approaches. In the doctoral thesis, the focus of the decisions is always on the individual, their thinking, experiences and resulting perceptions and actions. To illustrate the holistic and multidisciplinary framework of this research, consider Figure 1. The basic purpose of the figure is to illustrate the interrelationship between the factors that this research - and future research - intends to explore. The direction of the relationships and the extent of their impact are not part of the presentation, only their context is relevant for now, and are therefore shown with double arrows and dotted lines.

Figure 1: The research framework (own editing)



c. The hypotheses

Research hypotheses:

H1 Cognitive mathematical skills are related to problem-solving and decision-making skills:

- H1a The more educated someone is in relevant mathematical skills related to their profession, the more likely they are to be a problem solver and decision maker.
- H1b The mathematical competences acquired have an impact on thinking skills and behaviour; thus, indirectly leading to more successful decisions.

H2 The quality of mathematics teaching plays a decisive role in the perception of the usefulness of the subject:

- H2a Traditional mathematics teaching methodologies are not suited to prepare students in a sustainable way to meet the cognitive challenges of the 21st century.
- H2b Educational innovation in mathematics education can lead to sustainable thinking if it increases the self-esteem of future generations, who will then be able to empower themselves as decision-makers.

d. Sampling and questionnaire

The research used a non-random sampling procedure. This is because the sampling was explicitly designed to select subjects with a good chance of being selected who are likely to have spent more time in the education system learning mathematics, and thus, have direct experience of its short-, medium- and long-term effects. Hence, the technique used was snowball sampling (Sajtos and Mitev, 2007), which is a non-probability sampling procedure. The explicit aim was to select research subjects belonging to specific groups.

The application and adoption of the snowball method is discussed in a number of scientific publications, such as Goodman (1961), who examined the technique in one of his early studies, or Biernacki and Waldorf (1981), who analysed in detail the potential and limitations of the method. These works help to understand the theoretical basis and practical application of snowball sampling, as well as its effectiveness and limitations in different research contexts.

One of the main disadvantages of the sampling procedure in the present study is that it does not provide random sampling, so the results obtained may not be generalisable to the larger population. In addition, there is a risk of sample bias, as the sample is often limited to the social networks of the initial participants, which may affect the objectivity of the results.

The sampling procedure sought to compensate for these limitations in two ways:

1. On the one hand, the starting point for the sampling was not only personal social platforms (Facebook and LinkedIn), where the researcher first reached out to subjects in his own personal and professional network, but also to online articles related to the research, which provided access to the questionnaire to independent subjects;
2. Given the large number of respondents (sample of over 500), sampling error is reduced and reliability is increased (Alreck-Settle, 1995).

It is important to stress, however, that the research is exploratory in nature, so its short-term aim is to test the research questions and their associated hypotheses and to prepare the ground for follow-up research that the present findings justify. For this reason, the fact that the questionnaire was filled in by those who felt that they were in some way related to its subject matter, given the way in which the sample was drawn, is of particular importance. At this stage, it

was not intended to survey those who were not related to mathematics. Data collection was carried out in the year 2021.

The questionnaire consisted of 5 blocks. In the first, the demographic background of the respondents was assessed. The second focused on free associations with mathematics. The third was to recall experiences of learning mathematics. The fourth block asked about the relationship between career choices, work and mathematics. Finally, personal opinions were explored, the extent to which respondents thought mathematics was needed in the context of each occupational area.

The research sample was reduced to 505 people after data cleaning. Respondents who only started but did not complete the questionnaire or dropped out at some point (did not complete the questionnaire) were excluded from the analysis. The reduced sample size of 505 respondents is informative for the research: for a mathematics questionnaire of this depth, it is challenging to relate to mathematics and to do a focused piece of work in this context. The reduced sample of 505 respondents answered almost all the questions, for their age group. However, there were a few incomplete fields, which, when analyses are presented on the data for the full sample, in some cases leave the total number of respondents a few values short of 505.

III. Results of the thesis

a. Featured results

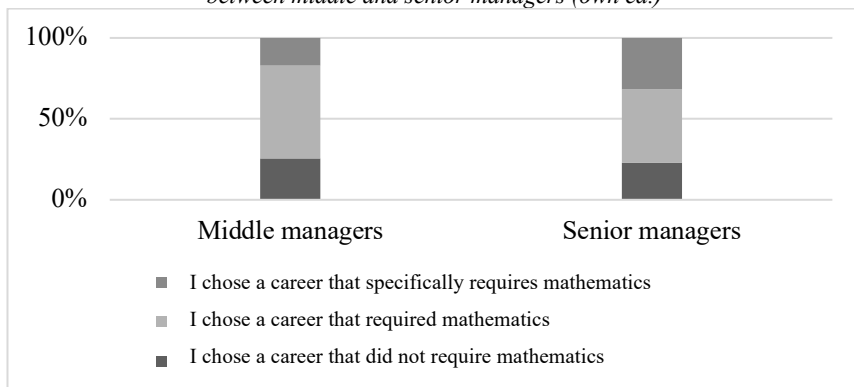
- When analysing the indicators related to the relationship between cognitive mathematical skills and problem-solving and decision-making skills, the results show that among those who perform well in mathematics (excellent or good, i.e. 5 and 4), there is a higher proportion of middle and senior managers.

While for the high achievers it is 41.55%, for the (low) achievers it is only 30.17%. In other words, it can be said that in the sample, respondents who are more successful in mathematics are more likely to become middle or senior managers. In fact, if we look only at the managerial classification, almost half (49.3%) of the high performers hold a managerial position, i.e. one in two respondents are managers. And if we look at the results of the high performers, the figure is 36.2%, which means that only nearly one in three respondents hold a managerial position. It follows that mathematical success shows a positive relationship with job function, but to nuance this picture we need to look at the results of other correlations between variables (H1).

- For those who have a prior need to take maths because of their profession (either because it is necessary for further study or because it is the direction they want to go in), the impact of being successful in maths is even more pronounced. When viewed in the light of success in mathematics at secondary school, those who are more successful in mathematics are much more likely to choose a career that is mathematics-demanding. Figure 2 also shows that nearly 90% (88.47%) of high school students who achieve an A (5) in mathematics choose a career that requires mathematics. This

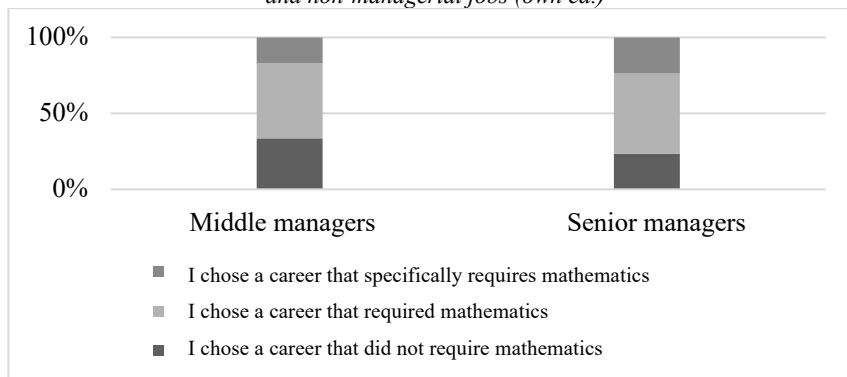
percentage drops towards the fair (2) achievers, where it is only 34.37%. So, while only one in ten of the high achievers in maths do not choose a maths track, only one in three of the low achievers go into a track that requires maths.

2Figure 1: Distribution of career choice and respondents' job type between middle and senior managers (own ed.)



- The results of the analysis show that the more successful 'maths students' are more likely to choose a maths-related further education pathway, while almost two thirds (67.5%) of all respondents said that they needed maths in their career choice, i.e. maths is not bypassable. And when comparing senior and middle managers (Figure 3), the sample shows that the proportion of senior managers who specifically chose a career requiring mathematics (31.92%) is almost twice as high as the proportion of middle managers (16.95%).

3Figure 1: Distribution of career choice and respondents' job type between managerial and non-managerial jobs (own ed.)



In other words, there is indeed a predominance of those who are more successful in the mathematical skills specific to their profession (H1a).

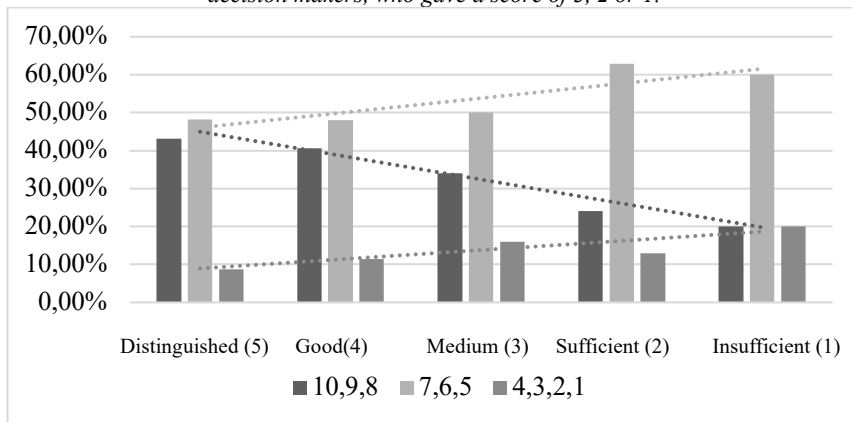
- In terms of competences acquired, a significant proportion of respondents (74.1%) said that it was situationally dependent on whether they used a mathematical approach to decision-making. The majority of confident respondents, 17.6%, do not use mathematics and only 6.3% think they do.
- Another measure of acquired competences was the extent to which respondents were influenced by numbers in their decisions. Interestingly, the proportion of confident respondents is much higher in this respect: 51.9% of them are influenced by numbers in their decisions. If we look at the answers to these two questions using a cross-tabulation analysis, it is clear that those who always base their decisions on mathematics are also influenced by numbers. Among those who said that it depends on the decision whether or not they use a mathematical approach (374 respondents), a significant proportion (56.95%) are those who are influenced by numbers in their decisions. And even among those who do not use a mathematical approach,

one in five respondents (21.34%) said that they are nonetheless influenced by numbers.

- 25.9% of respondents agree that a mathematical approach helps them to make decisions and only 11.5% think they would not make a different decision if they used a mathematical approach.
- In terms of acquired competences, if we take into account the success in higher education, the percentage of those who consider themselves rational decision-makers is the highest for those who are A (5): 43.21%. Typically, this percentage decreases in a strictly monotonic way as the success rate in mathematics related to the profession decreases, up to 20%. Thus, those who are more successful in mathematics consider themselves more rational decision-makers than those who are not, and twice as many unsuccessful than successful people consider themselves intuitive decision-makers.

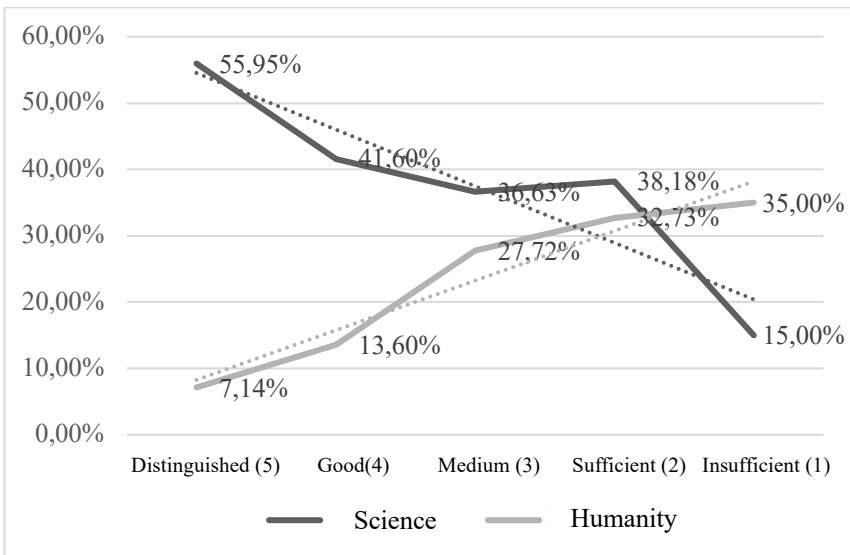
Plotting the average of respondents, the trend is clearly visible in Figure 4.

4Figure 1: Distribution of respondents according to how mathematical success in higher education shapes the distribution of respondents by type of decision-maker. The first group, the more rational decision makers, gave themselves a score of 10, 9 or 8. The second, those in the middle, gave a score of 7, 6, 5 or 4. And finally, the intuitive decision makers, who gave a score of 3, 2 or 1.



- This dichotomy is even more apparent when looking at the intersection of personal attitudes and mathematical success. If we focus only on the science and humanity responses (Figure 5), it is very striking to see the extent to which successful mathematical outcomes dominate real attitudes for the marked (5) performance, and as mathematical performance deteriorates, the human approach takes over (H1b).

5Figure 1: The relationship between mathematical success in higher education and realistic or humanistic personal attitudes (own ed.)

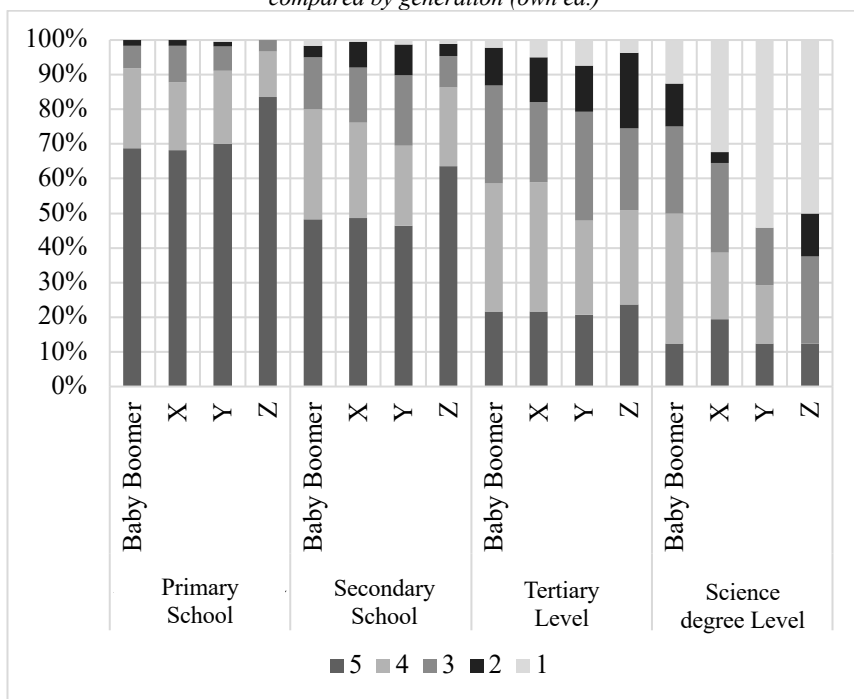


- The quality of mathematics teaching plays a crucial role in determining the usefulness of the subject:
 - (1) The extent to which respondents are engaged in the class is strongly positively correlated with their liking of the subject at all levels of education. The magnitude of this correlation is 0.72 at primary school,

0.78 at secondary school, 0.71 at tertiary level and 0.77 at science degree level.

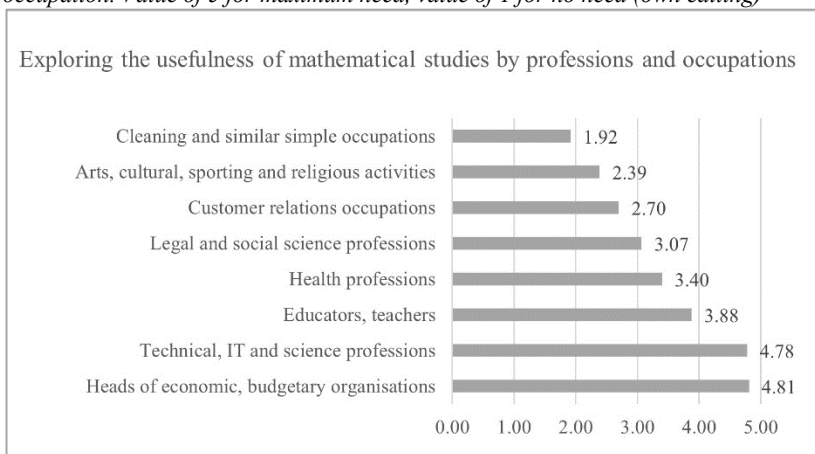
- (2) If respondents are successful in mathematics, they also like learning mathematics. This correlation is strongest in secondary school, i.e. before choosing a career, when it is 0.78, i.e. a strong positive correlation.
- (3) Analysing success in mathematics across generations, the last four generations have similar results in mathematics success, with no increase in the proportion of successful students, despite a number of methodological innovations (Figure 6).

6Figure 1: Success rate of respondents by educational level compared by generation (own ed.)



- (4) In primary and secondary school, nearly one in two of those who had a private teacher managed to understand mathematics. The other half were not given any methodological support other than what they were taught in class, and this did not help them to succeed in mathematics.
- In the 21st century, the question of which profession/career to choose is particularly challenging. But it is important to see how this choice is linked to mathematics. In response to questions about professions and the extent to which mathematics is relevant to them, respondents clearly put professions that deal with numbers in the category where the study of mathematics can be very useful, as expected. In parallel, it can also be seen that jobs that are typically linked to humanities or simple skilled workers and administrators were downgraded in the direction of the usefulness of mathematics, according to the stereotypes they had learned (Figure 7).

7Figure 1: Perceptions of the usefulness of studying mathematics, grouped by occupation. Value of 5 for maximum need, value of 1 for no need (own editing)



This assessment also shows that, in relation to the performance in terms of participation in the education system, for the professions linked to a degree (exceptions are culture, arts and sport), respondents are expected to have a

mathematical qualification. This is particularly true in the field of management, so when discussing how to develop management models that ensure sustainability, it is worth bearing in mind that the basic expectation is that those who make decisions about sustainability should be mathematically literate. This can only be achieved if we can change the current benchmarks and are able to methodologically innovate mathematics education.

- As related research has shown, performance is closely linked to mathematical anxiety, which can be resolved by increasing self-esteem through success. The results of this research clearly show that it is success that strengthens our connection to mathematics and as a result we are more likely to become prepared decision makers.
- The results of the exploratory research show that
 1. In the sample, respondents' cognitive mathematical abilities, measured through mathematical success, are related to their problem-solving and decision-making abilities, i.e. their job function.
 2. The more educated a person is in relevant mathematical skills related to their profession, i.e. more successful in mathematics in higher education, the more likely they are to be a problem solver and decision maker, because the proportion of people with higher mathematical success is higher among those in managerial jobs.
 3. The mathematical competences acquired have an impact on thinking skills, because those who are more successful in mathematics are more likely to be rational decision-makers and realistic than those who are not. Furthermore, they are more likely to take a mathematical approach and take numbers into account when making decisions; therefore, this has an impact on their

behaviour, indirectly making more successful decisions and holding higher job positions.

Hypotheses H1 and H2 were confirmed in the research sample.

b. Conclusion

Mathematics, mathematical thinking, has always had a decisive significance in the history of science as the embodiment of rationality and logical thinking. It is also a strong, positive call to action in the context of our decisions. It is the mathematical knowledge which is used in the preparation of management decisions, behind economic analyses, behind data analysis or artificial intelligence.

The aim of this research is to provide a basis for rethinking our relationship with mathematics. What happens, why does our positive relationship with mathematics change after initial success? The results presented show that the initial relationship (in primary school) is strong and successful. Later on (in secondary school and higher education) this relationship deteriorates, and the process may be closely correlated with the person of the teacher and the success achieved. This relationship, based on the analysis of the sample, is the same for generations, one could say it is hereditary. How can this be changed? How can we preserve the initial positive experiences, successes, positive perceptions and put mathematics at the service of developing problem-solving and decision-making skills? Many people understand the function and benefits of mathematics, that it teaches us to think, which is important for decision making. However, as studies become more complex, we are moving away from mathematics, despite the fact that its importance is not in question. So here is the challenge: how do we maintain a good relationship with mathematics?

One possible solution is to make this discipline sustainable. It is necessary to be able to change constantly and to innovate mathematics education in order to change the trends resulting from research: innovative mathematics education leads to sustainable thinking without lowering the self-esteem of future generations, so that they are able to self-actualise. This requires adherence to three principles:

- (1) The first is that what we want to teach must not exceed the receptive and processing capacity of the students.
- (2) The second is that what is expected as performance should not exceed the performance of the students.
- (3) Third, when a problem is solved incorrectly and students' self-esteem declines, it should not exceed the rate at which students can be brought to a level of understanding so that they can experience self-actualization.

The results suggest that the teacher is a key player in this process. The right methodological approaches and the right trainers need to be found. In the future, further research is needed to deepen our understanding, comprehension and development of our relationship with mathematics, and thus to further develop our decision-making skills.

IV. Main references

Alreck P. L. & Settle R. B. (1995) *The Survey Research Handbook: Guidelines and Strategies for Conducting a Survey*. IRWIN Professional Publishing, New York.

Biernacki, P., & Waldorf, D. (1981): Snowball Sampling-Problems and Techniques of Chain Referral Sampling. *Sociological Methods & Research*, 10, 141-163. <https://doi.org/10.1177/004912418101000205>.

Dourish, P., & Gómez Cruz, E. (2018) Datafication and data fiction: narrating data and narrating with data. *Big Data & Society*, 5(2), 1-10. <https://doi.org/10.1177/2053951718784083>

Goodman, L.A. (1961): Snowball Sampling. *Annals of Mathematical Statistics*, 32, 148-170. <https://doi.org/10.1214/aoms/1177705148>.

Kahneman, D. (2012) *Thinking, Fast and Slow*. New York: Farrar, Straus and Giroux

Loukissas, Y. A. (2019) All data are local: thinking critically in a data-driven society.

PISA (2007): 2006 Summary Report 2007 Budapest, Education Office (https://www.oktatas.hu/pub_bin/dload/kozoktatas/nemzetkozi_meresekek/pisa/pisa2006_jelentes.pdf)

Sajtos, L., & Mitev, A. (2007) *SPSS research and data analysis manual*. Budapest, Hungary : Alinea Kiadó, 402 p. SPSS SPSS and SPSS Tools for the Analysis of Social and Economic Research.

Zoltay Paprika, Z. & Farkas-Kis, M. (2021): The Myth of Maths in Decision Making, in Matteo, Cristofaro (Eds.) *Emotion, Cognition, and Their Marvellous Interplay in Managerial Decision-Making* (pp. 142-161.) Cambridge Scholars Publishing

V. List of own publications on this topic

Scientific books, book excerpts

Farkas-Kis M. (2023): Digital Revolution and Sustainability Without Well-founded Mathematical Education. In: K, Hemachandran; Rodriguez, Raul V., Artificial Intelligence for Business: An Implementation Guide Containing Practical and Industry-Specific Case Studies, New York, Amerikai Egyesült Államok, Productivity Press (2023) pp. 15-28., 14 p.

Zoltay Paprika, Z. & Farkas-Kis, M. (2021): The Myth of Maths in Decision Making. In: Matteo, Cristofaro (Eds.) Emotion, Cognition, and Their Marvellous Interplay in Managerial Decision-Making, Newcastle, Egyesült Királyság / Anglia, Cambridge Scholars Publishing (2021) 268 p. pp. 142-161., 20 p.

Professional journal articles

Farkas-Kis M. (2023): Rationality, Mathematics & Self-Esteem - Innovation In Mathematics Education Is The Key For Sustainability. In: Gómez Chova, L., Martínez, G., Lees, J. (szerk.) EDULEARN23 Proceedings, International Academy of Technology, Education and Development (IATED) (2023) pp. 7669-7677., 9 p.

Farkas-Kis, M. (2022): Decision making in the shadow of mathematical education. Journal of Decision Systems, 31: Supplement 1 pp. 168-180. , 13 p.

Farkas-Kis, M. (2022): A racionalitás végvárai: A számszerűsítés megjelenítése az üzleti döntésekben. Vezetéstudomány Budapest Management Review, 53(3), 73–82. 10p

International conference presentations

Farkas-Kis M. (2023): Rationality, Mathematics & Self-Esteem - Innovation In Mathematics Education Is The Key For Sustainability. EDULEARN23 (15th annual International Conference on Education and New Learning Technologies)

Farkas-Kis M. (2023): Mathematics & sustainable future - Why mathematics is the key to sustainability? VI. BBS International Sustainability Student Conference, Budapest Business University

Farkas-Kis, M. (2022): Decision Making in The Shadow of Mathematical Education. IFIP WG 8.3 Decision Support

Farkas-Kis, M. (2021): Count on it? How numbers and mathematical education influence business decisions and managerial thinking. International Doctoral Seminars about Inovations in Economics and Management, The Doctoral School of Wroclaw University of Economics and Business

Farkas-Kis, M. (2021): There are three types of people in business, who can count and who cannot. International New Horizons in Business and Management Studies, Corvinus University of Budapest

National conference presentations

Farkas-Kis M. (2024): Öröklött sémák - a matematikaoktatás tanult tehetetlensége. XV. Taní-tani Online Nemzetközi Tudományos Konferencia

Farkas-Kis M. (2022): Felejtsd el a számokat, tanulj gondolkodni – a fenntarthatóság kulcsa a matematikaoktatás innovációja. Hidak és utak konferencia, Budapesti Corvinus Egyetem, ISBN: 978-963-503-934-0

Farkas-Kis M. (2022): Létezhet-e digitális forradalom és fenntarthatóság a matematikaoktatás innovációja nélkül? Beyond Financial Reporting – Fenntarthatóság: Integrált gondolkodás és integrált vállalati jelentés konferencia, Budapesti Gazdasági Egyetem