Doctoral School of Economics

Thesis on

Kálecz-Simon András

Strategy and Behavior in Oligopolistic Markets

Ph.D. Dissertation

Supervisor:

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Department of Macroeconomics

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Price discrimination in asymmetric Cournot oligopoly

Bakó, B – Kálecz-Simon, A., 2012, Price discrimination in asymmetric Cournot oligopoly. *Economics Letters*, Elsevier, vol. 116(3), pp. 301-303

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On the equivalence of quota bonuses and quantity bonuses

3.1 Introduction

Even though a wide variety of bonuses were analysed in recent years, quota bonuses – though often applied in practice – were usually ignored as a nonlinear scheme. We try to demonstrate that this bonus system is closest to sales bonuses; in some sense, they could be understood as „local" sales bonuses. Since they are more focused, the expected cost of this scheme is lower than that of sales bonuses, assuming risk neutral managers.

In practice, nonlinear elements, e.g. quota bonuses are not uncommon. According to an empirical study by Joseph and Kalwani (1998), 5 percent of the firms participating in the survey used fixed salaries for their salespeople, 24 percent used only commissions, while the overwhelming majority included some kind of bonus payment in their compensation packages. By far the most important factor in determining bonus payments was the comparison of actual sales and a predetermined quota. As Oyer (1998) remarks, executive contracts also often include quota-like features. The behavior of salespeople – eg. their attitudes toward risk, as demonstrated by Ross (1991) – is strongly influenced by how quotas are set.

Oyer (1998) also points out a potential dynamic problem with the use of quotas. This could lead to uneven effort during the year, since agents increase their effort when the deadline for determining quota bonus is near. Executives or salespeople could behave in an opportunistic manner and engage in "timing games", ie. rushing sales or using creative accounting to ensure the quota bonus. However, the findings of Steenburgh (2008), based on analysing individual-level salesforce data, seem to indicate that timing games are less common in practice and the main effect of quotas are to increase the efforts of the agents.

3.2 The model

Our model features a Cournot duopoly. Each firm has an owner, maximizing their[1] profit, and a manager, maximizing their salary. For simplicity we assume away costs for either firm.

Products are homogenous, thus inverse demand is: $P = 1-Q$, where P is price and Q is industry output. We further assume that there is some uncertainty regarding actual sales within the period. This could be due to involuntary timing issues, such as delays in contracts, or last-minute sales. This quantity shock is drawn from a normal distribution with with zero mean and σ^2 variance. The shocks to the firms are independent. Thus if the manager of firm i chooses to sell q_i units and the manager of firm j chooses to sell q_j units, then the realized sales within the period are $q_i + \varepsilon_i$ and $q_j + \varepsilon_j$ respectively, where $\varepsilon_i \sim N(0, \sigma^2)$, $\varepsilon_j \sim N(0, \sigma^2)$ and $Cov(\varepsilon_i, \varepsilon_j) = 0$.

We assume both owners and managers to be risk neutral[2]. In section 3.4, we discuss the possible implications of risk averse actors.

We investigate three possible bonus systems.

- Pure profit evaluation: in this case, the variable part of the manager's salary is proportional to the profit of the firm. Thus the manager maximizes the expected profit, ie. the manager of firm i maximizes

  $$E[(1-(q_i+\varepsilon_i)-(q_j+\varepsilon_j))(q_i+\varepsilon_i)] = (1-q_i-q_j)q_i - \sigma^2$$

  .

- Sales bonus: here the variable part of the manager's salary depends both on the profit of the firm and the quantity sold. The manager

[1]From here on in I am using the singular they when referring to actors in order to maintain gender neutrality.

[2]Similarly to Ferstmann and Judd(1987).

of firm i therefore maximizes

$$E[(1-(q_i+\varepsilon_i)-(q_j+\varepsilon_j))(q_i+\varepsilon_i)+\lambda_i(q_i+\varepsilon_i)]=$$
$$=(1-q_i-q_j)q_i-\sigma^2+\lambda_i q_i,$$

where λ_i is the bonus coefficient as determined by the owner of firm i.

- Quota bonus: in this setup, the variable part of the manager's salary depends on the profit of the firm, but fulfulling the prescribed quota means a further lump-sum bonus. Hence the manager of firm i maximizes

$$E[(1-(q_i+\varepsilon_i)-(q_j+\varepsilon_j))(q_i+\varepsilon_i)+\lambda_i P[(q_i+\varepsilon_i)\geq\bar{q})]=$$
$$=(1-q_i-q_j)q_i-\sigma^2+\lambda\left(\frac{1}{2}+\frac{1}{\sqrt{\pi}}\int_0^{\frac{q_i-\bar{q}}{\sqrt{2}\sigma}}\mathrm{e}^{-t^2}\mathrm{d}t\right),$$

where λ_i is the bonus coefficient and $\bar{q}$ is the quota as determined by the owner of firm i and $P[(q_i+\varepsilon_i)\geq\bar{q}]$ is the probability that actual sales exceed the quota, given that the manager intends to sell q_i units.

We posit the following game. In period 0 owners announce the respective managers' share of profit and hire the managers[3]. In period 1 the owners fix the bonus system they are going to use. In period 2 the owners choos the size of the bonus, as well as its conditions, if necessary. In period 3 managers choose their planned output respectively, shocks are realized and markets clear.

We assume that managers maximize their salary. We also assume – in line with the existing literature – that owners maximize their gross profit, ie. their profit before paying bonuses. However, we posit that if two methods achieve the same gross profit, the owner would prefer the one with lower expected cost of implementation.

3.3 Results

3.3.1 Cases not involving quota bonuses

The following results are well-known and repeated here for later comparison.

[3] Note that due to uncertainty and symmetry all firms offer the same profit share.

Lemma 3.1 *If both owners apply pure profit evaluation, then we have a classical Cournot duopoly, thus the respective outputs and profits are:*

$$
\begin{aligned}
q_1 &= \frac{1}{3} \\
q_2 &= \frac{1}{3} \\
\pi_1 &= \frac{1}{9} \\
\pi_2 &= \frac{1}{9}
\end{aligned}
$$

Lemma 3.2 *If the owner of firm 1 applies pure profit evaluation, while the owner of firm 2 applies sales bonus, then the bonus coefficient, respective outputs and profits are corresponding to those of a Stackelberg duopoly* [4]*:*

$$
\begin{aligned}
q_1 &= \frac{1}{4} \\
q_2 &= \frac{1}{2} \\
\pi_1 &= \frac{1}{16} \\
\pi_2 &= \frac{1}{8} \\
\lambda_2 &= \frac{1}{4}
\end{aligned}
$$

Lemma 3.3 *If both owner applies sales bonuses, then the respective outputs, profits and bonus coefficients are*[5]*:*

$$
\begin{aligned}
q_1 &= \frac{2}{5} \\
q_2 &= \frac{2}{5} \\
\pi_1 &= \frac{2}{25} \\
\pi_2 &= \frac{2}{25}
\end{aligned}
$$

[4] See Basu (1995).
[5] See eg. Vickers (1985).

$$\lambda_1 = \frac{1}{5}$$
$$\lambda_2 = \frac{1}{5}$$

3.3.2 Pure profit vs quota

Let us now consider the case, when the owner of firm 1 applies pure profit evaluation, whilst the owner of firm 2 applies quota bonus.

Since there is no strategic decision on the behalf of Owner 2, we can guess that just like in the case of the sales bonus[6], owner 1 is able to set incentives in order to commit their manager to produce the Stackelberg leader output.

The manager of firm 1 maximizes[7]:

$$S(q_1) = q_1(1 - q_1 - q_2),$$

thus chooses quantity according to the first-order condition:

$$\frac{\partial S(q_1)}{\partial q_1} = 1 - 2q_1 - q_2 = 0.$$

The manager of firm 2 maximizes

$$S(q_2) = q_2(1 - q_1 - q_2) + \lambda_2 \left(\frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_0^{\frac{q_2 - \bar{q}}{\sqrt{2}\sigma}} \mathrm{e}^{-t^2} \mathrm{d}t \right)$$

thus chooses quantity according to the first-order condition:

$$\frac{\partial S(q_2)}{\partial q_2} = 1 - q_1 - 2q_2 + \lambda_2 \frac{\mathrm{e}^{-\frac{(\bar{q} - q_2)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} = 0.$$

We could obtain the outputs and thus the profits by solving the system of equations consisting of the above equations, but that is by no means trivial. So we first guess the incentives set by the owner of firm

[6]As well as the case of market share bonus (see Jansen et al.(2007)) or the case of relative profits performance bonus (see van Witteloostuijn et al.(2007).

[7]Henceforth I omit the variance terms, since they do not effect the first-order conditions

2, then we verify, that they are indeed optimal.

It is easy to check that if Owner 2 chooses the following incentive system:

$$\bar{q} = \frac{1}{2}$$
$$\lambda_2 = \frac{1}{2}\sqrt{\frac{\pi}{2}}\sigma$$

then the respective outputs are:

$$q_1 = \frac{1}{4}$$
$$q_2 = \frac{1}{2}$$

Since these are the Stackelberg output levels, firstly the respective profits are:

$$\pi_1 = \frac{1}{16}$$
$$\pi_2 = \frac{1}{8}$$

and secondly, we have shown that the above incentive system is optimal.

Proposition 3.1 *If the other firm chooses pure profit evaluation, then choosing sales bonus or quota bonus leads to the same outcome. However, since*

$$q_s * \lambda_s = \frac{1}{2} * \frac{1}{4} > \frac{1}{2} * \frac{1}{2}\sqrt{\frac{\pi}{2}}\sigma = P[(q_q + \varepsilon_q) \geq \bar{q})]\lambda_q$$

if $\sigma < \sigma^* \approx 0.398942$, *for sufficiently low* σs, *the system of quota bonus is less costly to implement.*

3.3.3 Sales bonus vs quota

Now let us investigate the case when the owner of firm 1 applies sales bonus and the owner of firm 2 applies quota bonus.

The manager of firm 1 maximizes:

$$S(q_1) = q_1(1 - q_1 - q_2) + \lambda_1 q_1,$$

thus chooses quantity according to the first-order condition:

$$\frac{\partial S(q_1)}{\partial q_1} = 1 - 2q_1 - q_2 + \lambda_1 = 0.$$

The manager of firm 2 maximizes

$$S(q_2) = q_2(1 - q_1 - q_2) + \lambda_2 \left(\frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_0^{\frac{q_2 - \bar{q}}{\sqrt{2}\sigma}} \mathrm{e}^{-t^2} \mathrm{d}t \right)$$

thus chooses quantity according to the first-order condition:

$$\frac{\partial S(q_2)}{\partial q_2} = 1 - q_1 - 2q_2 + \lambda_2 \frac{\mathrm{e}^{-\frac{(\bar{q} - q_2)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} = 0.$$

We see that in the latter case we cannot express the best-response function of Manager 2 in a closed form. However, under certain conditions, we can invoke the Implicit Function Theorem.

We can apply the theorem if the Jacobian of the partial derivatives of the first-order conditions with respect to the quantities is not zero, ie.[8]:

$$|J| = \begin{vmatrix} \frac{\partial F_1}{q_1} & \frac{\partial F_1}{q_2} \\ \frac{\partial F_2}{q_1} & \frac{\partial F_2}{q_2} \end{vmatrix} = \begin{vmatrix} -2 & -1 \\ -1 & -2 - \lambda_2 \frac{\mathrm{e}^{-\frac{(q_2 - \bar{q})^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} \frac{q_2 - \bar{q}}{\sigma^2} \end{vmatrix} \neq 0$$

The first-order condition of Owner 1 is:

$$\frac{\partial \Pi_1}{\partial \lambda_1} = (1 - 2q_1 - q_2) \frac{\partial q_1}{\partial \lambda_1} - q_1 \frac{\partial q_2}{\partial \lambda_1} = 0$$

and that of Owner 2:

[8]From now on, we will refer to the above two first-order conditions as (3.1) and (3.1) as F_1 and F_2, respectively.

$$\frac{\partial \Pi_2}{\partial \lambda_2} = (1 - q_1 - 2q_2)\frac{\partial q_2}{\partial \lambda_2} - q_2 \frac{\partial q_1}{\partial \lambda_2} = 0$$

Assuming that previous inequality holds, the partial derivatives can be found using the Implicit Function Theorem.

$$\frac{\partial q_1}{\lambda_1} = \frac{\begin{vmatrix} \frac{\partial F_1}{\lambda_1} & \frac{\partial F_1}{q_2} \\ \frac{\partial F_2}{\lambda_1} & \frac{\partial F_2}{q_2} \end{vmatrix}}{|J|} = \frac{\lambda_2 \frac{\mathrm{e}^{-\frac{(q_2-\bar{q})^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} \frac{q_2-\bar{q}}{\sigma^2} - 2}{|J|}$$

$$\frac{\partial q_2}{\lambda_1} = \frac{\begin{vmatrix} \frac{\partial F_1}{q_1} & \frac{\partial F_1}{\lambda_1} \\ \frac{\partial F_2}{q_1} & \frac{\partial F_2}{\lambda_1} \end{vmatrix}}{|J|} = \frac{1}{|J|}$$

$$\frac{\partial q_1}{\lambda_2} = \frac{\begin{vmatrix} \frac{\partial F_1}{\lambda_2} & \frac{\partial F_1}{q_2} \\ \frac{\partial F_2}{\lambda_2} & \frac{\partial F_2}{q_2} \end{vmatrix}}{|J|} = \frac{\frac{\mathrm{e}^{-\frac{(\bar{q}-q_2)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}}{|J|}$$

$$\frac{\partial q_2}{\lambda_2} = \frac{\begin{vmatrix} \frac{\partial F_1}{q_1} & \frac{\partial F_1}{\lambda_2} \\ \frac{\partial F_2}{q_1} & \frac{\partial F_2}{\lambda_2} \end{vmatrix}}{|J|} = \frac{-2\frac{\mathrm{e}^{-\frac{(q_2-\bar{q})^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}}{|J|}$$

Substituting the partial derivatives into owners' first-order consitions, we get the following equations:

$$\frac{\partial \Pi_1}{\partial \lambda_1} = 3q_1 + 2q_2 - 2 + (1 - 2q_1 + q_2)\lambda_2 \frac{\mathrm{e}^{-\frac{(q_2-\bar{q})^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} \frac{\bar{q} - q_2}{\sigma^2} = 0$$

$$\frac{\partial \Pi_2}{\partial \lambda_2} = 2q_1 + 3q_2 - 2 = 0$$

However, notice that from a previous first-order condition:

$$\lambda_2 \frac{e^{-\frac{(q_2-\bar{q})^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} = q_1 + 2q_2 - 1$$

so the first-order-condition for Owner 1 can be expressed as:

$$\frac{\partial \Pi_1}{\partial \lambda_1} = 3q_1 + 2q_2 - 2 + (1 - 2q_1 + q_2)(q_1 + 2q_2 - 1)\frac{\bar{q} - q_2}{\sigma^2} = 0$$

Let k be equal to $\frac{\bar{q}-q_2}{\sigma^2}$. Optimal k cannot be negative, since in that case choosing $-k$ provides the same incentives to the manager, while decreasing the expected cost of the bonus system.

Assuming positive k, solving the above equations simultaneously yields[9]:

$$\begin{aligned} q_1 &= \frac{5k - 3\left(5 - \sqrt{25 - (6-k)k}\right)}{8k} \\ q_2 &= \frac{5 + k - \sqrt{25 - (6-k)k}}{4k} \end{aligned}$$

Thus Owner 2 maximizes $\frac{\left(5+k-\sqrt{25-(6-k)k}\right)^2}{32k^2}$.

However, the first derivative of the above expression is negative for all positive ks, thus the optimal k is zero. Hence

$$\begin{aligned} q_1 &= \frac{2}{5} \\ q_2 &= \frac{2}{5} \end{aligned}$$

and thus

$$\bar{q} = \frac{2}{5}$$

This implies

$$\begin{aligned} \lambda_1 &= \frac{1}{5} \\ \lambda_2 &= \frac{1}{5}\sqrt{2\pi}\sigma \end{aligned}$$

[9] We ignore the solutions that would imply negative output and/or negative sales bonus.

Proposition 3.2 *If the other firm chooses sales bonus, then choosing sales bonus or quota bonus leads to the same outcome. However, since:*

$$q_s * \lambda_s = \frac{2}{5} * \frac{1}{5} = \frac{2}{25} > \frac{1}{2} * \frac{1}{5}\sqrt{2\pi}\sigma = P[(q_q + \varepsilon_q) \geq \bar{q})] * \lambda_q$$

if $\sigma < \sigma^* \approx 0.319154$*, the expected cost of implementing a quota bonus is lower for the owner of firm 2, for sufficiently low* σ*s.*

3.3.4 Quota vs quota

Finally let us discuss the case when both owners rely on quota bonuses.

The manager of firm 1 maximizes:

$$S(q_1) = q_1(1 - q_1 - q_2) + \lambda_1 \left(\frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_0^{\frac{q_1 - \bar{q}_1}{\sqrt{2}\sigma}} \mathrm{e}^{-t^2} \mathrm{d}t \right)$$

thus chooses quantity according to the first-order condition:

$$\frac{\partial S(q_1)}{\partial q_1} = 1 - 2q_1 - q_2 + \lambda_1 \frac{\mathrm{e}^{-\frac{(\bar{q}_1 - q_1)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} = 0.$$

The manager of firm 2 maximizes:

$$S(q_2) = q_2(1 - q_1 - q_2) + \lambda_2 \left(\frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_0^{\frac{q_2 - \bar{q}_2}{\sqrt{2}\sigma}} \mathrm{e}^{-t^2} \mathrm{d}t \right)$$

thus chooses quantity according to the first-order condition:

$$\frac{\partial S(q_2)}{\partial q_2} = 1 - q_1 - 2q_2 + \lambda_2 \frac{\mathrm{e}^{-\frac{(\bar{q}_2 - q_2)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} = 0.$$

Invoking the Implicit Function Theorem yields[10]:

$$\frac{\partial q_1}{\lambda_1} = \frac{\mathrm{e}^{-\frac{(\bar{q}_1 - q_1)^2 + (\bar{q}_2 - q_2)^2}{2\sigma^2}} \left(\lambda_2(\bar{q}_2 - q_2) - 2\mathrm{e}^{\frac{(\bar{q}_2 - q_2)^2}{2\sigma^2}} \sqrt{2\pi}\sigma^3\right)}{2\pi\sigma^4}$$

[10]To simplify things, we are not giving the actual partial derivatives here, but the right hand sides are multiplied by $|J|$

$$\frac{\partial q_2}{\lambda_1} = \frac{\mathrm{e}^{-\frac{(\bar{q}_1 - q_1)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}$$

$$\frac{\partial q_1}{\lambda_2} = \frac{\mathrm{e}^{-\frac{(\bar{q}_2 - q_2)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}$$

$$\frac{\partial q_2}{\lambda_2} = \frac{\mathrm{e}^{-\frac{(\bar{q}_2 - q_2)^2 + (\bar{q}_1 - q_1)^2}{2\sigma^2}}(\lambda_1(\bar{q}_1 - q_1) - 2\mathrm{e}^{\frac{(\bar{q}_1 - q_1)^2}{2\sigma^2}}\sqrt{2\pi}\sigma^3)}{2\pi\sigma^4}$$

Using the partial derivatives and some simplification leads to the following first-order conditions:

$$\frac{\partial \Pi_1}{\partial \lambda_1} = \lambda_2(2q_1 + q_2 - 1)(q_2 - \bar{q}_2) + \mathrm{e}^{\frac{(\bar{q}_2 - q_2)^2}{2\sigma^2}}\sqrt{2\pi}(3q_1 + 2q_2 - 2)\sigma^3 = 0$$

$$\frac{\partial \Pi_2}{\partial \lambda_2} = \lambda_1(2q_2 + q_1 - 1)(q_1 - \bar{q}_1) + \mathrm{e}^{\frac{(\bar{q}_1 - q_1)^2}{2\sigma^2}}\sqrt{2\pi}(3q_2 + 2q_1 - 2)\sigma^3 = 0$$

Next, notice that from the first-order condition we can conclude that

$$\begin{aligned}
\lambda_1 &= \sqrt{2\pi}\sigma(2q_1 + q_2 - 1)\mathrm{e}^{\frac{(\bar{q}_1 - q_1)^2}{2\sigma^2}} \\
\lambda_2 &= \sqrt{2\pi}\sigma(q_1 + 2q_2 - 1)\mathrm{e}^{\frac{(\bar{q}_2 - q_2)^2}{2\sigma^2}}
\end{aligned}$$

Using these equations we can rewrite the owners' first-order conditions as:

$$\frac{\partial \Pi_1}{\partial \lambda_1} = (1 - q_1 - 2q_2)(2q_1 + q_2 - 1)\frac{\bar{q}_2 - q_2}{\sigma^2} + (3q_1 + 2q_2 - 2) = 0$$

$$\frac{\partial \Pi_2}{\partial \lambda_2} = (1 - 2q_1 - q_2)(2q_2 + q_1 - 1)\frac{\bar{q}_1 - q_1}{\sigma^2} + (3q_2 + 2q_1 - 2) = 0$$

Let k_1 be $\frac{\bar{q}_1 - q_1}{\sigma^2}$ and let k_2 be $\frac{\bar{q}_2 - q_2}{\sigma^2}$ Firstly, realize that if firm i chooses k_i to be 0, then we are back to the previous case (equivalent to sales bonus vs. quota bonus) and the best response of the other firm is going to be setting k_{-i} equal to zero. Limiting our solution set to the symmetric solutions, it is easy to see that this is going to be the case. If one of the firms would choose a positive k, then choosing a similar k would lead to individual output levels higher than $\frac{2}{5}$, thus choosing k to be equal to zero is preferable.

Therefore

$$\begin{aligned} q_1 &= \frac{2}{5} \\ q_2 &= \frac{2}{5} \\ \bar{q}_1 &= \frac{2}{5} \\ \bar{q}_2 &= \frac{2}{5} \\ \lambda_1 &= \frac{1}{5}\sqrt{2\pi}\sigma \\ \lambda_2 &= \frac{1}{5}\sqrt{2\pi}\sigma \end{aligned}$$

Thus we can state:

Proposition 3.3 *If the other firm chooses quota bonus, then choosing sales bonus or quota bonus leads to the same outcome. However, since:*

$$q_s * \lambda_s = \frac{2}{5} * \frac{1}{5} = \frac{2}{25} > \frac{1}{2} * \frac{1}{5}\sqrt{2\pi}\sigma = P[(q_q + \varepsilon_q) \geq \bar{q})] * \lambda_q$$

if $\sigma < \sigma^* \approx 0.319154$*, the expected cost of implementing a quota bonus is lower for the owner of firm 2, for sufficiently low* σ*s.*

3.4 Discussion

We have seen that quota bonus is going to lead to the same outcome as sales bonuses, however, at a lower expected cost. So we can conclude that assuming risk neutral actors quota bonus is preferable to sales bonuses. However, one can speculate that in the case of risk averse actors the advantage provided by quota bonuses can diminish or become a disadvantage. This can further explain the coexistence of sales bonuses and quota bonuses. Firms with less risk averse actors are going to choose quota bonus systems, while firms with more risk averse actors can choose sales bonuses.

Chapter 4

Strategic segmentation

4.1 Introduction

Picture an industry where consumers who differ in their quality valuation and price elasticity. Will the firm producing high quality good leave the low valuation segment? If yes, how will this demarketing affect prices and welfare?

We consider the following set-up: there are two segments of consumers differing in their valuation of quality and price-elasticity. We show that as the price-sensitive segment decreases the equilibrium prices increase. Hence, the high quality firm may benefit from excluding some of its most price-sensitive consumers. Our main finding suggests that a high-quality firm quits the low-end market entirely if the quality valuation is high enough and the price-sensitive segment size is sufficiently low.

Rodrigues et al. (2014) present a model with vertical and horizontal differentiation to explain the phenomenon of pseudo-generics in the pharmaceutical industry. Our model answers a similar question, however with a different approach and somewhat different conclusions[1]. While the authors focus on the competitive aspect of introducing pseudo-generics, we show that segmentation might play an even more important role. Our results do not contradict their proposition that introduction of generics

[1]Regarding this paper, a technical question might arise regarding assumptions about costs and locations; linear transportation costs would not be consistent with locations chosen at endpoints. To avoid this problem, we used quadratic costs.

and pseudo-generics lead to price increases[2], however we show that due to re-positioning, it could increase social welfare. We aim to contribute to this literature, believing that studies of the pharmaceutical industry (e.g. Grabowski and Vernon (1992)) support the emphasis on our focus on the segmentation of the markets.

4.2 The model

Consider a mass of consumers with a high-end (H) and a low-end (L) segment. Each consumer group is uniformly distributed on the $[0,1]$ interval. The mass of high-end market is normalized to 1 and the total number of consumers in the low-end market is μ. In order to consume, each consumer has to travel to a manufacturer where the desired product can be purchased, and we assume that transportation costs are quadratic in distance. The two groups differ fundamentally in (a) their travel cost and (b) their valuation for the quality of service they receive while shopping. The high-end segment has a transportation cost of t_H, and the low-end group of t_L, and consistent with the above mentioned $t_H > t_L > 0$. That is, the low-end consumer group is more price sensitive than the high-end group. Furthermore, we assume that consumers from the high-end group value the service as s_H while the price-sensitive group as s_L, where $s_H > s_L$. Consumers in H demand only a product with complementary service, while consumers from the low-end group are indifferent between a product with or without service. Both consumer groups have a reservation utility of v for the product and each consumer demands at most one unit. We assume that v is high enough to ensure that all consumers buy one product in equilibrium.[3] To simplify our calculation we normalize the value of t_H to 1 and set s_L to zero. Morover, we assume that $s_H - s_L > t_H - t_L$, hence consumers are more differentiated in the way they value the services as they are in travel costs.

We consider the following game. First firm choose their location, then set a price subject to market regulations, finally the market clears. We solve the game for its subgame perfect equilibrium using backward induction.

[2]Consistent with the findings of Ward et al. (2002) in the food industries.
[3]In the subsequent analysis we give the exact lower bound of such a v.

4.2.1 Competition for the low valuation segment

Suppose, there is a firm located at $a \in [0,1]$ producing a product and selling it by providing a complementary service to it without being able to price discriminate between the consumers. Also consider that a low-quality firm, l, with no marginal cost is also present in the market and offers a product without any additional service. In the further analysis we refer to the product without any complementary service as low-quality product, and to the product with complementary service as high-quality product.

In this duopoly game, the two firms make their decision on both location and pricing. Tackling the first question, we make use of

Lemma 4.1 *In location games with quadratic transportation costs the equilibrium locations are the two extremes.*

Proof: See d'Aspremont *et al.* (1979). ■

Without loss of generality we assume that firm l is located at 1, while the incumbent firm (from now on denoted as firm h) is located at 0. Notice that unlike in the monopoly case, we see maximum product differentiation here.

Since consumers in H demand only the product with an additional service they keep purchasing the product from firm h, and the surplus of a consumer located at x obtained from consumption is

$$CS_H = \begin{cases} v + s_H - x^2 - p_h & \text{if she buys from firm } h \\ 0 & \text{if she buys from firm } l \end{cases}$$

where p_h is the price of the product with complementary service.

Consumers in L value both products similarly, and for that reason they are indifferent which product to consume as far as their price is equal. Denoting the price of the low-quality product by p_l, the utility of a consumer in L at x can be given as

$$CS_L = \begin{cases} v - t_L x^2 - p_h & \text{if she buys from firm } h \\ v - t_L(1-x)^2 - p_l & \text{if she buys from firm } l \end{cases}$$

Consumers purchase the product which yields them to the highest surplus. Thus, the consumer i from the low-end market located at x

buys from firm h if $x_i \leq \frac{1}{2} - \frac{p_h - p_l}{2t_L}$, otherwise she buys from firm l. Hence, the demand functions of the firms are as follows

$$D_H(p_h, p_l) = 1 + \mu\left(\frac{1}{2} - \frac{p_h - p_l}{2t_L}\right)$$

and

$$D_L(p_h, p_l) = \mu\left[1 - \left(\frac{1}{2} - \frac{p_h - p_l}{2t_L}\right)\right]$$

Using the above demand equations, the profit functions of the firms can be given as

$$\pi_h = \left[1 + \mu\left(\frac{1}{2} - \frac{p_h - p_l}{2t_L}\right)\right](p_h - c)$$

$$\pi_l = \mu\left(\frac{1}{2} + \frac{p_h - p_l}{2t_L}\right)p_l$$

Solving the first-order conditions, leads to

Lemma 4.2 *In equilibrium firms charge*

$$p_h^D = \frac{1}{3}\left[3t_L + 2c + \frac{4t_L}{\mu}\right] \qquad \text{and} \qquad p_l^D = \frac{1}{3}\left[3t_L + c + \frac{2t_L}{\mu}\right].$$

These are equilibrium prices only if the market is fully covered. For that we need the surplus of the consumer from group H located at 1 to be non-negative with the given prices. By evaluating this we set the lower bound of v consistent with the model. Thus, we need, that

$$v + s_H - 1 - \frac{1}{3}\left[3t_L + 2c + \frac{4t_L}{\mu}\right] \geq 0$$

Simplifying the above expression yields

$$v \geq \underline{v} \equiv 1 + t_L + \frac{2}{3}c + \frac{4}{3}\frac{t_L}{\mu} - s_H$$

That is, if the above condition is satisfied, the market is fully covered in equilibrium and prices given by our previous lemma are indeed the equilibrium prices.

Corollary 1 *More differentiation results in higher equilibrium prices.*

Proof:

$$\frac{\partial p_j^D}{\partial t_L} > 0 \qquad \text{for every} \quad j = h, l.$$

■

Corollary 2 *If the price sensitive segment is increasing the equilibrium prices are decreasing.*

Proof:

$$\frac{\partial p_j^D}{\partial \mu} < 0 \qquad \text{for every} \quad j = h, l.$$

■

The intuition behind these corollaries is that as the differentiation between products increases the substitution is becoming more difficult which softens the competition in the market. This gives the firms the incentives and the possibilities to increase their prices. However, as the more elastic group is becoming more dominant relative to the less price sensitive segment the equilibrium prices drop.

Substituting the equilibrium prices into the profit functions yields

Lemma 4.3 *In equilibrium firms profits are*

$$\pi_h^D = \frac{\mu}{18t_L}\left(3t_L - c + \frac{4t_L}{\mu}\right)^2 \quad \textit{and} \quad \pi_l^D = \frac{\mu}{18t_L}\left(3t_L + s - c + \frac{2t_L}{\mu}\right)^2$$

4.3 Strategic de-marketing

In fact, under certain conditions the high quality firm has the incentive to deviate from the equilibrium given in our previous lemma. To illustrate this consider the following. From out previous lemma on the equilibrium profits we have

Corollary 3 *The high-quality firm benefits from excluding some consumer of the most price sensitive segment if the size of this segment is less than moderate.*

Proof:

$$\frac{\partial \pi_h^D}{\partial \mu} = \frac{1}{18t_L}\left[(3t_L - c)^2 - \left(\frac{4t_L}{\mu}\right)^2\right]$$

This is negative whenever $\mu < \mu^S \equiv \frac{4t_L}{3t_L - c}$. ■

This corollary suggests that the high-quality producer might be better off by quitting the more elastic segment. In this case prices and profits can be easily calculated, since in both segments only a specific firm operates and therefore it will charge a price which binds consumers reservation utility.

Formally, the firms profits can be given as follows

$$\pi_h = (p_h - c)D_H(p_h) \qquad \text{and} \qquad \pi_l = p_l D_L(p_l)$$

where $D_H(p_h)$ and $D_L(p_l)$ stands for the demands faced by firm h and l, respectively. Since consumers' reservation utilities are high enough to provide non-negative surplus even for the consumer farthest away from the company she buys from, in equilibrium firms charge prices that consumers with the biggest distance from the company can still afford.

Notice that instead of a duopoly, we have in fact two separate monopolies in two separate markets. The choices of location therefore will aim to minimize the distance from the farthest consumer of the respective customer, each firm setting product characteristics to cater to the median customer.

Formally, we can state the following

Lemma 4.4 *Suppose firm h quits the low-end segment. In equilibrium firms will locate at the middle of the unit interval and equilibrium prices and profits are as follows:*

$$p_h^S = v + s_H - \frac{1}{4} \qquad p_l^S = v - \frac{t_L}{4}$$

and

$$\pi_h^S = v + s_H - \frac{1}{4} - c \qquad \pi_l^S = \mu\left(v - \frac{t_L}{4}\right)$$

Comparing the profits in the two cases we can determine conditions under which strategic demarketing is indeed an equilibrium. For this we need

$$\frac{\mu}{18t_L}\left(3t_L - c + \frac{4t_L}{\mu}\right)^2 < v + s_H - \frac{1}{4} - c \tag{4.1}$$

A different way to write this is

$$s_H > s_H^S \equiv \frac{\mu}{18t_L}\left(3t_L - c + \frac{4t_L}{\mu}\right)^2 - v + \frac{1}{4} + c \tag{4.2}$$

Hence, we have the following result

Proposition 4.1 *The high-quality firm stops serving the low-end segment if the consumers differ fundamentally in their complementary service valuation and if the more price-sensitive segment size is sufficiently low.*

The intuition behind this statement is the following. To serve any of the consumers from L firm h has to lower its price below the reservation utility of the least valuable consumer from H. The price decrease is more significant if the service provided by the firm is more valuable to the consumers. Hence, there is a significant consumer surplus what the high-end consumers obtain because of the low prices. By quitting the low-end segment, firm h is not facing any competition from the low-quality firm and therefore can set its price higher. However, if the low-segment is remarkable is size quitting the price-sensitive group can hurt the firm‘s profit, since the price increase cannot offset the loss caused by the major demand loss. Actually, the same happens when consumers reservation utility is high enough. Softening the competition by leaving a segment and operating only on one segment, drives prices higher. As the demand loss is not significant, the profit rises as well.

Notice that when strategic de-marketing is profitable it always leads to higher average prices as well. This is because the low-end prices are not decreasing after the demarketing and the high-end consumers pay more for their products.

Additionally, choosing de-marketing has essentially different implications regarding product characteristics. The entry of a low quality provider would most likely lead to maximum product differentiation. In the case of de-marketing, however, both firms will cater to the tastes of the median consumers of their respective segments. Notice that due to convex costs, in this case we end up with lower aggregate transportation costs.

This latter result also has consequences regarding social welfare. In our model, lower aggregate transportation costs necessarily mean higher aggregate welfare. Hence de-marketing could lead to higher social welfare than just competition for the low valuation segment.

Proposition 4.2 *If the high-quality firm stops serving the low-end segment average prices are going to increase, however, due to the repositioning of the products, social welfare will increase.*

4.4 Conclusion

If the price sensitive segment is not significant in size the manufacturer is better of by quitting the low-end market entirely. To achieve this goal the incumbent could (1) forbid the price-sensitive consumers to purchase its product, (2) pursue a negative de-marketing campaign or (3) launch a low quality product by itself and segment its consumers effectively. Thus the high quality firm should not necessarily get involved in price competition but rather focus on (de-)marketing strategies.

Even though this demarketing will lead to higher prices, this is countered by the effects of product repositioning and thus social welfare can increase. Our findings therefore carry a caveat that in certain cases demarketing could be considered desirable by regulators.

Chapter 5

Anchoring in an oligopolistic market

5.1 Introduction

Even though economic models usually posit rational actors, behavioral economics have established the existence of quite a handful of well-researched decision-making biases. One of these is anchoring, which refers to the phenomenon that original guesses for a certain question could act as anchors and could influence our final answers. For example if we are asked first whether we would be willing to pay \$10 for a watch, our valuation for it could be lower then if we are first asked a similar question but with \$1000. Of course, this is clearly not consistent with our model of the rational consumers who derive their valuations from their system of preferences.

Many studies tried to deepen our understanding of this regular quirk of consumer behavior. Tversky and Kahneman (1974) asked subjects about the percentage of African nations in the United Nations. However, firstly a wheel of fortune (with numbers between 0 and 100) was used to obtain an initial guess and before giving their own guess, the subjects had to answer whether the percentage is higher or lower than the one drawn. This chance number has clearly influenced the final guess given by the subject. Early research on anchoring and reference points and the connection of these phenomena with prospect theory is presented by Kahneman (1992).

Northcraft and Neale (1987) have shown that experts are also susceptible to this phenomenon. Students and real estate agents had to make pricing choices about properties they were shown. According to the results of the experiment, subjects in both groups were influenced by the other listings provided before the decision.

Kalyanaram and Winer (1995) have found three general conclusions based on the previous empirical literature. Reference prices do have a non-neglectable effect on consumer valuations, past prices play an important role in shaping this reference price and in a way not inconsistent with loss aversion, there is an asymmetrical reaction to price increases and price decreases.

Ariely et al. (2003) carried out ground-breaking experiments on how anchoring affects consumer valuation. They have found that the last digits of the social security numbers – used in a similar fashion as the wheel of fortune in the experiment by Tversky and Kahneman – could be used to influence the subjects' willingness to pay. At the same time, the valuation of related products is also influenced in a consistent fashion. To use the example of one of the experiments detailed in the article: recalling the last two digits of the Social Security Number in a priming question influences how much someone is willing to pay for a bottle of average wine, but everyone is willing to pay more for a bottle of "rare" wine than for the "average" one. Subjects acted in a somewhat similar way when their willingness-to-accept was tested. The originally provided anchor influenced how much they accepted to endure a 30-second high-pitched voice, however the sums accepted for a 10-second or a 60-second voice were consistent with this. The authors thus find that valuations are originally resilient. After the encounter with an anchor however, they have been "imprinted", and they create a system of valuations that is internally consistent, even though its foundation (the anchor) was arbitrary.

Simonson and Drolet (2004) investigated whether there is an asymmetric anchoring effect on willingness-to-pay and willingness-to-accept. They have found that although smaller differences might exist, the impacts of anchoring are very similar in these cases. In the experiment some subjects set selling prices under the assumption that they want to sell their item, while others were instructed to assume that they are not sure whether they want to sell. The experiment has shown that anchoring effects are the strongest if there is an uncertainty in the desire to trade. Nunes and Boatwright (2004) argued that anchors can effect the willingness-to-pay in case of unrelated goods as well. In their experiment they have found that displaying a T-shirt with an expensive ($80) or a cheap ($10) price tag at their stand affected how much visitors are willing to pay for the CD they were selling. They used the term "incidental prices" for the advertised or observed prices of completely unrelated products which were still able to influence consumer decisions.

Amir et al. (2008) asked the question whether there is a strong relationship between predicted pleasure (utility) and reservation prices. Subjects had to answer survey questions about a hypothetical concert where different cues where given about the details of the event. They have found that there is no such relationship: some cues (like the production costs) would affect the reservation price, other factors (like the details about the temperature in the auditorium) would affect predicted pleasure. This further hints towards the fact that numerical data which does not affect utility (such as past prices) can affect consumers' willingnesses to pay and thus demand. Beggs and Grady (2009) have shown that data from art auctions strongly supports the existence of the anchoring effect amongst buyers in this market.

Baucells et al. (2011) based on their laboratory experiment tried to estimate how subjects create reference prices. According to their model, early and most recent data gets a larger weight, while intermediate data gets a lower weight. Adaval and Wyer (2011) found that extreme prices can serve as anchors not only for related goods, but unrelated products as well if anchoring occurs unconsciously, when consumers encounter prices by chance. On the other hand, if the consumer consciously seeks out information on prices, the anchors will only influence the valuations of similar products.

However, Fudenberg et al. (2012) raised questions regarding the robustness of anchoring results. Their laboratory experiments regarding common market goods and lotteries have found only very weak effect on the subjects' willingness to pay. Mazar et al. (2013) on the other hand argue that market dependent valuations[1] support the hypothesis that consumers focus on other factors then the utility obtained from consuming the product and thus could hint at the significance of anchoring. In their experiments, they exposed potential buyers of mugs and gift vouchers to different a priori price distributions before soliciting their valuations. They have found that the exposure to different price distributions had significant effect on the subjects' willingnesses-to-pay.

As seen above, the study of price anchoring as a phenomenon has a very expansive literature, however, the logical follow-up of behavioral economics findings (as seen for example in Kőszegi and Rabin (2006), Schipper (2009), or in Jansen et al. (2009)) would be to extend our previous models of consumer behavior and markets using these results.

[1] I.e. the phenomenon that the valuation of the consumer is influenced by the prices encountered in the market.

The first step in this direction was taken by Nasiry and Popescu (2011), who have investigated the effect of anchoring on the dynamic pricing problem of the monopoly and found that ignoring the behavioral effects can lead to under- or overpricing. Under the peak-end rule they applied (ie. the reference price is a combination of the lowest price and the last price), they have shown that optimal price path will always be monotone; thus the monopoly will employ skimming or penetration pricing.

In this paper we continue this direction by incorporating the effects of anchoring into oligopoly models. Even though one might expect that firms can exploit anchoring to increase their revenues, as they are able to do in other cases of consumer bias[2], we find that in our finite-horizon Bertrand game, anchoring can lead to lower prices on average. Furthermore, we find stronger price-decreasing effect in less competitive markets, thus the existence of anchoring in some sense protects the consumers from firms taking advantage of product differentiation.

In the following section, we briefly introduce our model and consider two versions. Firstly, we show a two-period game that focuses on the dynamic price changes due to anchoring. Then we analyze the steady state of an infinite-horizon game, directing our attention to the long-term incentives created by anchoring. We end our paper with concluding our results.

5.2 The model

Suppose that n firms produce differentiated products with zero marginal costs.[3] Demands are given by $(i = 1, 2, \ldots, n)$[4]:

$$D_{i,t}(\mathbf{p_t}, r_t) = d_{i,t}(\mathbf{p_t}) + h_t(r_t, p_{i,t})$$

where $\mathbf{p_t} = (p_{1,t}, p_{2,t}, \ldots, p_{n,t})$, $d_{i,t}(\mathbf{p_t}) = 1 - p_{i,t} + \sum_{j \neq i} \beta p_{j,t}$ and $0 < \beta < 1$, while $t = 1, 2, \ldots$. Furthermore, $h_t(r_t, p_{i,t})$ captures the price anchoring effect, with r_t representing the reference price in period t. We assume, that $h_t(r_t, p_{i,t}) = \lambda(\sum_i p_{i,t-1}/n - p_{i,t})$, where $\lambda \in (0, 1)$ and $h_1(\cdot, \cdot) = 0$.[5] That is, we assume that the effective reference price in

[2]See e.g. Heidhues et al. (2012) or Wenzel (2014).

[3]All our results would hold if we assume positive marginal costs, however the expressions would be more complicated. Therefore, for simplicity, we assume symmetric firms with zero marginal costs.

[4]Our demand function is based on Nasiry and Popescu (2011).

[5]We restrict our attention to cases when gains and losses have symmetric effects. That is, we use the same λ even when the actual price is higher or lower

period t is the industry average price of period $t-1$.[6] Each firm seeks to maximise its sum of discounted profit $\Pi_i = \sum_{t=1}^{\infty} \delta^{t-1} p_{i,t} D_{i,t}(\mathbf{p_t}, r_t)$, with $\delta \in (0,1)$ the common discount factor.[7]

To give an intuition, first we solve the game for the finite case of two periods and $n = 2$ assuming no discounting, and then proceed to the general game described above.

5.2.1 The finite duopoly case

To solve this game we use backward induction. Firms' profit functions in period 2 can be written as $(i = 1, 2)$:

$$\begin{aligned} \pi_{i,2}(\mathbf{p_2}) &= p_{i,2} D_{i,2}(\mathbf{p_2}, r_2) \\ &= p_{i,2}\left[1 - p_{i,2} + \beta p_{j,2} + \lambda\left(\frac{\sum_i p_{i,1}}{2} - p_{i,2}\right)\right] \end{aligned}$$

Maximizing this expression with respect to $p_{i,2}$ and imposing symmetry, we have that:

$$p_{i,2}^* = \frac{\lambda \sum_i p_{i,1} + 2}{2[2(1+\lambda) - \beta]} \qquad \text{for} \quad i = 1, 2. \tag{5.1}$$

Firms' objective functions in the first period are $(i = 1, 2)$:

$$\begin{aligned} \Pi_i(\mathbf{p_1}) &= \pi_{i,1} + \pi_{i,2} = p_{i,1} d_{i,1}(\mathbf{p_1}) + p_{i,2} D_{i,2}(\mathbf{p_2}, r_2) \\ &= p_{i,1}(1 - p_{i,1} + \beta p_{j,1}) + \\ &+ p_{i,2}\left[1 - p_{i,2} + \beta p_{j,2} + \lambda\left(\frac{\sum_i p_{i,1}}{2} - p_{i,2}\right)\right] \end{aligned}$$

Plugging into this $p_{i,2}$ given above and maximizing it with respect to $p_{i,1}$, yields:

Lemma 5.1 *Equilibrium prices and profits are as follows:*

$$p_{i,1}^* = \frac{(1+\lambda)[4(1-\beta) + 5\lambda] + \beta^2}{(2-\beta)^3 + 4(2-\beta)^2\lambda + (7-4\beta)\lambda^2 - \lambda^3}$$

than the average price of the previous period.

[6] As pointed out by Biswas et al. (2011), the competitors' prices can also influence the reference price for a product.

[7] One may think of this infinite horizon game as a finite game where the game continues with a probability of δ after each period.

$$p_{i,2}^* = \frac{(2-\beta+\lambda)[2(1+\lambda)-\beta]}{(2-\beta)^3+4(2-\beta)^2\lambda+(7-4\beta)\lambda^2-\lambda^3}$$

and

$$\begin{aligned}\pi_i^* &= \frac{\beta^4(2+\lambda)-\beta^3(1+\lambda)(16+5\lambda)+8\beta^2(1+\lambda)^2(6+\lambda)}{[(7-4\beta)\lambda^2+4(2-\beta)^2\lambda+(2-\beta)^3-\lambda^3]^2} \\ &\quad -\frac{\beta(1+\lambda)^2[\lambda(68+3\lambda)+64]-(1+\lambda)^3[(32-\lambda)\lambda+32]}{[(7-4\beta)\lambda^2+4(2-\beta)^2\lambda+(2-\beta)^3-\lambda^3]^2}\end{aligned}$$

for $i = 1, 2$.

Comparing equilibrium prices we have that firms set higher prices in the first period than in the second period. The intuition behind this is that in the first period they give up sales in order to provide a high anchor for the second period, where they can finally reap what they have sown, so to speak. More formally:

Remark 1 $p_{i,2}^* < p_{i,1}^*$ *for* $i = 1, 2$, *whenever* $\beta \in (0, 1)$.

To examine the effect of price anchoring, let us consider the case when there is no price anchoring. In this case firms' per period profits can be given as ($i = 1, 2$):

$$\pi_{i,t}(\mathbf{p_t}) = p_{i,t}d_{i,t}(\mathbf{p_t}) = p_{i,t}(1 - p_{i,t} + \beta p_{j,t}) \tag{5.2}$$

Maximizing this with respect to $p_{i,t}$ ($i, t = 1, 2$), straightforward computation yields to:

Lemma 5.2 *With no price anchoring in equilibrium firms choose* $p_{i,t}^{**} = \frac{1}{2-\beta}$ *in each sequence of period and profits can be given by:*

$$\pi_i^{**} = \frac{2(1+\beta)}{(2-\beta)^2}$$

for $i = 1, 2$.

This leads to the following result.

Proposition 5.1 *If* β *is sufficiently small the average price of the two periods is lower compared to the no-anchoring case.*

Proof: To show this, we need:

$$\frac{\sum_{t=1}^{2} p_{i,t}^{*}}{2} < p_{i,t}^{**}$$

Plugging into this the equilibrium prices we have that

$$\frac{-2\beta^2 + \beta(7\lambda + 8) - (\lambda + 1)(7\lambda + 8)}{2\left[(4\beta - 7)\lambda^2 - 4(\beta - 2)^2\lambda + (\beta - 2)^3 + \lambda^3\right]} < \frac{1}{2 - \beta}$$

This inequality holds whenever:

$$\lambda < 1 - \beta$$

■

This result is depicted on Figure 5.1. The shaded area corresponds to the cases when anchoring yields lower average prices.

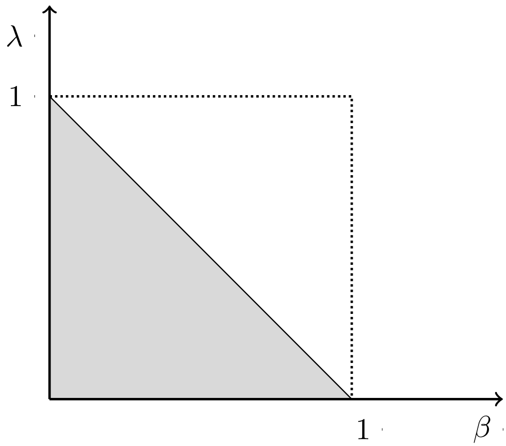

Figure 5.1: Change of average prices.

Remark 2 *Notice, that the output-weighted average price is even lower than the average price, since with anchoring the second period equilibrium prices are lower and the equilibrium quantities are greater than in the first period.*

The intuition behind the above result is that in the first period, firms are increasing prices in order to create a favourable anchor for the second period where they can make up for the lost sales. However, prices in a Bertrand setup are strategic complements, hence when demands are more interrelated, this leads to a more significant price increase in the first period. Of course this implies that the firms are also able to charge a higher price in the second period as well. Therefore the average price increases if the products are close substitutes and decreases when demands are relatively independent of each other.

5.2.2 The general case

In this section we consider a more general case with $n \geq 2$ firms playing an infinite horizon game. In this case firms aim to maximise their discounted profits of $\Pi_i = \sum_{t=1}^{\infty} \delta^{t-1} p_{i,t} D_{i,t}(\mathbf{p_t}, r_t)$, with $\delta \in (0,1)$. Therefore, the respective Bellman equations for this problem can be given as:

$$V_{i,t}(p_{i,t-1}) = \max_{p_{i,t}} \left\{ p_{i,t} \left[1 - p_{i,t} + \sum_{j \neq i} \beta p_{j,t} + \lambda \left(p_{r,t} - p_{i,t} \right) \right] + \right. \\ \left. + \delta V_{i,t+1}(p_{i,t}) \right\}$$

for every $i, j = 1, 2, \ldots, n$ $(j \neq i)$, where $p_{r,t} = \frac{\sum_i^n p_{i,t-1}}{n}$. Dropping time subscripts from the value function,s $V_{i,t}(p_{i,t-1})$, these simplify to:

$$V_i(p_i) = \max_{p_i} \left\{ p_i \left[1 - p_i + \sum_{j \neq i} \beta p_j + \lambda \left(p_r - p_i \right) \right] + \delta V_i(p_i) \right\}$$

Let us assume that $V_i(p_i) = A + Bp_i + Cp_i^2$. In this case first-order conditions yield:

$$\left[1 - p_i + \sum_{j \neq i} \beta p_j + \lambda \left(\frac{\sum_{j \neq i} p_j - (n-1) p_i}{n} \right) \right] + \\ + p_i \left(-1 - \frac{(n-1)}{n} \lambda \right) + \delta B + 2\delta C p_i = 0$$

for every $i = 1, 2, \ldots, n$. Imposing symmetry we have that:

$$p_i^* = \frac{1 + \delta B}{2 - (n-1)\beta + \frac{(n-1)}{n}\lambda - 2\delta C}$$

for every $i = 1, 2, \ldots, n$. For this p_i^* the Bellman equation simplifies to:

$$A + Bp_i^* + C{p_i^*}^2 = p_i^* \left(1 - p_i^* + \sum_{j \neq i} \beta p_i^*\right) + \delta \left(A + Bp_i^* + C{p_i^*}^2\right)$$

or

$$(1-\delta)(A + Bp_i^* + C{p_i^*}^2) = p_i^* - [1 - (n-1)\beta]{p_i^*}^2$$

From this we have that:

$$A = 0 \qquad B = \frac{1}{1-\delta} \qquad C = -\frac{1-(n-1)\beta}{1-\delta}$$

Plugging these into our previous equation yields:

$$p_i^* = \frac{1}{2 - (1+\delta)(n-1)\beta + (1-\delta)\frac{(n-1)}{n}\lambda}$$

Solving the no-anchoring case for n firms we have that firms choose $p_{i,t}^{**} = \frac{1}{2-(n-1)\beta}$. Comparing this to the prices given by the previous equation we can show that:

Proposition 5.2 *Price anchoring yields lower prices if* $\lambda > \frac{\beta\delta n}{1-\delta}$.

Proof: Let us define $\Delta_p \equiv p_i^* - p_i^{**} = \frac{\delta(n-1)\beta - (1-\delta)\frac{n-1}{n}\lambda}{[2-(1+\delta)(n-1)\beta + (1-\delta)\frac{(n-1)}{n}\lambda][2-(n-1)\beta]}$. We shall prove that $\Delta_p < 0$. Since the denominator in Δ_p is positive we need that:

$$\delta(n-1)\beta - (1-\delta)\frac{n-1}{n}\lambda < 0$$

This implies that:

$$\lambda > \frac{\beta\delta n}{1-\delta}$$

■

Proposition 5.3 *Price anchoring yields lower profits if and only if it yields lower prices.*

Proof: Plugging the equilibrium prices into the respective profit functions we have that in the case of anchoring profits are:

$$\pi_i^* = \frac{1 - \delta(n-1)\beta + (1-\delta)\frac{(n-1)}{n}\lambda}{[2 - (1+\delta)(n-1)\beta + (1-\delta)\frac{(n-1)}{n}\lambda]^2}$$

and without anchoring they equal to

$$\pi_i^{**} = \frac{1}{[2-(n-1)\beta]^2}$$

for each $i = 1, 2, \ldots, n$. Let $\Delta_\pi \equiv \pi_i^* - \pi_i^{**}$. This is negative if:

$$[2-(n-1)\beta]^2[1 - \delta(n-1)\beta + (1-\delta)\frac{(n-1)}{n}\lambda] <$$
$$< [2 - (1+\delta)(n-1)\beta + (1-\delta)\frac{(n-1)}{n}\lambda]^2$$

or

$$-[(1-\delta)\lambda - \beta\delta n][(1-\delta)\lambda + \beta(2-\delta-\beta(n-1))n] < 0$$

which simplifies to:

$$\lambda > \frac{\beta\delta n}{1-\delta}$$

Notice that this condition is the same as the one derived in the previous proposition. ■

It is quite interesting that this time we obtain a lower bound for λ, opposite to our result in the two-period game. The explanation lies in the fact that the steady-state incentives are somewhat different than the ones that exist in a dynamic pricing game. The firms are not setting prices in order to exploit the anchor in the future. Rather the existence the anchoring in some sense "pushes down" the price limit in this Bertrand competition, since a stronger anchoring effect makes it more profitable to decrease prices further.

Proposition 5.4 *The price decreasing effect of anchoring is more apparent if fewer firms are active in the market. The same applies if firms produce highly differentiated products (i.e. $\beta \to 0$) or if firms value future earnings less (i.e. $\delta \to 0$)*

Proof: The proof follows trivially from the previous proposition since the right-hand side of the inequality condition is increasing in n, β and δ, respectively. ■

If there are fewer firms in the market, one firm can influence the average price more, hence they have a larger incentive to lower their prices. If the products are more differentiated, the firms own demand is less affected by the price decreases of the other firms, therefore there is more room to decrease prices. If the firms value future earnings less, then they are willing to lower prices more even when this does not lead to future gains.

5.3 Conclusion

Previous literature warns us that in certain cases firms are able to exploit consumer bias to increase their profits, while harming their consumers. Anchoring is well-known and well-researched bias for psychologists as well as marketing professionals. Little research was done however on the issue how price anchoring affects the conclusions of our market models. To at least partially answer this question, we investigated these effects within a finite horizon Bertrand game with differentiated products. We assumed that the average price of the previous period serves as an anchor for the consumers, furthermore we assumed that this fact is common knowledge for the firms. Solving our model, we find that in the case of anchoring, the consumer bias might lead to lower prices. Somewhat surprisingly, we also find that this price-lowering effect is more likely in more differentiated markets, thus firms with higher market power are even less likely to exploit anchoring. In our more general infinite-horizon model, we have further found that if there are less firms in an industry, it is more likely that price anchoring will lead to lower prices, also pointing in this direction. Even though the effects of anchoring on equilibrium oligopoly prices is ambiguous, we have shown that it can lead to lower prices, especially in those cases when the firms have higher market power.

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