CORVINUS UNIVERSITY OF BUDAPEST

REAL OPTION DECISION MODEL APPLICATIONS FOR A EUROPEAN UNION EMISSIONS TRADING SYSTEM PARTICIPANT GAS-FUELED POWER GENERATOR

Ph.D. Dissertation

Tamás Nagy

Budapest, 2013.

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PARTICIPANT GAS-FUELED POWER GENERATOR

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CORVINUS UNIVERSITY OF BUDAPEST

MANAGEMENT AND BUSINESS ADMINISTRATION
DOCTORAL SCHOOL

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I. INTRODUCTION

While managing the risk associated with the various financial instruments is a daily routine in the financial sector, the need for professional risk management is still something new for the power plants active in the electricity market. Hungarian power stations primarily deal with technical risks (failures, malfunctions, catastrophes), financial risk management is in its very early stages (Western European power plants are ahead of us in this regard). The situation seems particularly problematic considering that the electric energy industry is, in a certain respect, characterized by a higher level of risk than banking: the price of electricity is extremely volatile, and very difficult to model (the different time-of-use periods – that is, the electric energy delivered during those periods – may be considered different products) and moreover, the electricity market has a rather high total turnover and is very concentrated (the number of market actors being small, they can more easily influence the prices).

Recently, the development and the spreading of energy market risk management techniques has been facilitated by a number of factors. One is the liberalization of electricity markets (one of the most significant consequences of which was the appearance of the spot price) resulting in the previously prevailing system of long-term production contracts and fixed-schedule production becoming gradually replaced by market (profitability) dependent operation. Another important factor is the start-up of the EU ETS (EU emissions trading system), whereby the new risk factor represented by the emission units further complicates our risk models.

In the present phase of the emissions trading system, the total volume of allowances that the authorities allocate to European corporations free of charge adds up to approximately 2 billion tonnes[1], distributed among some ten thousand participating businesses. As a consequence, some actors might possess large amounts of "unbound" allowances after the allocation. It was the need to manage these enormous free-of-charge assets that first made the individual companies face the question how many allowances they will actually need, and how many they should sell or buy, and when they should do so. While the

[1] Assuming an allowance price of 7 EUR/tonne, this means a 14 billion euro market.

(economically sound) storage of the electricity generated is hardly possible and that of the fuel (gas) necessitates large-scale investments, a specialty of emission units is that they are not directly required for the production itself (only when surrendering them afterwards) and that they can be stored at practically no cost. Consequently, during the year, power plants are free to adjust their allowance positions as they wish, which gives space to various hedging and speculative transactions.

Experience from the first years of the EU ETS also called attention to the importance of the total volume of allowances: in spring 2006, market actors witnessed a two-thirds drop in the price of quotas within 8 days (24 Apil 2006: 29.43 EUR/ton, 2 May 2006: 10.9 EUR/ton). The reason behind the sudden price dip was the publication of the actual emissions data, in the light of which market actors felt it more and more likely that too many allowances had been allocated in total. That price drop meant a huge loss for any actor with a surplus of quotas, while a substantial profit for the ones in a lack of allowances. Following the allocation – and still before the price drop – many actors decided to sit out, that is, not make any transactions in the market. This sitting-out, this lack of exchange transactions was nothing else but speculation: actors with a quota surplus (i.e. in a long position) bet on an increase in the allowance price, while those in a shortage of quotas (i.e. in a short position) put their bet on a price drop.

This unintended sort of speculation, caused by the lack of quota position management, was not limited to market actors only: though only a few are aware of this fact, but the Hungarian state itself was one of these actors. In its national allocation plan, Hungary made use of its right not to distribute a portion of the allowances free of charge, but through auctions open to participating companies. Setting the amount to be sold at these auctions meant that afterwards, the government held an open (i.e. exposed to price risk) position of a substantial value. If the Hungarian government does not enter appropriate hedging arrangements after the decision has been made, that means they – unintentionally – bet on a rise in the price: in possession of an amount of salable allowances, they hold an open (long) position, that is, they bet that with time, the present value of their position comprised of auctionable quotas will exceed the price that prevails at the time the decision about the auctions is made. This state of unintentional speculation lasts until they either open a hedge position or close their open position by completing the auction. During the very first ten months of operation of the allowance market (right until the price drop), the price remained above 20 euro. The first auction in Hungary was held on

11 December 2006; the Hungarian government sold allowances equivalent to 1.2 million tonnes at a price of 7.42 euro per tonne (Kaderják, 2007), generating an income of about 8.88 million euro (about HUF 2.3 billion at the then current exchange rate). The second auction took place on 26 March 2007, with a total sales volume of 1.18 tonnes at a 0.88 euro per tonne price generating about 1.04 million euro in revenue (approx. HUF 256 million). Thus the total revenue from the auctions amounted to about 9.92 million euro (HUF 2.5 billion). Following the decision on the auctions, it would have been reasonable for the state to open a hedge position to cover their future allowance sales, which would have prevented most of the "loss" they made because of the price drop. Assuming a sufficiently liquid market with a 20 euro per tonne strike price and the absence of hedging transaction costs, the state's revenue could have reached nearly 48 million euro or HUF 12 billion (which is approx. 38 million euro or 10 billion HUF more than what was actually realized).

Though the speculative aspect and the order of magnitude of the loss were (and probably still are) not really clear to the actors and the public, it was a general perception that the postponement of the auction and the drop in the quota price must have caused a loss of profit for the state. The ministry responsible for environmental protection put the blame on the Ministry of Finance for delaying the auction. Noteworthy is, however, that there was not one single critical remark about the state's open – i.e. risk-exposed – position being unhedged and the inherent, though unintended, speculation it implied. Obviously, the emissions trading system was a novelty not only for market actors, but for the authorities, as well, which is why it is not fully justified to emphasize the deficiencies of their proceedings. The case nevertheless calls attention to the fact that not even regulators had a full understanding of the relevant risk management issues, and the nature of allowance (and energy) market risk and how it might be kept under control.

It must be underlined that the hedging of an open (risk-exposed) position does not warrant a profit, but reduces the volatility of future revenues (cash flow). Had their position been hedged, the state would have suffered a far smaller loss caused by the price decrease. The profit to be made on the hedging transactions would have compensated for the damages caused by the price dip. Of course, there are always two sides to a coin: the hedging arrangement does not only reduce a potential loss casused by price change, but also "eliminates" the profit that could be made on a price change. Had the position been hedged, a hypothetical increase in the market price would have resulted in the loss from the hedge diminishing the profit from the delay and the price increase. In which case, an unhedged position would have yielded a significant additional revenue for the state (i.e. the state would have made the right bet). (In this theoretical scenario, it might have been the Ministry of Finance that would blame the environmental department for urging the auction.)

The dissertation aims at employing a combined approach, comprising environmental economics, corporate economics, real options, simulation and stochastic finance elements, to the European Union Emissions Trading System (EU ETS) participant electric power generator company, and at providing power plants with practicable risk management techniques.

I.1. Structure of the Dissertation

The rest of the first chapter deals with the major environmental economics theories related to the emissions trading scheme, the path that, through a series of international conferences, led to the introduction of the EU ETS, and the basic features of the actual scheme.

In the second chapter, I will develop a real option decision model for a EU ETS participant (i.e. obliged to comply) power generator company. I will show that a rational, profit maximizing business operates conditionally: they only generate power if the spread (gross margin) per unit of production is positive. Technological constraints (e.g. minimum up and down times) were not considered in the model.

As we will see, the company's emissions can be deduced from the real option model: the output of a future period can be interpreted as the payoff function of a three-asset digital spread option. While the realized gross margin of a future production day can be interpreted as the payoff function of a three-asset (electric energy, gas and emission allowances) spread option. The value of the power plant's revenue generating capacity is equal to the sum of the values of all the spread options for the given future interval.

I extended the three-asset model (incorporating electric energy, gas and emission allowances) to a four-asset model by accounting for the price of electricity in a dual way:

for peak and off-peak periods separately. The advantage of the four-asset model is that it can better approximate the phenomenon that gas power stations, which have higher production costs but are more flexible to operate, are typically kept idle during off-peak hours (characterized by lower electricity prices), but do generate power in peak periods (with higher electricity prices).

The third chapter presents the technical background of the calculations. First, I will introduce the mean-reverting stochastic model for the prices of the four underlying commodities, which I fitted to price data from the German power exchange. Subsequently, I will provide an overview of major spread option pricing methods, along with a detailed account of the analytical approach and Monte Carlo simulation employed in the dissertation.

The fourth chapter demonstrates the first application of the real option model, used to predict the company's carbon dioxide emissions. The expected volume of emissions is of particular importance to both energy market actors and the regulator (just think of the price dip in spring 2006). Beyond the expected value of emissions, its distribution, its probability density function will also be given, which shows the probability of each emission level. Relying on these results, the value of the carbon credits the power plant has to buy for the year in question – that is, the maximum cost of their compliance – can be calculated for a given confidence level. With reference to this chapter, the appendix also includes a description of a numerical method built upon recombining binomial trees, which can be used to approximate the probability density function of future emissions.

The fifth chapter is devoted to discussing the real option applications related to the value of the future realized spread. I will examine the sensitivity of the spread options that represent the value of the power plant to changes in various technological and market factors and, based on the results, I will explore the dependence of the power station's 30-year revenue generating capacity on different factors.

The value of the power plant is exposed to the risk arising from potential changes in the price of the underlying commodities, against which it should therefore be hedged. I will suggest a four-asset dynamic delta hedging strategy, which requires the power plant to hold, at any given moment, an amount of hedging positions that ensures the resultant portfolio (generation capacity, any potential supply agreements and hedging transactions) has a deltas of zeroes. At the same time, we will also arrive at the recommended “optimal” amount of emission allowances at that given point in time. As we will see, this is a quantity close to the expected emissions volume; more specifically, it can be quantified based on the aggregate of the delta parameters of the remaining days’ spread options with respect to emission units.

Relying on the power plant valuation model, the value of a theoretical efficiency improvement project can easily be determined. I will identify the factors that have a significant influence on the value of the savings from more efficient production.

Power stations frequently enter long-term production agreements with fixed or resource-dependent selling prices. I will show that given the model’s assumptions (absence of technological constraints, sufficient liquidity in the marketplace), power plants’ decisions whether to run or idle should still be based on spot prices even if they have such agreements in place. If their production follows a fixed schedule, a portion of the option value from their flexibility will be lost. The loss is related to the production days when the spread is negative: the power plant would be better off stopping production and covering its supply obligations through market transactions instead. The amount of the profit lost can be estimated by the difference between the values of the spread options and the swap transactions for the period in question. I will examine how this loss is influenced by various technological and market factors.

The sixth chapter aims at assisting power plants in defining an efficient auction strategy. Based on the options for the spread without the emission units, I will derive the reservation price of the allowances, which will be used to arrive at the company’s MNPB (Marginal Net Private Benefit) and individual allowance demand functions. Using these, then again, one can determine the highest per-unit price that is worth paying for a given amount of allowances at the auction by a given company. I will also examine how changes in thermal efficiency and in the parameters of the price model affect the MNPB and individual demand functions.

The last chapter comprises my conclusions.

I.2. Introduction to the Theory of Emissions Markets – from an Environmental Economics Point-of-View

This section provides a brief overview of the *mainstream* environmental economics theories related to emissions trading.

In reality, economic activities are almost certain to unintentionally affect, either in a positive or a negative way, one or more third parties. These consequences are, for the most part, externalities (external economic effects), which by definition (Kerekes, 2007, p. 118) have the following characteristics:

- The activity affects the welfare of some third party
- The effect on the third party is unintended
- The third party is not compensated for the damages / does not have to share the benefits.

A well-known example of a positive externality (the external effect increases the third party's welfare) is the case of the orchard and the beekeeper, where the positive external economic effect is bi-directional: the beekeeper's activities enhance the orchard owner's welfare by the bees pollinating the trees; at the same time, the beekeeper also benefits from the orchard, because the nectar harvested from the flowers is what the honey they produce is made from.[2]

A negative externality means that the economic activity causes – unintentional – damage to some third (external to the economic transaction) party, and they receive no compensation for that. A telling example is carbon dioxide emitting economic activities: the greenhouse gases emitted show no respect for state boundaries, they spread throughout the atmosphere and hence affect the climate of the entire Earth, probably resulting in a reduction in global welfare; the effect is unintentional; and the affected parties are not (yet) compensated for the damages suffered.

[2] In practice, the beekeeper case does not always involve an externality: beekeepers often pay for using the bee pasture, and orchard owners, as well, frequently pay for the pollination. In which case the third party is compensated, and thus there is no externality any more.

The mainstream microeconomic model of external economic effects is that of Pearce and Turner (1990, p. 90), also discussed in detail by Kerekes and Szlávik (2003, pp. 92–93). The model involves two measures, marginal revenue and marginal cost, both interpreted as a function of production/pollution quantity:

- The MNPB (Marginal Net Private Benefit) function shows how much the company's profits are increasing by a unit increase in production (or pollution). In the most basic interpretation, the MNPB function can be represented by a downward sloping straight line, and given as the difference between the horizontal marginal revenue (MR) function – equivalent to the market price – and the upward sloping (growing) marginal cost (MC) curve.
- The MEC (Marginal External Cost) function shows how the additional social costs (i.e. those affecting external parties) of pollution change with its total level. In this model, the function's graph is an upward sloping straight line: the rate at which costs grow increases with pollution level.

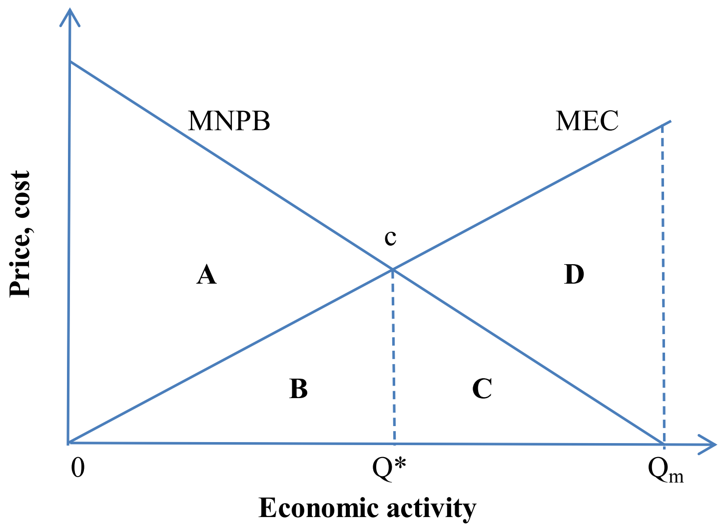

Figure 1: Optimal level of economic activity in case of externality (Kerekes, 2007, p. 125.).

In the absence of regulation, the traditional model suggests that the profit maximizing company will continue to increase its production until its *MNPB* function is positive. The individual optimum is at the point of intersection Q_m of the *MNPB* function and the horizontal axis. At that point, the company's profit corresponds to the area *A*+*B*+*C*, total external cost to *B*+*C*+*D*, and total social benefit to *A*-*D*.

If however external costs are also taken into consideration, the socially optimal production (pollution) level of the company will be given by the point of intersection Q^* of the two functions *MNPB* and *MEC*. The company's profit is equal to the area $A+B$, external cost equals B, and total social benefit corresponds to area A. Moving away from this social optimum in any direction will result in a drop in total social benefit. The model also illustrates the finding – probably rather surprising for the layman – that we should not aim at the total elimination of environmental pollution, but rather at its reduction to the level that ensures maximum social benefit (Kocsis, 1998). That is, an environmental pollution level above zero may well be rational.

Unlike the traditional model, Löfgren (2000) opts for a horizontal MNPB function. The author presumes the company's market to be competitive, and also accounts for its capacity constraints. From the company's perspective, the selling price of its product is an exogenous factor, and hence constant – which is a similarity with the mainstream model. A difference is, however, that the average variable cost of production is constant throughout its entire capacity range and, consequently, so is its marginal cost (in contrast to the increasing marginal cost assumption of the traditional model). The real option model in the dissertation applies to power generators yields an MNPB similar to that proposed by Löfgren in the short run – within a given time-of-day interval –, yet a downward sloping MNPB curve in the long run (for details, see Chapter VI).

In order to shift towards the social optimum, external costs need to be internalized. *Command and control* type methods (Kerekes - Szlávik, 2003, pp. 109-116) are a top-down, administrative approach to influencing the company's pollution level. The two main types are taxes on pollution (*Pigouvian tax*) and *quantity restrictions*. A Pigouvian tax is a predetermined charge imposed by the authorities on each unit of production (or, possibly, pollution) in order to reduce production and, hence, pollution by an extent sufficient to reach the social optimum. An advantage of this solution is that companies' cost of compliance is predictable, while a drawback is that the total amount of emissions remains uncertain. The other *command and control* type method is *quantity restriction*, which sets separate maximum pollution levels for each company falling under the regulation. Its advantage is the high certainty in determining the total amount of emissions; on the other hand, a disadvantage is that the total social cost of compliance is difficult, if at all possible, to predict. Polluters have to limit their emissions to a given extent irrespective of their cost structure, therefore pollution reduction does not necessarily happen where it would be the cheapest to achieve, thus the social optimum is certain to be not met. An overall problem with *command and control* type methods is that in order to use them efficiently and to arrive at the social optimum, one would need to know the individual *MNPB* and *MEC* functions of all the companies and set quantity restrictions on a company-by-company basis – neither of which is feasible in practice.

Another possible way of dealing with externalities is Coase's *property rights approach* (1960), according to which the appropriate definition and allocation of property rights allows for the social optimum to be achieved through negotiation (given that transaction costs are sufficiently low). If the polluter has the right to pollute, then those exposed to the externalities caused will pay the polluter for reducing the pollution. If, however, the third party exposed to the pollution has got the right to live in a completely clean environment, then it will be the company that has to pay for the possibility to pollute. From a Coasean point of view, the problem with the externality related to the excessive emission of greenhouse gases originates in the atmosphere not having an exactly defined owner, that is, in the lack of one given person or organization designated to make decisions in air pollution matters. An important precondition for the Coasean solution is the possibility of bargaining at low transaction costs. Is the number of stakeholders too high, agreements become rather hard to negotiate.

Building on the Coasean foundations, it was Dales (1968) who developed the concept of pollution permit markets, which the operation of the European Union Emissions Trading System (EU ETS) is based upon. It is not the atmosphere exposed to pollution the property rights of which are defined by the model, but rather the permits for emitting a unit of pollution. Pollution permits can be freely traded, which ensures that emission reduction happens where it is the cheapest to achieve. If the price at which a company can sell their pollution permits in the market is higher than what they could earn by causing a unit of pollution, they will terminate production and sell their quotas instead (or buy less of them). If however the profit they make on the activity causing the pollution exceeds the quota price, they will increase production (and hence pollution) and buy more permits. If transaction costs are low and market actors are well-informed, the system will ensure that the social optimum is achieved.

In a *cap and trade* system, the first step is to lay down the "permitted" total amount of pollution (cap) and create the corresponding number of pollution quotas. Subsequently, the trade in these pollution permits will ensure that targets are met in an efficient way. Rubin (1996) built a model for the emissions market and the behavior of quota prices. He concluded that a market comprised of profit maximizing companies indeed leads to emission reduction being realized at the lowest possible cost. Soleille (2006) underlines, nevertheless, that the efficiency of the emissions trading scheme does not arise from the nature of the instrument alone, but also depends on how strict the global emissions target is. A weak spot of the system is the setting of the correct *cap* (total amount): if too many emission units are issued, they will suffer a price drop and thus cease to motivate for pollution reduction; if there are too few of them, on the other hand, that might lead to an excessive restriction of production and, consequently, recession.

When employing a complex environmental policy comprising several different types of instruments, one also has to take into account the interactions between the emissions trading system and other regulatory measures. According to Sorrell and Sijm (2003), the application of complementary instruments in addition to an emissions trading scheme is only acceptable if they improve the efficiency of the latter or if they serve some further regulatory purposes. They also emphasize that the parallel use of two environmental policy instruments increases the system's costs, and the complementary instrument does not necessarily contribute to the further reduction of emissions. Böhringer et al. (2006) call attention to the fact that if the emissions trading scheme does not cover all industries, then the remaining industries need to be regulated by some other environmental policy instrument in order to meet the relevant emissions targets. Based on a simulation for Germany, the authors conclude that a possible lack of efficiency in the emissions market should be ascribed to the lobbying activities of influential sectors, which aim at being exempted from the regulation or at being allocated more than their reasonable share of emission units.

The first significant emissions market was created in the United States of America, a country with a long tradition of stock markets, and operates within the framework set forth in the *Clean Air Act* of 1990, enacted as part of the *Acid Rain Program* of the US Environmental Protection Agency (EPA). The first phase of the program started in 1995, the second in 2000. Though chronologically only the second, the emission market that ranks first in terms of volume and international significance is the European Union Emissions Trading System (EU ETS), launched in 2005 – a detailed description of which follows below.

I.3. European Union Emissions Trading System (EU ETS)

As we were approaching the end of the 20th centrury, it became more and more unquestionable that the significant amount of carbon dioxide emissions that large-scale industrialization brought about would result in long-term global climate change. The pollution and its effects both being global, the issue may only be effectively addressed through international cooperation. The first agreement of decisive importance was the United Nations Framework Convention on Climate Change (UNFCCC), basically a result of the 1992 *Rio Earth Summit*. Those signing the convention admitted that greenhouse gases indeed act to disrupt the Earth's ecosystem. The declared purpose of the agreement was to stabilize the atmospheric concentrations of greenhouse gases at levels that could still prevent human-induced, dangerous climatic changes from occurring. The parties furthermore agreed to hold, starting in 1995, annual conferences (Conference of Parties, COP) to discuss the most important advancements in their fight against climate change and to elaborate concrete steps in order to avoid the worst consequences.

The third conference (COP 3) in the Japanese city of Kyoto in December 1997 adopted the *Kyoto Protocol* (The Kyoto Protocol on Climate Change), in which industrialized and transitional (*Annex B*) countries committed to an average 6 to 8 percent reduction in their emissions for 2008-2012 relative to their 1990 levels. The condition for the protocol itself to enter into force was that the countries it gets ratified by account for at least 55 percent of global greenhouse gas (GHG) emissions. Implementation was put in jeopardy by the withdrawal from the protocol in 2001 of the largest polluter, the long-hesitating US (the United States would have had to pledge to a 7 percent reduction). At the 2001 meeting in Bonn (COP 6), the US delegation remained on the sidelines as observers. The agreement adopted in Bonn dealt with the various *flexibility mechanisms* (ET, JI, CDM) and the matter of *carbon sinks*, as well. The deadlock over the Kyoto Protocol was only resolved in 2004, when another dominant polluter – namely Russia – decided to sign the treaty. The uncertainties about the protocol's coming into effect and the seven-year delay are telling examples of the strength of the conflicts between individual countries' short-term economic interests and the long-term global interests of mankind.

Concerning climate change and climate talks, there are two significant pieces of work that have had very remarkable influence on public and political opinion and, thus, must be mentioned here: the periodic report (Assessment Report) of the Intergovernmental Panel on Climate Change (IPCC) and the *Stern Review*. IPCC released four reports so far (in 1990, 1995, 2001 and 2007), and the next one is due in 2014 (IPCC Fifth Assessment Report: Climate Change 2014). The reports summarize the findings of three workgroups, discussing atmospheric physical, ecological and economic matters. Nicholas Stern was commissioned by the UK government to prepare his 700-page *Stern Review* (2006). One of its most important conclusions is that in the absence of appropriate action, mankind may lose 5 percent of global GDP each year (should more significant collateral consequences occur, losses might even amount to some 20 percent). The report also states that the most significant negative effects might be possible to avoid by devoting approx. 1-2 percent of our annual GDP to the matter. Beyond delivering a “diagnosis”, the report also urges to create, through international cooperation, global prices for emission units, “broadly similar” within the individual regions, as an essential condition for efficient emissions reduction.

Thanks to these reports, the public of the more developed countries became more and more supportive of climate protection efforts. It may be both interesting and instructive to review how the role of the European Union in the Kyoto process has changed with time (for details, see Convery et al., 2008). The first significant stage of community-level pro-environmental cooperation was the *Single European Act* of 1986, which put emphasis on the cost-efficient, community-level management of environmental challenges. In order to limit greenhouse gas emissions and to internalize external effects, the European Commission suggested in 1992 that a community-wide carbon dioxide tax be introduced. The proposal was not implemented after all, on the one hand because some member states interpreted a European-level tax as a limitation of their fiscal autonomy, on the other hand because the energy industry, having an adverse interest, possessed significant lobbying power. The Union assumed a leading role in the negotiations that laid the foundations of the *Kyoto Protocol*. Its initial stance was the introduction of a uniform 15 percent reduction and the refusal of emissions trading. From the European point of view, the negotiations were far from successful given that neither one of these two suggestions were incorporated in the protocol after all. Following the signature of the treaty and the failure of the European lobby, there was a turnaround in the EU’s strategy: given the absence of the US, it took over the leading role in international climate protection. The former adversary of the emission market turned into the initiator of the currently largest carbon permit market.

The EU as a community committed itself to an 8 percent reduction in the protocol, while the determination of the individual countries' share was left to the EU's own discretion. Member states' emissions targets (through which the Union's target will be met) were defined in the *Burden Sharing Agreement*. The more developed member states pledged to larger reduction percentages, while the emissions of less developed countries were allowed to rise. As regards the protocol, the transitional economies of the former "Eastern Bloc" (including Hungary) had a special position, for their previously significant industrial output, energy consumption and greenhouse gas emissions all fell back at the end of the eighties due to the economic transition. These countries were allowed not to choose the year 1990 as a basis (compared to which their reduction targets are interpreted), but one of the earlier years. Concerning Hungary, this means that the 6 percent reduction we committed to needs to be met on a 1985-87 basis, which will be possible without any particular additional effort. Hungary is expected to have a quota surplus until 2020 (Szabó et al., 2010), but in the long run, we will also have to assume a more active role in carbon dioxide emission reduction.

The foundations of the European Union Emissions Trading System (EU ETS), established in order to meet emissions targets in a cost-efficient way, were laid by Directive 2003/87/EC of 13 October 2003. The EU ETS is a *cap and trade* system, where countries determine the permissible amount of emissions in a top-down process, and issue a corresponding amount of freely tradable quotas. The quota price fluctuates depending on market processes, therefore the total cost of emission reduction is impossible to determine in advance. The low level of transaction costs (a result of free tradability and liquidity in the quota market) ensures that emission reduction – or savings – occurs where the associated costs are the lowest.

With its launch in 2005, EU ETS preceded the 2008-2012 period – which the Kyoto commitments concern – by three years. In line with that, the startup of the trading system was performed in two stages: the first stage (*Pilot Phase, First Phase*) lasted from 2005 until 2007, with the gaining of experience as the main purpose, while the second stage, until 2012 (*Kyoto Phase, Second Phase*), was intended to ensure the cost-efficient compliance with the set targets. The third stage (*Third Phase*) covers the period between

2013 and 2020 and includes the further reduction of the emissions of all types of greenhouse gases, and the gradual transition from the free allocation of permits to auctioning.

The European Union Emissions Trading System is the world's first international system for the exchange of carbon dioxide emission units. It covers the total carbon dioxide emission volume of 2 billion tonnes of approximately 10,500 institutions in the 27 member states of the EU plus the three associated states (Iceland, Lichtenstein, Norway). From amongst all the greenhouse gases, the present system almost exclusively covers carbon dioxide. The only exception is nitrous oxide emitted in Norway and the Netherlands (European Commission, 2009), though the respective amount is rather limited.

It might be worth comparing the volume covered by the EU ETS to total global emissions. Comparability is compromised by the fact that emissions data available for the different country categories (developed, developing) pertain to differing periods. The total emission of developed (*Annex I*) countries in 2007 was equivalent to 18.1 billion tonnes of carbon dioxide (UNFCCC, Flexible Queries, 2010), with 5 billion tonnes ascribed to the EU-27. The total emission of the 122 developing countries (non Annex I) was, based on 1994 data, equivalent to 11.7 billion tonnes of carbon dioxide (UNFCCC, 2005), while the size of the EU ETS was equated to the amount allocated in 2009 (CITL, 2010):

	CO2 emissions (million tonnes)	**As a ratio of the category before**	**Compared to total emissions**
Total	29 848	-	*100.0%*
Annex-I	18 112	60.7%	*60.7%*
EU(27)	5 032	27.8%	*16.9%*
EU ETS	**1 967**	**39.1%**	***6.6%***

Table 1: Relative size of the EU ETS.

As the emissions data evince, the currently largest emission market covers 39.1 percent of the emissions of the EU-27 and 6.6 percent of total global emissions (if we assume that the current total emission of developing countries exceeds the figure of 1994, the share of the EU ETS would be even lower).

If we were interested in which industries the 61 percent of European emissions not covered by the system comes from, it might be useful to take a look at the 2007 industry breakdown of the EU-27:

Industries	**million tonnes**
1.A.1 Energy Industries	1 604
1.A.2 Manufacturing Industries and Construction	642
1.A.3 Transport	980
1.A.4 Other Sectors	665
1.A.5 Other (Not elsewhere specified)	10
1.A Fuel Combustion - Sectoral Approach	3 901
1.B Fugitive Emissions from Fuels	87
1 Energy	3 988
2 Industrial Processes	430
3 Solvent and Other Product Use	12
4 Agriculture	461
5 LULUCF	-407
6 Waste	141
7 Other	0
Total	**4 625**
Total (without LULUCF)	**5 032**

Table 2: Emissions of EU(27) by industries, energy industry is detailed further.

In order to minimize per-unit transaction costs, it is imperative that the facilities causing the highest emissions be incorporated in the trading system first. The largest polluter, the energy industry, is indeed among those falling under the regulation. Industries with a significant total emission level yet including a relatively large number of individual polluters have not (yet) been incorporated into the EU ETS – a good example is transportation, where an immense amount of individual pollution sources altogether emit a very significant total volume of greenhouse gases (nearly 1 billion tonnes of carbon dioxide, based on 2007 data). Another non-covered area is agriculture with its near half-billion tonne annual emissions.

Of course, it cannot be simply concluded that certain areas were omitted from the regulation to begin with, for the commitments made by the individual countries in the protocol pertain to total country-level emissions. Even though there are other (regulatory) means of facilitating the achievement of national targets (e.g. an excise tax on fuels, setting mandatory minimum requirements for buildings' heat insulation), it would be more fortunate if the already existing carbon dioxide price could "permeate" a wider range of prices already existing in the economy, which then again calls for the expansion of the emissions trading system (the inclusion of the aviation industry points to the same direction).

Moreover, the EU ETS, as the very first multi-national emissions trading system, might prove out to be an invaluable source of experience to assist in the creation of a future global permit trading scheme. Buchner et al. (2006) and Ellerman (2008) discuss it in more detail how far the EU ETS can be interpreted as a prototype of a future international system. Ellerman argues that the Union is, in a sense, rather similar to the global community: member states are only connected by a weak federal structure and there are significant disparities in economic development between them. The differences in development between Europe's East and West are not as remarkable as those across the world, but experience from the EU ETS might be very helpful in designing a future system. The author concluded that the trial stage was very useful for the participants because of the significant body of experience they gained on the operation of the system. Based on actual data he established that, contrary to expectations, the EU ETS did not generate a significant volume of international capital flow: the majority of permits got surrendered in the same country in which they had been issued.

Quota Supply and Allocation

The basic unit of the EU ETS is an EUA (EU Allowance), by which one is entitled to emit into the atmosphere one tonne of carbon dioxide (or the equivalent of that from other greenhouse gases). The quota has a special validity: if banking is prohibited, then it is only valid for a limited period (the given year), before and after which it is worthless. If transfers between years are allowed, then permits are only voided when used to cover an actual unit of emission.

Besides the basic unit of the European Union Emissions Trading System, there are three more emission units worth mentioning: AAU, CER and ERU. An AAU (Assigned Amount Unit) is the basic unit of the Kyoto Protocol. An EUA actually corresponds to a "labelled" AAU, which the governments gives into private ownership within the framework of the national allocation plan (NAP). If a company from one member states sells an EUA to a participant in another country, then both the EUA and the AAU are handed over to the destination country. Therefore, the free allocation of permits actually means a loss of potential revenue for the state: if they did not give away EUAs to their corporations free of charge, they could sell them in the marketplace in the form of AAUs (supposed that the total emissions level of the country allows for that). In the second trading phase, it was already allowed – to a limited extent – to fall back to other Kyoto units for compliance. CER (Certified Emission Reduction) units are created through realized *Clean Development Mechanisms*, that is, when an investor from a developed (Annex I) country completes an emission reduction project in a developing (non Annex I) country. CER units, likewise marketable, are issued after the actual amount of carbon savings has been verified. ERUs (Emission Reduction Units) are created in so-called *Joined Implementation* projects – projects realized by two developed (Annex I) countries and resulting in carbon savings. In order to avoid duplication, the issuance of ERUs is accompanied by the cancellation of an equal amount of AAUs.

Emission units are a special type of good, the special characteristics of which might be best understood by comparing it to other, more usual exchange-traded instruments. In their quota vs. share comparison, Benz and Trück (2009) conclude that while share prices primarily depend on companies' profit expectations, the non-zero price of the quota is basically a consequence of its scarcity, its limited supply. Thinking along the same lines, we may also compare them to energy sources: emission units are special also because while resources need to be extracted at rather high costs, quotas are created through an administrative act: governments issue, based on their national allocation plan, a given number of emission units each year. Considering traditional resources, demand is satisfied from the reserves that can be efficiently extracted given the then current price level. Price increases along with demand, and – with the exhaustion of resources not taken into account – so do economically extractable reserves, thus supply is flexible to changes in market price (demand). The supply of emission units, on the other hand, is set at a fixed level by the regulator. A growing demand does induce a price hike, yet supply remains fixed. The situation is less clear-cut if we take into account emission reduction investments and the various Kyoto flexibility mechanisms, which in a sense act to expand supply. While the former increase companies' – freely tradable – quota surpluses by lowering their per-unit-production emissions, Kyoto units that originate from Kyoto flexibility mechanism projects directly increase supply, though there are administrative limits to their utilization.

The supply of permits is provided by the national authorities, the current practice of which is to allocate the emission units to the participating companies free of charge, for the most part. This initial system of free quota allocation contributed to market actors' acceptance of the emissions trading system. Participants did not suffer a sudden cost boom by entering the system (particularly compared to the other option: a carbon tax). In their dissertation, Lesi and Pál (2004) examined the effect of the different types of allocation (free distribution, auctioning), among others. They concluded that if transaction costs are low, the efficiency of the system is not affected by the method of quota allocation (free of charge or auctioning). The authors argue that free distribution may result in the resistance of market actors being lower, yet it is these quota-owners, as well, that earn the entire amount of the revenue from the scarcity of the units. Whereas in the case of a partial auction, part of the revenue goes to the government, yet participants' willingness to cooperate will not be that high, either. According to the authors, the theoretically desirable share of auctions is, depending on the parameters, somewhere between 16-28 percent.

In the long run, free allocation will be replaced by auctioning. The policymaker intends the resulting proceeds to provide government funding for environmental investments and tax cutting measures.

The Rules of and Experience from Compliance

In the current free allocation system, the distribution of units is performed each year by the end of February latest. Companies have to report on their actual annual emissions by March 31 next year, and within one month's time (by April 30), they have to surrender to the competent authority (compliance) an amount of emission units equal in number to the certified volume of their emissions during the previous year. Important settlement-related dates are shown on the time-line below:

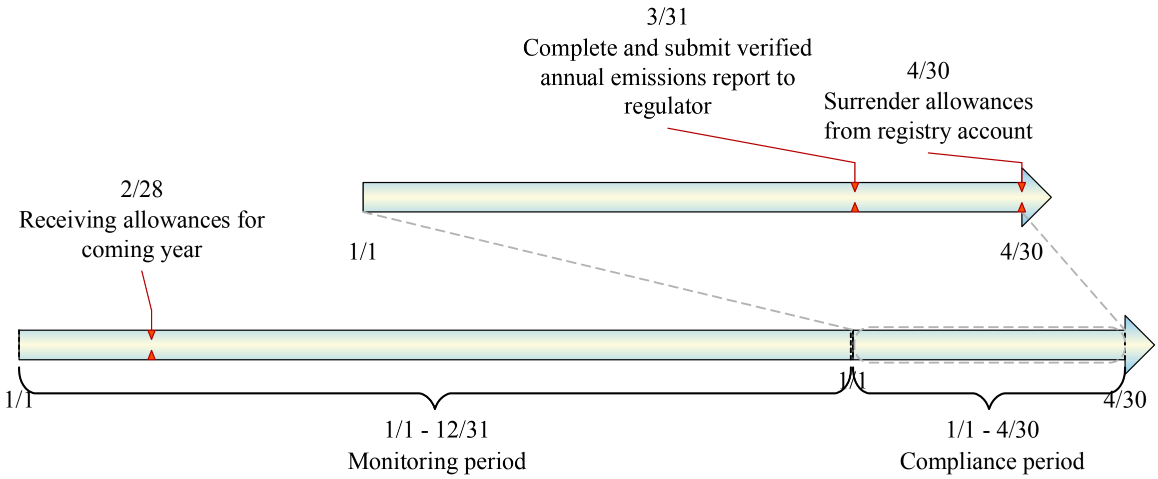

Figure 2: Periods and deadlines of EU ETS compliance.

Non-compliance occurs whenever a company fails to surrender enough quotas to cover their entire amount of emissions. In such a case, the deficit is transferred to the next year – that is, the amount of the shortfall is deducted from their allowance for the next year – and they also have to pay an additional 100 EUR/tonne fine (40 EUR/tonne in the Pilot Phase).

Relying on CITL[3] as a data source, Ellerman and Buchner (2006) analyzed the EU ETS system based on facility-level emission and allowance figures from 2005. The authors compared baseline emissions, the number of permits allocated and actual emissions. As a result, they proposed a measure that would allow for estimating the extent of actual pollution reduction as a function of emissions data, economic activity and energy intensity.

In her dissertation, Fazekas (2009) examined the impact the EU ETS has had on Hungary. According to her, the most significant problem of the Pilot Phase was that the majority of installations had not measured, calculated and kept a record of their carbon-dioxide emissions prior to the EU ETS start-up. Hungarian companies' attitude towards the emissions trading system was explored via a series of personal interviews. They typically perceived the system as an administrative burden, did not recognize the opportunity cost of the emission units, but only strived to minimize the costs to be incurred. According to

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[3] Data from CITL (Community Independent Transaction Log) can be used to analyze the allocated amounts and actual emissions by facility. Transactions affecting the ownership of emission units can only be examined indirectly, for individual deals remain confidential for a period of five years.

the author, Hungarian facilities' primary goal during the Pilot Phase was to make the most out of the allocation process, that is, to acquire the largest possible amount of free-of-charge units, instead of focusing on the total volume of avoidable emissions.

Buchner et al. (2006) found that policymakers believe production cuts to be the primary key to the reduction of carbon dioxide emissions. Therefore, in order to prevent an economic recession, actors usually received enough permits to be able to keep up their current production volume without any disturbances. The exception was the energy industry, which had a great potential for emission reduction: the fuel switch from coal to gas.

To shed further light upon the situation, let us have a look at how allocated amounts and actual emissions data from 2009 relate to each other:

Comparing of allocation and emissions in 2009 (million tonnes)	**Allocation**	**Emissions**	***Emissions surplus***	
Combustion installations	1 262.7	1 377.1	*114.3*	*9.1%*
Cement clinker or lime	213.8	151.5	*-62.3*	*-29.1%*
Pig iron or steel	185.0	95.5	*-89.5*	*-48.4%*
Mineral oil refineries	153.6	146.2	*-7.4*	*-4.8%*
Pulp, paper and board	38.8	27.9	*-10.9*	*-28.2%*
Glass including glass fibre	25.6	19.4	*-6.2*	*-24.1%*
Coke ovens	22.4	15.8	*-6.7*	*-29.7%*
Metal ore roasting or sintering	22.0	11.0	*-11.0*	*-49.8%*
Ceramic products by firing	19.2	9.1	*-10.1*	*-52.7%*
Other activity opted-in	24.2	19.9	*-4.3*	*-17.7%*
TOTAL	**1 967.4**	**1 873.3**	***-94.1***	***-4.8%***

Table 3: Allocation and emissions by industries.

As the table shows, companies were allocated emission units worth 1.97 billion tonnes of carbon dioxide in total in 2009, while the actual volume of emissions turned out to have been (partly "thanks" to the economic crisis) 94 million tonnes less than that. It was only one single industry where the difference between the allocated and the actually emitted amount was positive (i.e. emissions exceeded the allocated volume): power plants. Thus one might draw the – somewhat simplified – conclusion that the trading system brought about additional costs for the power plants and extra revenues for all the other industries. The data indeed seem to reflect the regulatory intent not to make the emissions trading system a source of competitive disadvantage for the industries that are exposed to international competition; thus the costs of compliance were born by the largest sector, one that produces primarily for the European market: the electric energy sector.

Experience About the Quota Market

The quota market has been growing at a quick pace ever since the launch of the EU ETS. The figure below shows the distribution of monthly trading volume and value[4] over a period of five years:

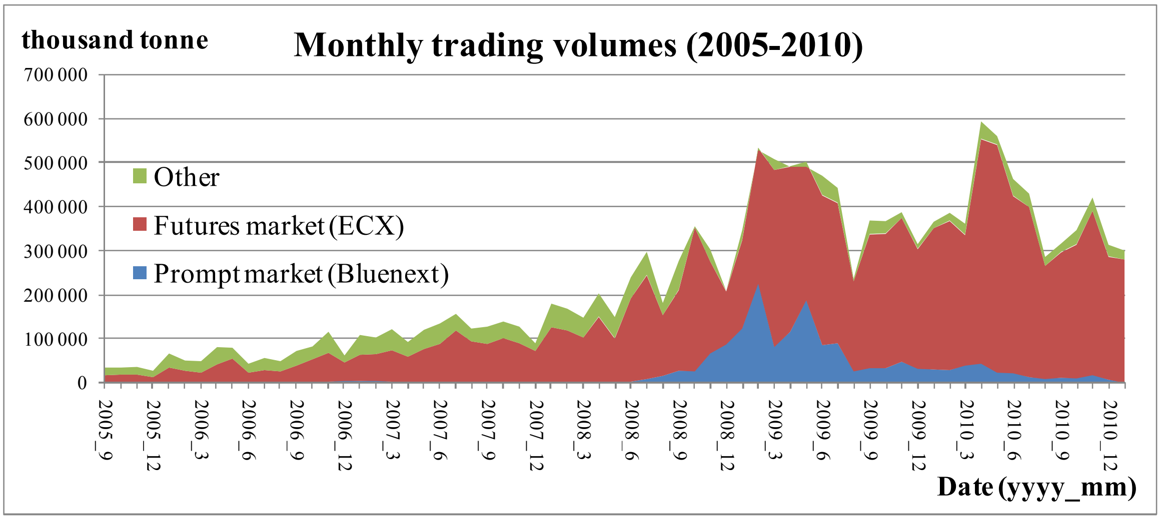

Figure 3: Monthly trading volumes (2005-2010).

As the graph shows, trade volume quintupled in those five years: at the beginning of the period in question, the monthly turnover of the entire market was had been 35 million tonnes, while in March 2010, it was already 361 million tonnes. Transaction were made in the futures market for the most part, the practical reason for which may be that it is sufficient to hold the quotas no sooner than the date of settlement, and hedging transactions are easier to do in the futures market, as well.

After the start-up of the emissions trading system, the quota market and the quota price assumed an increasingly important role in corporate decision making. In their comprehensive work, Convery et al. (2008) discuss the most important experiences from the first trading phase. In their view, national authorities were faced with three major problems in the Pilot Phase: the time frame for preparing their respective national allocation plans was rather short, facility-level emissions data were not available and the coverage principles of the system (which companies of which industries it pertains to)

[4] Source of data: www.cdcclimat.com

were not yet elaborated in full detail. In spite of these problems, the Pilot Phase was successful, insofar as it did meet its declared objective: the system was up and running, market actors gained experience and the market now had a price for carbon dioxide emissions. The system did not have a significant effect on international competitiveness: due to the technological limitations of power transmission, the electric energy sector, the industry most affected by the regulation, did not have to face competition from foreign markets, while all the other, internationally competing industries received more quotas than what they required. The EU ETS also had an external global impact: it was joined with the Norwegian system and through the Kyoto mechanisms, it also generated emission reduction investments in other countries. Given all the differing national interests of the different EU member states behind the EU ETS, experience from the system's operation will certainly be useful in designing a future global system.

In the emissions trading system, compliance always pertains to the given year. Banking means the transfer of unsurrendered allowances to the next year. Borrowing, on the other hand, is the utilization of allowances yet to be issued (in the coming years). While banking within the given trading phase was allowed, banking across two different trading phases was basically prohibited. Schleich et al. (2006) put forward the opinion that banking reduces the costs associated with the system by providing intertemporal flexibility. By way of simulation, they demonstrated that the prohibition of inter-period banking deteriorated the system's efiiciency. Employing a game theory model, they showed that in order to avoid the prisoners' dilemma, member states need to coordinate their banking-related decisions.

The ban on banking across the different phases had far-reaching consequences. The next figure shows the graphs of closing and settlement prices from the Bluenext (spot prices) and ECX (futures) exchanges for the period between June 2005 and May 2010. The reason for including futures prices is that they make it easier to compare prices across the first and second trading phases (futures prices have not been adjusted for time value):

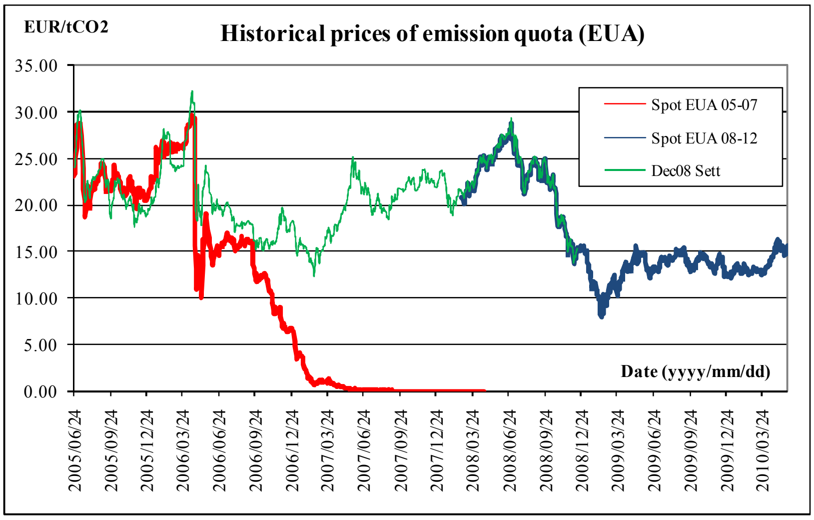

Figure 4: Historical prices of emission quota.

Due to the lack of banking, the permits of the two phases counted as two different assets. As evinced by the graphs, prices for the two periods followed more or less the same pattern until March 2006, the time when the pilot-phase price began to diverge from that of the Second Phase. The difference became particularly large following the publication of emissions data in 2006, as it became fairly likely that the number of emission units on the market exceeds the expected total volume of emissions. Given that, apart from a very few exceptions (Convery - Redmond, 2007), permits for the 2005-2007 period could only be utilized in that given period, any excess first-phase quotas lost their value right after the settlement of the year 2007. Which points out one of the specialities of *cap and trade* systems: over-allocation devalues the quotas, which therefore lose their potential for promoting emission reduction.

Based on prices from the two periods, Ellerman and Parsons (2006) derived a variable expressing the probability of over-allocation. The index was built on the assumption that if there is a quota surplus on the market at the end of the First Phase, then it is worthless (price is zero), while if there is a shortage of quotas, then – presuming an arbitrage-free market – a first-phase quota will have the same value as the alternative of non-compliance (sum of the next-period emission unit price and the fine).

Daskalakis et al. (2009) detected that the ban on inter-period banking also had an influence on the pricing of carbon credit derivatives. There are different types of derivative instruments depending on whether the term of the futures contract extends over the current trading phase or not. The pricing of those that “remain” within the given phase (intra-phase futures) follows the “usual” futures pricing formula, with zero convenience yield. For the pricing of futures stretching over two consecutive phases (inter-phase futures), however, it was models incorporating stochastic convenience yields that proved out to be appropriate. Considering inter-phase contracts, investors have to face a new risk factor: the amount to be allocated during the next phase is uncertain. Higher risk means lower liquidity, and hence a drop in market efficiency. It is of utmost importance, according to the authors, that inter-phase banking not be banned or restricted, and that this consideration be taken into account when developing new emissions markets.

Parsons et al. (2009) analyze the performance of cap and trade systems as a function of operating rules. They examined the two existing systems (the EU ETS and the US sulfur dioxide allowance market) with a primary focus on how the banking and borrowing of permits affects the operation of the market. They concluded that the restriction of these two operations negatively affect market liquidity. They also noted that free allocation keeps the opportunity of banking from being exploited to the full extent, reduces trade volume and thus limits market liquidity. As a consequence, the effects of short-term shocks get amplified and, due to the sub-optimal degree of emission reduction, the system-level cost of pollution reduction is increased.

Publications on the Stochastic Modeling of Allowance Prices

The publications that are in a closer relation to the topic of the dissertation employ various models to analyze the stochastic process governing quota prices. Daskalakis et al. (2009) tested different pricing models on a set of price data from 2005-2007. It was the geometric Brownian motion model with normally distributed jumps that best fit spot price data; the authors recommended against using mean-reverting models – quite widespread in commodity market applications – for quota prices. Seifert, Uhrig-Homburg and Wagner (2008) developed a stochastic equilibrium model for such purposes. They argue that the EU ETS differs from other more established emissions markets (for example the SO2 emission allowance market of the US), thus their models cannot be used without certain adjustments. They call attention to one of the features of immature markets, namely that a strong impact (shock) might have a significant influence on prices. The authors mention the price drop in spring 2006 as an example for a shock. Because of the immaturity of the market, they reason, the model should not be built upon characteristics observed on historical data: it would be more practicable to rely on the regulatory system of the EU ETS and related micro-economic relationships instead. Their analysis primarily aims at determining whether there is seasonality in the time series, whether there are upper or lower price boundaries, whether prices follow a mean-reverting or a random walk process and, also, how volatility behaves. Their most important findings were the lack of seasonality in CO2 prices and the dependence of the model's volatility on time and price.

Some other modeling efforts focused on what other factors the price of emission units was related to. Alberola et al. (2008) performed econometric analyses on the first trading phase between 2005 and 2007. The authors found that the most significant drivers influencing the quota price were the price of electricity, extreme changes in weather and political/regulatory decisions. They employed multiple linear regression to analyze the period from 2005 to 2007 and concluded that it was the extreme cold and the weather being cooler than the relevant seasonal value that had a significant influence on the allowance price. Furthermore, the authors also sought to identify structural breaks within the said interval: points, when there were changes in how the individual factors affected the quota price, that is, in how market actors behaved. They detected two such breaks: the first occurred in April 2006, as the actual emissions data were published (compliance break); while the second structural break comprised of the European Commission's communication in October 2006 that the amount of quotas to be allocated would be significantly cut back for the 2008-2012 period.

Benz and Trück (2009) distinguished two large groups of price drivers: policy (regulatory) factors and fundamental factors. The former affect the price in the long run, while the latter have a short-term impact, for the most part. The authors argue that the cheapest emission reduction alternative for Europe is the switch from coal to gas, that is, from a high-carbon-intensity fuel to one with lower carbon dioxide emissions: the higher the price of the allowances or that of coal, the more willing the industry is to switch to gas. If, however, coal becomes relatively cheaper, that increases its utilization and thus leads to higher emission levels. A good example for a policy factor is official communications about national allocation plans (NAP), which act to alter the actual amount of allowances or expectations thereof. Another determinant of the amount of available emission units are adjustments to the rules on inter-period transfers (banking) and on the relationship with the Kyoto flexibility mechanisms. The authors analyze short-term fluctuations in the EUA price, review the most important stylized facts about the quota market and hence develop a model to describe price behavior. According to their observation, the quota price fluctuates because the changes in regulatory environmental factors and weather factors (extreme cold, extreme heat, too much or little precipitation) occur at points relatively distant from each other in time. This necessarily leads to volatility clustering. They suggest that Markov regime-switching models and AR-GARCH models, both of which can handle the properties mentioned above (skewed distribution, heavy-tail phenomena and heteroscedasticity), should be used for modeling price behavior.

Reilly and Paltsev (2005) relied on the EPPA-EURO (Emissions Prediction and Policy Analysis) model to give an estimate for the first-phase quota price; they arrived at 0.6-0.9 EUR/ton. Whereas the actual per tonne value in mid 2005 was 20-25 euros. The authors ascribe this rather significant discrepancy to the expansion in the use of coal due to rising oil and gas prices, and to the drought, which restricted the availability of hydro energy and escalated the risks associated with nuclear energy.

Mansanet-Bataller, Pardo and Valor (2007) examined the influence financial and weather factors had on the quota price. With regard to meteorological factors, they relied on weather data from Germany (from amongst all EU ETS participant countries, Germany is the number one in terms of allocated quota amount, and their energy market is the largest, as well). *Deutscher Wetterdienst* provide daily minimum, average and maximum temperatures and precipitation amounts for a number of locations. The authors used these meteorological data to derive a population-weighted index for the purpose of further analyses. Expectations were that high temperatures would increase electricity consumption (air conditioning), while cold weather would yield higher heating needs. Both effects were expected to increase both the demand for and the price of emission units. Concerning precipitation, larger amounts act to expand hydro power capacities and thus to reduce the utilization of fossil fuels, which has a moderating effect on quota price.

Employing a panel-GARCH approach, Oberndorfer (2009) showed that changes in the quota price were positively related to changes in European power generation companies' stock prices, yet the relationship differed country by country (the relationship detected was negative for Spain, but positive for Germany and Great Britain). Based on these observations, the author concludes that the EU ETS has a palpable impact on financial markets and on the value of participant corporations. Veith et al. (2009) examine the relationship between carbon dioxide price and the stock prices of European power generation companies. The quota being a new factor of production and an additional cost, the correlation between the two was expected to be a negative one. Based on the authors' evaluation of empirical data, however, the relationship is positive, which means that even though the allowances constitute an additional cost item for power plants, they managed to pass on the burden to their consumers. They also note that the *windfall profits* from the free allocation of allowances may also have contributed to the growing valuation of power generators.

In his analysis of the relationship with energy source prices, Kanen (2006) found that it is the price of oil that determines gas price, which then again affects the prices of both electricity and carbon allowances. Convery et al. (2007) also argued that it is energy source prices that determine the price of carbon dioxide. A concept related to the relationship between energy source prices and the price of carbon permits is the *switching price* (Delarue – D'haeseleer, 2007). Given that the carbon intensity (i.e. carbon dioxide emissions per unit of energy generated) of coal-fired power plants exceeds that of gas-fired ones, a rising quota price diminishes the cost advantage of coal-fired facilities, or eliminates it altogether. The switching price of the quotas is the price under which the gross margins realized by coal-fired and gas-fired power plants are equal. Plotting the switching price as a function of the gas/coal price ratio yields positively sloped straight lines, depending on thermal efficiency. An allowance price above (below) the line means that gas-fired (coal-fired) generators should be preferred.

Given that allowances constitute a new factor of production for electric power plants, the question arises: to what extent do changes in the quota price influence the price of electricity? Both Linares et al. (2006) and Smale et al. (2006) demonstrated that as far as power generation companies are concerned, the quota price can be interpreted as part of their direct production costs. Consequently, the introduction of the EU ETS necessarily brings about a price hike in electricity. Based on their own model, Lesi and Pál (2004) conclude in their dissertation that auction-only allocation clearly acts to increase the price of electricity. According to them, Hungarian electricity tariffs will reflect the price-effect of CO2 costs even if the government continues to distribute the allowances free of charge. Based on a simulation, Chen et al. (2008) found that the "propagation" of carbon price through electricity tariffs was influenced by the competitive situation in the marketplace and the elasticities of demand and supply. Having analyzed the relationship between the quota price and the price of electricity futures, Zachmann and von Hirschhausen (2008) found that changes in the price of carbon dioxide have an asymmetric effect on the price of electricity futures: an increase in the quota price generates a stronger effect in the electricity price than a decrease does. The explanation, they argue, is two-fold: first, the market is not mature enough and second, power plants may take advantage of their power in the marketplace. Certain authors (Kara et al., 2008) also quantify this spillover effect on electricity prices, and expect, for the 2008-2012 period, a 1 EUR/tonne quota price increase to induce a 0.74 EUR/MWh hike in the price of electricity in the northern region.

Following the above theoretical introduction to emissions trading and the most important features of the EU ETS, we now proceed to the discussion of the real option model to be employed in later parts of the dissertation.

II. THE REAL OPTION MODEL OF THE GAS-FIRED POWER PLANT

From the introduction, we now have a basic understanding of the EU ETS framework, and we know that early in the next year, companies have to cover their current year emissions with emission units. In this section, we will examine how an EU ETS participant gas-turbine power plant makes its production decisions. With some minor adjustments, the methodology derived herein is also applicable to power stations with different technologies, and to companies that participate in an industry other than the electric energy sector. The approach outlined in the dissertation is especially practicable in cases where product and resource prices are highly volatile, the role of technological constraints is less significant and operation is flexible.

The focus of the chapter is the decision model of the individual company. There have been relatively few EU ETS related publications on this topic, especially if compared to the number of studies dealing with the rules of the emissions trading system and with quota price trends. Concerning Hungarian authors, Dobos (2002) used a comparative static model to examine the impact of the system of tradable pollution permits on a standard micro-economic (price-taking, profit-maximizing) firm. His model relies on two types of functions: the monotonically increasing and strictly concave production function and the monotonically increasing and strictly convex emission function. The production function gives the amount produced as a function of input volume (i.e. shows how efficient the production process is: how much fuel is needed to produce a given amount of output). The author defines investments in efficiency improvement to alter the production function of the firm. The emission function gives the emission volume as a function of output, that is, it shows the amount of emissions per unit of production. Investments into reducing emissions per unit of production, therefore, alter the emission function. A good example is end-of-pipe technologies, which prevent the pollutants generated from entering the environment. The author compares four different cases in his work: no change in technology; change in technology to improve efficiency; to reduce emissions per unit of production (pollution abatement); both projects are realized. The author’s conclusion from his model is that the company can achieve maximum profit if they realize both types of improvement, that is, if their production process becomes both more efficient and less polluting.

In their dissertation, Lesi and Pál (2004) primarily looked into the matter of efficient regulation and the system's effect on Hungarian power stations. They modeled power plants' decisions with the IID-MEH model, which was originally developed, on commission from the Hungarian Energy Office, to analyze the impact of electricity market liberalization. Based on their model, Hungarian companies are expected to be net sellers: to sell their quota surplus, about 2.7 - 6.1 million tonnes annually, in the European allowance market. The resulting revenues provide the funds for their carbon investments. Through a revenue-neutral, zero-windfall-profit allocation of emission units, the government should retain and sell about 4 to 7 million tonnes of CO2 permits each year, which would yield extra proceeds in the 5 – 35 billion HUF range annually between 2008 and 2012. Then again, the state could use this additional revenue to finance its climate change related tasks (flood protection or investments to facilitate the adaptation of drought-stricken areas, for example).

Delarue et al. (2010) analyzed the opportunities that the fuel switch induced short-term emission reduction constitute for the European electric energy sector. The analysis was performed using the E-Simulate model developed at the University of Leuven. The simulation provides an hourly breakdown of electricity generation for a one-year cycle in advance, at the level of the individual power plants. The system consists of inter-connected "zones", pertaining to a country or a group of countries. Inter-zone transfers are, just like in reality, limited. Demand is exogenous and given on an hourly basis; the model provides the least-cost production combination that can meet the demand from the zones. The model employs heuristics for generation dispatching, with some modifications in order to also account for certain technical conditions (minimum operating time, start-up and shutdown times). Each power plant is assigned an availability factor, which quantifies unforeseeable outages (required regular maintenance works do not fall into this category, as these are scheduled for low-demand periods). The model uses uniform energy source prices throughout Europe, and power plants' production decisions are independent from their existing contractual obligations. Simulation results suggest, according to the authors, that it is not only quota price, but also – and rather – the relative load on the system and the relative prices of gas and coal that the extent of abatement depends on. Their estimates for the volume of emission reduction induced by the switchover to a less carbon-intensive fuel were 35 million tonnes and 19 million tonnes for 2005 and 2006, respectively.

In the publications more closely related to the topic of this dissertation, authors develop real option models for the firm. They use the option techniques commonly applied in stochastic finance to model corporate decisions and their consequences. Real option models are to be used in valuation (decision) situations where the individual outcomes are uncertain, and we can assign probability values to the different cash flows. Also, they are a reasonable choice for situations with several inter-related future decisions. The results of traditional DCF (discounted cash flow) models tend to be unsatisfactory in such cases (for more details on real option, see Dixit and Pindyck (1994), and Bélyácz (2011)).

Herbelot (1994), as well, employs a real option model to model the decisions of a power generator company. One of his examples is about a coal-fired power plant that is obliged to comply with the Clean Air Act's provisions on sulfur dioxide emissions. The power station may buy SO_2 allowances or switch to low-sulfur coal or install end-of-pipe emission reduction equipment (a scrubber). The two stochastic variables, both of which follow a Wiener process, are the market price of sulfur dioxide allowances and the price difference between high-sulfur and low-sulfur coal. The author used a binomial model for pricing the two options: the switchover to low-sulfur coal and the installation of an end-of-pipe scrubber. He also examined the degree to which the different factors affect the value of the option. The second example discusses the effect of the installation of a coal gasification unit, with the two stochastic variables – both following a Wiener process, again – being the prices of gas and coal. The valuation was performed with several parameters being varied by the author.

Laurikka (2006) developed a stochastic real option simulation model to examine the influence of the EU ETS on a combined cycle (IGCC) power plant. He concludes that the DCF method is unsuited for the valuation of this type of investments because of the significant degree of risk that the EU ETS constitutes and the many real option situations it incorporates.

Abadie and Chamorro (2008) analyze a coal-fired power station, which has the opportunity to invest in a carbon capture (CCS) technology. The two stochastic variables are the price of the allowances and that of electricity. The authors use a two-dimensional binomial model to find the optimal investment decision. Based on the parameter values at the time of the study, immediate installation would not have been rational, yet the situation will change considerably if and when CCS technologies undergo significant improvements or governments decide to support such investments.

Hlouskova et al. (2005) present the real option model for a power generation firm operating in a liberalized energy market. The model was used to value the power plant and to identify the risk profile of its earnings. They concluded that the production decision of the company is independent of its entire portfolio (which also includes its long-term agreements). To model the price of electricity, they used a mean-reverting stochastic process with jumps, with seasonality being represented by a mean parameter that changes with time. The model did not incorporate any expenses for emission units, but did account for fuel costs and the various technical constraints (minimum up and down times, upper and lower capacity limits, startup and shutdown times and costs). Dynamic stochastic programming and Monte Carlo simulation were used to arrive at the optimum operating schedule.

Cragg et al. (2011) define the real option decision model of an emission market participant power plant as a function of three assets. They show that the company may significantly reduce its risks by complementing the traditional two-instrument (electricity and fuel price) hedging strategy with allowance transactions. Such hedging will reduce the variance of its profits to a significant degree.

The dissertation aims to contribute to the research avenue that relies on real options. The first and foremost goal is to derive the volume of emissions and to answer various questions related to the valuation of power plants. As an initial step, I will derive the gross margin (per unit of output), as the very foundation on which profit maximizing decisions are based. Subsequently, I will deduce the decision model of the rational power generation generation firm with respect to the utilization of its capacities vs. the suspension of its production. At the end of the chapter, I will show that the future emissions of the power plant may be interpreted as the sum of the digital options for the spread, and the value of the power plant as the sum of European spread options.

According to the model employed by Löfgren (2000) and already mentioned in the introduction in relation to the MNPB function, the maximum sum the company is willing to pay for the allowances is determined by the difference between the selling price of its product and its average variable cost excluding allowance costs. Is the allowance price lower than this value, production runs at full capacity, but if the emission unit price is higher, the firm's output is zero. The various technological constraints (for example the additional costs of starting up and shutting down the capacities) set aside, and given that the company's technology is fixed in the short run, the short run marginal cost curve can be taken as horizontal. The real option model applied in the dissertation makes use of Löfgren's approach, as well: for the interval considered, the technology of the firm to be modeled is taken as given, and the marginal cost curve of the power plant is considered horizontal up to its capacity limit for the time-of-day interval in question (and infinite otherwise). The changes in the prices of the electric energy produced and the company's inputs induce daily shifts in the horizontal MNPB function. If we consider the curve in the long run, we arrive at the downward sloping MNPB function that environmental economics models assume (I will elaborate on this in more detail at the end of the dissertation).

For the purposes of our subsequent analyses, let us assume a power generation firm, which generates electric power by burning gas and the production-related carbon dioxide emissions of which fall under the EU ETS regulation. We further assume that:

- The company's technological parameters (type of fuel used, efficiency, carbon-intensity) are fixed for the interval examined
- The company is a price-taker, that is, the prices of its product (electric energy) and inputs (gas, allowances) are exogenous factors, it has no influence whatsoever on them
- Markets are characterized by sufficient liquidity and zero transaction costs
- The price of electricity is constant within each peak and off-peak period of each day, while the price of allowances is constant within each day
- The company sells the energy produced on the spot market, and that is where the necessary inputs are procured, as well (i.e. holds no stocks)
- The impact of technological constraints (minimum up and down times, for example) is negligible, operation is flexible.

Let us see how, in view of the price of electricity, input prices and technological parameters, a rational power plant decides about the operation of its generator.

II.1. At the Heart of the Profit-Maximizing Decision: The Gross Margin Earned on Each Unit of Electric Energy Sold

The energy generation capacity of the power plant is a conditional "conversion device" that generates, depending on their production decision – i.e. conditionally –, outputs (electric energy) from the inputs (gas and emission allowances). To the short-term profit-maximizing decision, fixed costs are not relevant. Substantial variable costs were broken down into three basic categories in the model (cost of the fossil fuel, cost of covering the emissions with allowances, other variable costs).

The company's *spread (or gross margin)* can be calculated as follows:

Spread = Sales revenue from the energy generated – Cost of the required energy source – Cost of the required emission allowances – Other variable costs 1.

Let η be the *thermal efficiency* of the power plant, which shows how much electric energy the plant can generate from a unit of heat-content input. Its value is between 0% and 100% (a higher value indicates a higher efficiency). Let δ denote the *carbon intensity* of the fuel, that is, how much carbon dioxide is released when burning the given energy source (its unit being tCO2/MWh).

If prices are denoted by S (pow: electric power, gas: gas, eua: emission allowance) and *other variable costs* by v, then the spread per unit of energy generated will be:

$$spread_{pow} = S_{pow} - S_{gas}/\eta - S_{eua} \cdot \delta/\eta - v \qquad 2.$$

By rearranging the expression, we arrive at the spread realized on each unit of emission:

$$spread_{eua} = spread_{pow} \cdot \eta/\delta = \left(S_{pow} - v\right) \cdot \eta/\delta - S_{gas}/\delta - S_{eua} \qquad 3.$$

Several variants of the *spread* concept are being used in the energy market (for details, see Alberola – Chevallier – Cheze, 2008). These differ in, on the one hand, whether they pertain to coal or gas-fired facilities (the former is a *dark,* the latter is a *spark spread)* and, on the other hand, whether they account for the costs of procuring the quota amount required to cover the plant's emissions (if yes, *clean* is used as a prefix). Hence the four spread concepts used in the field: *dark spread, spark spread, clean dark spread, clean spark spread*. The spread concept I use throughout the dissertation basically corresponds to *clean spark spread*, with the difference that, first, it pertains to one individual power plant and, second, it includes an *other variable cost* term. The formula presented above differs from the approach of Hsu (1998) in two vital aspects: the profit function incorporates the cost of the emission allowances and, also, it includes an *other variable cost* term.

The profit-maximizing power plant generates power only if the *spread* to be realized is positive – otherwise, they are better off suspending production. This statement only holds true if the model does not take into account the additional technological constraints of power plants. Which include, for example, *minimum operating point*, which shows the minimum capacity at which it must be run as a percentage of its nominal capacity, the plant's *heat rate curve*, which gives the efficiency of operation at partial load levels, *minimum up and down times*, *startup and shutdown times*, *startup and shutdown costs*. Calculating the optimal production decision with all the technological constraints taken into account would necessitate a far more complex simulation technique. The "directions" of the conclusions of the dissertation will remain valid even if technological constraints are introduced to the picture – yet the actual figures will be different. The more flexible – that is, the easier to start up and shut down – the power plant, the closer the no-constraint model will be to reality. Hereinafter, the impact of technological constraints will not be considered.

The per-unit profit function π_{pow} of the profit-maximizing firm can be derived from the spread as follows:

$$\pi_{pow} = max\left(spread_{pow}, 0\right) = max\left(S_{pow} - S_{gas}/\eta - S_{eua} \cdot \delta/\eta - v, 0\right) \qquad 4.$$

The profit function can also be given in a per-unit-of-emission form (π_{eua}), which yields the following equation:

$$\pi_{eua} = max(spread_{eua}, 0) = \pi_{pow} \cdot \eta/\delta =$$

$$= max\left(\left(S_{pow} - v\right) \cdot \eta/\delta - S_{gas}/\delta - S_{eua}, 0\right) \qquad 5.$$

The conditional value of the profit function π resembles the payoff functions of options. Which fact allows us to rely on the tools of stochastic finance used for the pricing of conditional claims in modeling the related decision situations and in pricing the real assets in question. In addition to presenting relevant real option analogies, I will also introduce a fourth asset into our model in the next section (the day's price of electricity will be split into two prices).

II.2. The Four-Asset Real Option Model for the Future Spread

In reality, electric energy cannot be stored or, more precisely, can only be stored at a very high cost. Production is, in practice, adjusted to current and expected demand, and spot prices are quoted on an hourly basis. Fluctuations in the price of electricity demonstrate strong seasonality (Marossy, 2011). Within the day, electricity price changes according to human activities: high consumption in the „daytime" peak period results in a high price, while the low demand of evening hours induces a significant price dip. Typically, the power plants that operate during the low-demand (off-peak) period are the ones that generate electricity at a relatively lower per-unit cost. A weakness of theirs is that they tend to be less flexible (start-up and shutdown costs a significant amount of time and money). During the high-demand (peak) period, additional power plants join in: the ones that generate electric energy at a higher cost, but in a timing that matches demand. The gas-turbine facility to be modeled belongs to this latter type.

Regular fluctuations may be observed within the week as well (Ulreich, 2008, pp. 817-820.), the explanation for which is that a significant share of industrial production and service activities is concentrated to weekdays. Also detectable are some intra-annual cycles, primarily due to climatic factors: during periods of abundant precipitation, hydropower stations produce more energy, which acts to moderate the price of electricity. Particularly warm or cold seasons, nevertheless, increase consumption and, hence, electricity prices: during a hot summer, it is air conditioning, while in a cold winter, it is the increased heating need that leads to high levels of electricity consumption. Weekly and yearly cycles were not taken into consideration in the model.

Another typical electricity market phenomenon is the strikingly frequent occurrence of extreme values: a sudden hike in demand or a capacity shortage due to some technical failure cause significant upward spikes, while a sudden decline in demand induces a downward spike in the price level of electricity. This frequency of extreme values was incorporated into the model through the high volatility of electricity prices, yet I did keep the assumption about the log-normality of future prices (i.e. did not apply a heavy-tailed distribution).

In order to more accurately approximate the real-life situation, I decided to enhance the three-asset model to include a fourth asset, namely by splitting the day into two equal parts: the off-peak period comprises all the low-demand (and hence low-price) hours (8:00 p.m. - 8:00 a.m.), while the peak period covers higher-demand (higher-price) hours (8:00 a.m. - 8:00 p.m.). For the two time-of-day periods, there are two different electricity prices: an off-peak and a peak price.

The advantage of the four-asset model is that it facilitates a better approximation of the real decision situation of our gas-turbine power plant, where it is, depending on the future market situation and the current prices, typically idle during off-peak hours, but runs in the peak period. Hereinafter, I will use the expression *spread* to exclusively denote the gross margin realized on each unit of energy generated and hence refrain from using the index „pow"; the gross margin per unit of emission can be arrived at by a simple transformation.

The *spread* associated with any future day τ can be expressed as a function of the relevant future spot prices (denoted by $S(\tau)$). In the four-asset model, the peak and off-peak spreads can be given as:

$$spread_{peak}(\tau) = S_{peak}(\tau) - S_{gas}(\tau)/\eta - S_{eua}(\tau) \cdot \delta/\eta - v$$

$$spread_{off-peak}(\tau) = S_{off-peak}(\tau) - S_{gas}(\tau)/\eta - S_{eua}(\tau) \cdot \delta/\eta - v \qquad 6.$$

While the per-unit (of energy produced) profits π to be realized by the profit-maximizing firm on a future τ day are, respectively:

$$\pi_{peak}(\tau) = max\left[spread_{peak}(\tau), 0\right]$$

$$\pi_{off-peak}(\tau) = max\left[spread_{off-peak}(\tau), 0\right] \qquad 7.$$

The total volume of profit Π to be realized on a given day τ can be reckoned by multiplying the per-unit profit π pertaining to the given period by the daily maximum capacity Γ for the same period. As both periods constitute the exact half of a day, it follows that a day's total profit equals the product of the arithmetic mean of the per-unit profits and daily capacity:

$$\Pi(\tau) = 0.5 \cdot \Gamma \cdot \pi_{off-peak}(\tau) + 0.5 \cdot \Gamma \cdot \pi_{peak}(\tau) = \Gamma \cdot \frac{\pi_{off-peak}(\tau)+\pi_{peak}(\tau)}{2} \qquad 8.$$

II.3. The Real Option Model of Carbon Dioxide Emissions

A power plant operated in a profit-maximizing manner only produces energy if its revenue exceeds its variable costs, that is, if the value of the spread between the future spot prices is positive. Subsequently, I will assume that the company's carbon dioxide emissions are the direct, technologically determined consequence of its production, i.e. the company does not have an end-of-pipe cleaning technology in place that can be freely switched on or off. If and when production is running, the company releases carbon dioxide into the atmosphere, but whenever it stands still, it has a zero emission level.

In view of the above, it is possible to derive the emission volume of the company from the profit-maximization condition. Let there be a Bernoulli distributed, binary (0/1) production decision variable (Λ) such that:

$$\Lambda(\tau) := \begin{Bmatrix} 1 & if\ spread(\tau) > 0 \\ 0 & otherwise \end{Bmatrix} \qquad 9.$$

If the spread is positive, the firm can realize a profit on its production and thus the turbine is running ($\Lambda = 1$) and releasing carbon dioxide into the atmosphere; if, however, the spread is negative, the firm would realize a loss if it was producing electricity, therefore its generation capacity sits idle ($\Lambda = 0$) and emission is zero.

The probability that the turbine will operate at a given time τ in the future equals the probability that the then-current gross margin is positive:

$$\mathrm{E}[\Lambda(t)] = P(spread(\tau) > 0) \cdot 1 + P(spread(\tau) \leq 0) \cdot 0 = P(spread(\tau) > 0) \qquad 10.$$

The value of the production decision variable Λ corresponds to the payoff function bno^{PO} of a European-style binary option for the spread, with maturity τ and exercise price v.

$$\Lambda(\tau) = bno^{PO}(\boldsymbol{S}(0), \boldsymbol{w}, v, \tau) = \begin{Bmatrix} 1 & if\ \ \boldsymbol{w}' \cdot \boldsymbol{S}(\tau) > v \\ 0 & otherwise \end{Bmatrix} \qquad 11.$$

Where $\boldsymbol{S}$ is the column vector of the prices and $\boldsymbol{w}$ the column vector of the weights for the individual assets, with a positive weight for the first underlying and negative weights for the rest, namely:

$$\boldsymbol{w}' = [1 \quad -1/\eta \quad -\delta/\eta]. \qquad 12.$$

The payoff function of the option is an expression of future realization. As far as expected emissions are concerned, we are interested in the expected value of this expression.

Given that the price of the spread option itself is the present value of the payoff function's expected value, the expected value of future emissions can also be derived from the prices of the spread options that use the appropriate peak and off-peak electricity prices.

The volumes of carbon dioxide emissions are not exchange-traded, thus the conditions of arbitrage-free pricing do not hold. Consequently, it is not the risk-neutral, but the physical measure that we have to use in the option pricing formula used to estimate emissions[5]. In practice, this means that the drift parameter used to calculate the future distributions will not be derived from the risk-neutral rate of return, but from the parameters of the model fitted to observed price movements. Whereas in calculating the present value, we will have to use the sum r^* of the risk-free rate plus the risk premium.

The option's value equals the present value of the expected value, therefore the expected value can be given as the future value of the option's price. The expected value of the production decision variable Λ can be expressed in terms of the price bno^{Pr} of the binary option.

$$\mathrm{E}[\Lambda(t)] = \mathrm{E}[bno^{PO}(\boldsymbol{S}(0), \boldsymbol{w}, v, \tau)] = e^{r^*\tau} \cdot bno^{Pr}(\boldsymbol{S}(0), \boldsymbol{w}, v, \tau) \qquad 13.$$

The emission volume Q on a future day τ can be given as a function of the production decision variable Λ, the daily maximum capacity Γ, the carbon-intensity δ of the fuel and the thermal efficiency η as follows:

[5] For details on the considerations related to physical and risk-neutral measures, see (Medvegyev, 2009)

$$\mathrm{Q}(\tau) = \Lambda(\tau) \cdot \Gamma \cdot \delta / \eta \qquad 14.$$

We decided to split the day into two – a period of peak hours and a period of off-peak hours – in the four-asset model. In light of the above, the expected value of emissions on a future day τ will be:

$$\mathrm{E}[\mathrm{Q}(\tau)] =$$

$$\frac{\Gamma}{2} \cdot \delta/\eta \cdot e^{r^*\tau} \cdot \left[bno^{Pr}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right) + bno^{pr}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right)\right] \qquad 15.$$

While the expected value of the cumulative emissions for a longer period $Q_c(0, T)$ that lasts from now to date T is given by:

$$E[Q_c(0, T)] =$$

$$\frac{\Gamma}{2} \cdot \delta/\eta \cdot \sum_{\tau=0}^{T} e^{r^*\tau} \cdot \left[bno^{Pr}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right) + bno^{Pr}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right)\right] \quad 16.$$

Thus the expected emissions of the power plant for a given period may be calculated based on the expected payoffs or prices of the binary options for the spread and the daily maximum amount of emissions. In the four-asset model, the expected volume of emissions can be derived from the future value of the arithmetic mean of the prices of the two (peak and off-peak) three-asset binary spread options by cumulating these values over the future period of interest and multiplying the result by the daily maximum output.

Chapter IV will elaborate in more detail on questions related to the emission derived from the real option model. The emission's probability density function and the maximum cost of compliance at a given confidence level will also be determined, among others. A detailed analysis of the results' sensitivity to technological and market factors will also be provided.

II.4. Real Option Valuation of the Power Plant

The value of future profits depends on future prices. We, nonetheless, would like to evaluate the power plant in the present, which requires us to reckon the expected present value of future profits. To this end, we will use European-style spread options.

The payoff function spo^{PO} of a European-style three-asset spread option with exercise price v and maturity τ can be written as:

$$spo^{PO}(\boldsymbol{S}(0), \boldsymbol{w}, v, \tau) = \begin{Bmatrix} \boldsymbol{w}' \cdot \boldsymbol{S}(\tau) - v & ha\ \boldsymbol{w}' \cdot \boldsymbol{S}(\tau) > v \\ 0 & otherwise \end{Bmatrix} \quad 17.$$

The payoff function of the option can be used to express the profit realized on each unit of electric energy generated during peak and off-peak hours:

$$\pi_{off-peak}(\tau) = spo^{PO}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right)$$

$$\pi_{peak}(\tau) = spo^{PO}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right)$$

$$\boldsymbol{S_{off-peak}}(0) = \begin{bmatrix} S_{pow}^{off-peak}(0) \\ S_{gas}(0) \\ S_{eua}(0) \end{bmatrix} \qquad \boldsymbol{S_{peak}}(0) = \begin{bmatrix} S_{pow}^{peak}(0) \\ S_{gas}(0) \\ S_{eua}(0) \end{bmatrix} \quad 18.$$

Thus the sum total of the profit realized on a future day τ is given by:

$$\Pi(\tau) = \frac{\Gamma}{2} \cdot \left(spo^{PO}(\boldsymbol{S_{base}}(0), \boldsymbol{w}, v, \tau) + spo^{PO}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right)\right) \quad 19.$$

The option's payoff function is an expression of future realization. With the valuation of the facility in mind, we are interested in the expected present value of this expression.

Given that the price of the spread option itself is the present value of the payoff function's expected value, the expected value of future profits can also be derived from the prices of the spread options that pertain to the appropriate peak and off-peak electricity prices. If spo^{Pr} stands for the price of the option, the present value of the expected profits for a future day τ can be written as:

$$PV\left[E^Q[\Pi(\tau)]\right] =$$

$$= PV\left[E^Q\left[\frac{\Gamma}{2} \cdot \left(spo^{PO}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right) + spo^{PO}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right)\right)\right]\right] =$$

$$= \frac{\Gamma}{2} \cdot \left(spo^{Pr}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right) + spo^{Pr}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right)\right) \quad 20.$$

Concerning the valuation of the power plant's profits, the conditions for an arbitrage-free pricing would only be met if (among others) the facility as an asset would be perfectly divisible for the shares to be bought and sold at low transaction costs. Given that this is not practically feasible (even though a number of energy generation companies are indeed listed on some stock exchange, but they typically own more than one power plant and their balance sheet tends to include several additional assets, as well), the power plant is not sufficiently divisible for its shares to be traded at low transaction costs. Accordingly, the valuation, too, will be performed using the physical measure, instead of the risk-free one.

Relying on our option analogy, we can express the financial value of the power plant as an asset: it equals the sum total of the present values of all the expected gross margins to be realized over its entire lifetime *T*:

$$V = \frac{\Gamma}{2} \cdot \sum_{\tau=0}^{T} \left[spo^{Pr}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right) + spo^{Pr}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right) \right] \qquad 21.$$

As evinced by the above, the real option model can be used to calculate the revenue generation capacity of the power plant. In the four-asset model, the price of the power plant corresponds to the aggregate amount of the arithmetic means of the appropriate three-asset spread options. The calculation is based on the assumptions that: the cost of all repairs throughout the power plant's lifetime is included in the other variable costs term; tax effects are ignored; replacement investments are also included in other variable costs; end-of-life (residual) value is zero.

Chapter V will further analyze the questions related to the real option based valuation of the power plant. I will examine how sensitive the power plant's price is to changes in certain technological and market factors, and how the plant might reduce its risk arising from price fluctuations in the underlying commodities. I will provide an overview on a method for evaluating an efficiency improvement project, and on the factors that the result depends on. Afterwards, we will determine the extent of the loss our power plant would make if it was not operating in the spot-price-dependent, profit-maximizing fashion we have assumed so far, but enters long-term production agreements and runs continuously instead.

In the next chapter, I will discuss the pricing model applied to the underlying assets and the procedures used to evaluate the spread options.

III. THE PRICING MODEL AND THE PRICING PROCEDURES APPLIED

In order to make use of the real option model, we need a stochastic model to describe the price movements of the underlying assets. Subsequently, I will first introduce the geometric Ornstein-Uhlenbeck (GOU, Log Ornstein-Uhlenbeck, logOU or Exponential Vasicek) model we are going to use. I will present the mathematical relationships associated with the simulation runs (that is, the discrete realizations of the process), and the distributions of the prices for a future point in time. Afterwards, I will derive a relationship that will allow for the analytical pricing formulae pertaining to geometric Brownian motion to be applied to a weighted case of underlying assets that follow a GOU process. The stochastic model was fitted to market data from the German energy exchange. To round off the chapter, I will introduce the procedures used to evaluate the spread options.

The price movement of stocks is often presumed to follow a *geometric Brownian motion* (GBM). In this model, consecutive continuous returns are independent, future prices are log-normally distributed. As regards geometric Brownian motion, the standard deviation of the distribution of logarithmic returns increases proportional to the square root of time; in the model, as we progress in time, the distribution gets "wider": the range that the higher-probability price values cover is becoming larger and larger. The price movements of commodities are, however, most often modeled with mean-reverting stochastic processes. In these models, prices are "prone" to revert to a long-term mean after a series of swings. The distribution of the price stabilizes in the long run, its standard deviation becomes constant. According to the economic consideration behind the application of mean-reverting models, raw material prices are determined by the costs of extraction (supply side) and the gross margin that can be realized on the final product (demand side). On both sides, the factors are more or less stable in the long run, which is why the price, though swinging in the short run due to temporary fluctuations in supply and demand, reverts to a certain constant value with time.

The arithmetic mean-reverting process was defined by Ornstein and Uhlenbeck (1930), who employed their model to describe the velocity of the Brownian particle under the influence of friction:

$$dx_t = \lambda(\Theta - x_t)dt + \sigma dW_t$$

Where $\Theta > 0, \sigma > 0$ and W_t is a Wiener process 22.

The discrete realizations of the process can be simulated according to the following relationship (van der Berg, 2011):

$$x_{t+\delta} = x_t e^{-\lambda\delta} + \Theta\left(1 - e^{-\lambda\delta}\right) + \sigma\sqrt{\frac{1-e^{-2\lambda\delta}}{2\lambda}} N_{0,1} \quad 23.$$

At time *T*, the variable is normally distributed with the following parameters:

$$x_T \sim N\left[\Theta + (x_0 - \Theta)\exp(-\lambda T), \quad \frac{\sigma^2}{2\lambda}\left(1 - \exp(-2\lambda T)\right)\right] \quad 24.$$

Among the first ones to introduce the Ornstein-Uhlenbeck model to the field of stochastic finance was Vasicek, who used it to describe spot interest rate changes over time (1977). One of the specialties of the arithmetic base model is that the random variable to be modeled can take negative values, as well. This characteristic is not much of an advantage with respect to commodity markets, where negative prices are hardly possible[6].

The geometric Ornstein-Uhlenbeck process

In order to avoid prices below zero, it seems advisable to use the one-factor model of Schwartz (Schwartz, 1997), where future movements in commodity prices follow the process below:

$$dS = \lambda(\theta - lnS)Sdt + \sigma SdW \quad 25.$$

The model can be traced back to an arithmetic Ornstein-Uhlenbeck process. The model pertaining to the logarithm of the price can be derived by applying Itō's Lemma:

$$d\ln S = \lambda(\mu - \ln S)dt + \sigma dW$$

$$\mu = \theta - \frac{\sigma^2}{2\lambda} \quad 26.$$

---

[6] Electricity, however, is one of the very exceptions: because of the very limited and costly availability of storage and the absolute necessity of balancing supply and demand, it may eventually have a negative price.

In this model, the price distribution at maturity can be obtained from the distribution of the arithmetic model by adjusting the mean term:

$$lnS_T \sim N\left[\theta - \frac{\sigma^2}{2\lambda} + \left(lnS_0 - \theta + \frac{\sigma^2}{2\lambda}\right)\exp(-\lambda \mathrm{T}), \quad \frac{\sigma^2}{2\lambda}\left(1 - \exp(-2\lambda \mathrm{T})\right)\right] \qquad 27.$$

The process is interchangeably called one-factor Schwartz model, logarithmic Ornstein-Uhlenbeck, geometric Ornstein-Uhlenbeck (GOU) process, or exponential Vasicek model by others (Brigo et al., 2007).

In the real option model, the prices of the inputs are multiplied by technologically predetermined weight factors. A multiplication by a non-negative weight *w* corresponds to a parallel shift by a value *ln(w)* in the logarithmic model. In the arithmetic model, both the initial and the long-term mean parameters are shifted by this same value, the mean-reversion rate remains unchanged. By applying the following substitutions, we may express the price distribution at maturity for the weighted case:

$$\ln(S_0) \longrightarrow \ln(S_0) + \ln(w), \theta \longrightarrow \theta + ln(w)$$

The distribution of the weighted value at maturity date *T* will be:

$$ln(w \cdot S_T) \sim$$

$$N\left[\theta + ln(w) - \frac{\sigma^2}{2\lambda} + \left(lnS_0 - \theta + \frac{\sigma^2}{2\lambda}\right)\exp(-\lambda \mathrm{T}), \quad \frac{\sigma^2}{2\lambda}\left(1 - \exp(-2\lambda \mathrm{T})\right)\right] \qquad 28.$$

The real option model presented earlier incorporates several assets, therefore we will need the joint probability density function of the three-dimensional logarithmic Ornstein-Uhlenbeck process, which will be a multi-dimensional normal distribution with the following parameters:

$$N\left[\boldsymbol{\theta} + ln(\boldsymbol{w}) - \frac{\sigma^2}{2\lambda} + \left(\boldsymbol{lnS_0} - \boldsymbol{\theta} + \frac{\sigma^2}{2\lambda}\right)\exp(-\boldsymbol{\lambda} \cdot \mathrm{T}), \quad \boldsymbol{\Sigma}\right] \qquad 29.$$

Where vector $\boldsymbol{w}$ contains the weights, $\boldsymbol{\theta}$ the long-term means, $\boldsymbol{\lambda}$ the mean-reversion rates, $\boldsymbol{lnS_0}$ the logarithms of the initial prices and $\boldsymbol{\sigma}^2$ the variances, and $\boldsymbol{\Sigma}$ is the covariance matrix. Utilizing the formula of effective correlation as provided by Deng et al. (2008) and the variance of the distribution, the elements of the covariance matrix are given by:

$$\Sigma_{i,j} = \varrho_{i,j} \cdot \frac{\sigma_i \cdot \sigma_j}{\lambda_i + \lambda_j} \cdot \left[1 - \exp\left(-(\lambda_i + \lambda_j)\mathrm{T}\right)\right] \qquad 30.$$

where $\varrho_{i,j}$ is the correlation between the Wiener terms.

The Correspondence Between Geometric Brownian Motion and Geometric Ornstein-Uhlenbeck Motion

One of the models most widely used in stochastic finance is geometric Brownian Motion (GBM), where the price movements of the underlying asset follow the process:

$$dS = \mu S dt + \sigma S dW \tag{31.}$$

The distribution of the price's logarithm is normal, and the joint probability density function of the multi-dimensional variant is characterized by the following parameters ($\boldsymbol{S_T}$ is the vector of the prices at maturity, $\boldsymbol{\mu}$ is that of the expected returns, and the elements of covariance matrix $\boldsymbol{\Sigma}$ can be derived from the products of the correlation coefficients between the Wiener terms and the standard deviations thereof):

$$ln(\boldsymbol{S_T}) \sim N\left[\left(\boldsymbol{\mu} - \frac{\boldsymbol{\sigma}^2}{2}\right) \cdot \mathrm{T},\ \boldsymbol{\Sigma} \cdot \mathrm{T}\right] \tag{32.}$$

Regarding the pricing of European-style derivatives, the price of the derivative depends on the price of the underlying asset at maturity. The particular path followed by the price until maturity does not have an influence on the yield. A notable consequence of this attribute is that if we have an analytic pricing formula pertaining to geometric Brownian motion, the very same formula can, by way of parameter substitution, also be used for the valuation of European-style derivatives with underlying assets that follow a GOU process. It is important to note that the valuation is performed under the physical measure, therefore we do not need to address the issue of finding a risk-neutral measure for the geometric Ornstein-Uhlenbeck process. The correspondence between the two types of process can be created the following way: based on the parameters of the GOU model, we determine the multi-dimensional distribution at maturity, and find, by calculation, the parameters of the GBM process that yields the exact same distribution. Then, the resulting GBM parameters are substituted into the potentially available analytic formula.

Considering the multi-dimensional weighted case, the drift and variance vectors of the GBM process that corresponds to the GOU process are:

$$\sigma'^2 = \frac{\sigma^2}{2\lambda}\left(1 - \exp(-2\lambda\mathrm{T})\right)/\mathrm{T}$$

$$\mu' = \frac{\theta + ln(w) - \frac{\sigma^2}{2\lambda} + \left(lnS_0 - \theta + \frac{\sigma^2}{2\lambda}\right)\exp(-\lambda \cdot \mathrm{T})}{T} + \frac{\sigma'^2}{2} \qquad 33.$$

According to Deng et al. (2008), the formula of the correlation needed to calculate the covariance matrix can be obtained from:

$$\rho_{i,j} = 2\varrho_{i,j} \cdot \frac{\sqrt{\lambda_i \lambda_j}}{\lambda_i + \lambda_j} \cdot \frac{1 - \exp\left(-(\lambda_i + \lambda_j)\mathrm{T}\right)}{\sqrt{1 - \exp\left(-2\lambda_i \mathrm{T}\right)}\sqrt{1 - \exp\left(-2\lambda_j \mathrm{T}\right)}} \qquad 34.$$

where $\varrho_{i,j}$ is the correlation coefficient between the Wiener terms.

III.1. Fitting the Stochastic Model to Market Data

The starting point for the estimation of the model's parameters was the simulation equation of the geometric Ornstein-Uhlenbeck process (van der Berg, 2011):

$$\ln(S_{t+\delta}) = \ln(S_t)e^{-\lambda\delta} + (\Theta - \frac{\sigma^2}{2\lambda})\left(1 - e^{-\lambda\delta}\right) + \sigma\sqrt{\frac{1 - e^{-2\lambda\delta}}{2\lambda}} N_{0,1} \qquad 35.$$

The model fits to a linear equation:

$$\ln(S_{t+\delta}) = a \cdot \ln(S_t) + b + \epsilon \qquad 36.$$

The coefficients estimated by the least squares method are used to express the parameters of the stochastic model:

$$\lambda = \frac{\ln(a)}{\delta} \qquad \sigma = std(\epsilon)\sqrt{\frac{-2\ln(a)}{\delta \cdot (1 - a^2)}} \qquad \Theta = \frac{b}{1 - a} + \frac{\sigma^2}{2\lambda} \qquad 37.$$

The correlation matrix is calculated from the resulting residual terms ϵ.

The prices for the individual instruments were those of the EEX[7] energy exchange. The period examined was from 28 February 2008 to 31 May 2012. Only those days were

[7] www.eex.com

considered for which there was a price for all four assets, thus I had price data for 1010 observation days in the end. For emission units prices were only available for working days, weekends were left out of consideration altogether, and the time distance between two consecutive observation days was assumed to be $\delta = 1/252$ years. The very nature of the logarithmic model necessitated some further minor adjustments: days with a negative price (for off-peak power – there were four such occurences altogether) were excluded from the data set, and thus I had a final total of 1006 observations.

Having been acquainted with the parameter estimation procedure, we now proceed to the regression results with respect to the individual assets.

The Stochastic Model of the Emission Units

As regards the emission units, the model was fitted to prices from the second trading phase (2008-2012), more specifically from 26 February 2008 to 31 May 2012. The following figure illustrates, based on data from the EEX, the movements in the quota price during the said period:

Figure 5: Historical daily prices of EUA in the Second Phase.

As depicted by the curve, the initial price of nearly 20 EUR/tonne increased throughout the first half of 2008 and then, after the 2008 financial crisis unfolded, it began to fall sharply. The primary cause of the price drop was the demand for electricity being proportional to GDP: the expected production – and, hence, the expected carbon dioxide emissions – of energy-intensive industries dropped due to business expectations having been darkened by the crisis. Consequently, the demand for and the price of the allowances fell, too. Afterwards, the quota price hovered around 15 EUR/tonne for more than two years to once again take a steady downward path in mid-2011, gradually approaching the 6.26 euro value that was used as the initial price in the simulation.

Plotting the logarithms of the prices from consecutive trading days and performing the linear regression yields the following figure:

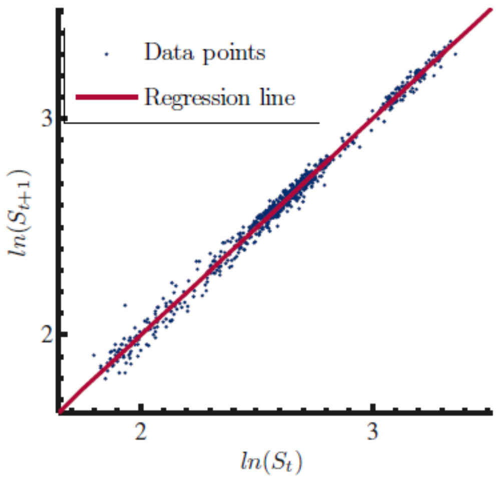

Figure 6: Linear regression on the logarithm of consecutive EUA prices.

It is clearly apparent that the regression line fits the data points quite well. The linearized model takes the following form:

$$\ln(S_{t+1/252}) = 0.9989 \ln(S\_t) + 0.0018 \qquad 38.$$

The gradient is very close to 1, which infers that autoregression only has a weak role in price developments (the theoretical gradient of GBM models is 1). The coefficient of determination is rather high ($R^2 = 0.9929$), which confirms the good fit of the model.

The parameters and the shape of the fitted model are:

$\sigma = 0.4375 \quad \theta = 1.9222 \quad \lambda = 0.2804$

$$\mathrm{dS} = 0.2804\,(1.9222 - \ln S)\,S\,dt + 0.4375\,S\,dz \qquad 39.$$

Annual standard deviation is high (43.75%), the logarithm of the price reverts to 1.92, the exponential equivalent ($\exp(\theta)$) of which is 6.84 EUR/ton. The initial value for the simulation is the price of the last day used in the fit: 6.26 EUR/ton.

The Stochastic Model of the Gas Price

Gas prices were acquired from the EEX exchange, as well; they pertain to the *day ahead* contracts. Even though prices are for delivery the following day, I will treat them as spot prices hereinafter. Movements in the price of gas during the period examined, based on data from the EEX, are shown on the following graph:

Figure 7: Historical daily prices of gas.

Plotting the logarithms of the prices from consecutive trading days and performing the linear regression yields the following figure:

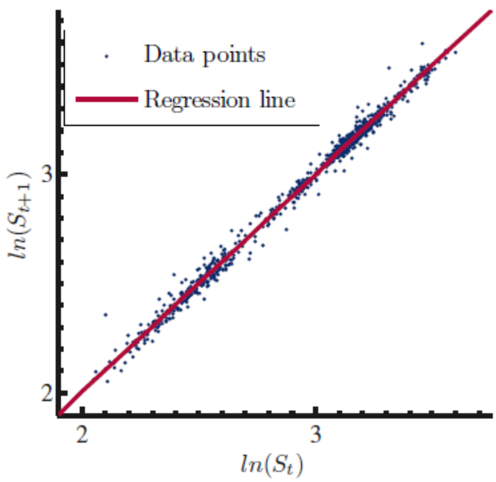

Figure 8: Linear regression on the logarithm of consecutive gas prices.

It is clearly apparent that the regression line fits the data points well. The linearized model takes the following form:

$$\ln\left(S_{t+1/252}\right) = 0.9967\,\ln(S\_t) + 0.0097 \qquad 40.$$

The gradient in the linear formula is close to 1, thus autoregression has a weak role in price developments. The coefficient of determination is remarkably high ($R^2 = 0.9934$) for gas, as well, which confirms the good fit of the model.

The parameters of the fitted model and the shape of the resulting GOU model are:

$$\sigma = 0.4545 \quad \theta = 3.0811 \quad \lambda = 0.8251$$

$$\mathrm{dS} = 0.8251\,(3.0811 - \ln S)\,S\,dt + 0.4545\,S\,dz \qquad 41.$$

Annual standard deviation is 45.45%, the logarithm of the price reverts to 3.0811, the exponential equivalent of which is 21.78 EUR/ton. The initial value for the simulation is 23.47 EUR/MWh.

The Stochastic Model of Peak and Off-Peak Power Price

I relied on the hourly closing prices (EUR/MWh) of the EPEXSpot. Historical peak and off-peak prices were determined as the arithmetic means of the hourly closing prices in the interval between 8:00 a.m. - 8:00 p.m. and 8:00 p.m. - 8:00 a.m., respectively. The following figure shows the price movements for electricity during the period in question:

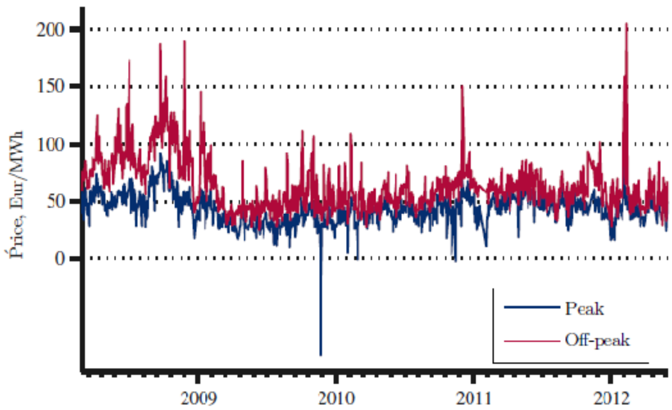

Figure 9: Historical daily prices of peak and off-peak electricity.

As it is obvious from the graph, the price of electricity is highly volatile, the time series is full of sudden spikes. Extreme lows are quite frequent, as well, there are even some negative values for off-peak power (which necessitated a minor adjustment to be made when fitting the model built upon the logarithm of the price). Another observation is that daily peak and off-peak prices tend to move in unison, the cause of which might be that demand and supply exercise a similar influence on these two types of daily price. Plotting the logarithms of the prices from consecutive trading days and performing the linear regression yields the following figures:

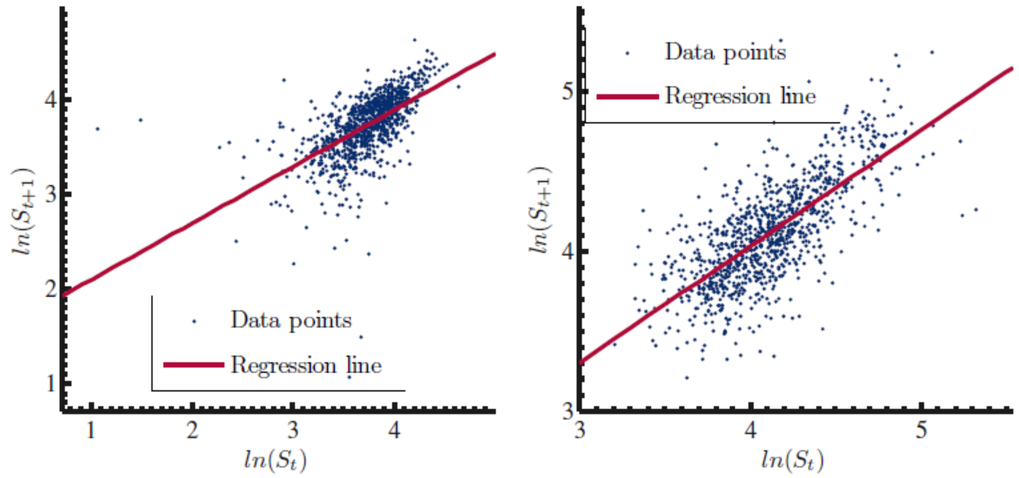

Figure 10: Linear regressions on the logarithm of consecutive off-peak (left) and peak (right) electricity prices.

The regression lines have the following shapes:

off-peak period: $\ln\left(S_{t+1/252}\right) = 0.5979\ \ln(\mathrm{S_t}) + 1.5005$

peak period: $\ln\left(S_{t+1/252}\right) = 0.7282\ \ln(\mathrm{S_t}) + 1.1184$ 42.

The coefficient of determination for the off-peak period is 0.3574, which infers a worse fit compared to that of gas and quota prices. The fit is, nonetheless, better for the peak period, with a coefficient of determination of 0.5303. To allow for the application of the analytic results, the assumption of the GOU model was kept for all underlying assets.

The shapes of the stochastic differential equations for electricity prices are:

Off-peak period $\sigma = 5.3291$ $\theta = 3.8409$ $\lambda = 129.6231$

$\mathrm{dS} = 129.6231(3.8409 - \ln\mathrm{S})\mathrm{S}\ \mathrm{dt} + 5.3291\ \mathrm{S}\ \mathrm{dz}$

Peak period $\sigma = 4.1001$ $\theta = 4.2203$ $\lambda = 79.9205$

$\mathrm{dS} = 79.9205(4.2203 - \ln\mathrm{S})\mathrm{S}\ \mathrm{dt} + 4.1001\ \mathrm{S}\ \mathrm{dz}$ 43.

In the model, the initial price used for off-peak power was 38.82 EUR/MWh, with a long-term mean of exp(3.8409)=46.57 EUR/MWh. As regards peak power, the initial price was 67.67 EUR/MWh, and the long-term mean equalled exp(4.2203)=68.05 EUR/MWh.

Correlation

The relationships between the price changes of the four assets can be determined by calculating the correlation matrix of the remainder terms from the model fitting. I performed a hypothesis test for the results, where the null hypothesis was that the correlation between the terms is zero, while the alternative hypothesis was that the correlation coefficient is not equal to zero. The resulting matrices were:

Correlation	**Off-peak**	**Peak**	**Gas**	**EUA**
Off-peak	1.0000	0.4830	0.0190	-0.0192
Peak	0.4830	1.0000	0.0275	-0.0051
Gas	0.0190	0.0275	1.0000	0.1655
EUA	-0.0192	-0.0051	0.1655	1.0000

p-value	**Off-peak**	**Peak**	**Gas**	**EUA**
Off-peak	1.0000	0.0000	0.5481	0.5439
Peak	0.0000	1.0000	0.3845	0.8717
Gas	0.5481	0.3845	1.0000	0.0000
EUA	0.5439	0.8717	0.0000	1.0000

Table 4: Correlation coefficients and the corresponding p-values.

There is a strong correlation between peak and off-peak electricity price (48.3%, p value is 0, the relationship is significant). The strong positive relation suggests that a part of the demand and supply factors that determine the price pertain to a given day, and thus influence the price of both periods. The absence of a perfect correlation, however, indicates that intra-day (possibly even hourly) market factors have a great role, as well. The relationship between gas and emission units is also positive and significant, though weaker (16.55%). Whenever the price of gas increases, the production share of power plants using fuels of higher carbon-intensities goes up, along with the emissions, and thus the quota price increases, as well. The significance of all the other relations was low, therefore the results and their interpretations are less reliable. The correlation coefficient of gas and electricity price is higher for the peak period (2.75%), than for off-peak hours (1.9%). Which might imply that a large number of gas power plants concentrate their production into the peak period. The growth in the demand for electric power is met by the more flexible, yet also more expensive gas-fired type of power plants, inducing a hike in the demand for and the price of gas. Emission units are weakly negatively correlated with electricity price. The explanation, once again, might lies in the production capacity mix. Short term supply is fixed, the growing electricity price acts to increase the share of gas power plants, which generate electricity in a cleaner way (with less carbon dioxide emissions); accordingly, this shift towards gas acts to reduce the demand for and the price of carbon credits. This negative effect is stronger (-1.92%) in the off-peak period (when the share of coal-fired power stations is relatively higher), and weaker during off-peak hours (-0.51%).

Simulation Parameters Used

For the purposes of the simulation, the year was broken down into 252 trading days, excluding weekend days from the model. The reason was that weekend days were not taken into account in the fitting of the model to market data, either, as for these dates, there were no historical prices available for all four assets.

The technological parameters of the power plant to be modeled cannot be observed through market prices, because every single facility is different. The model assumes an open cycle gas-turbine power plant, the thermal efficiency (the quotient of the energy output and energy input) of which is 38% (Comission of the European Communities, 2008). The energy content of the fuel used – natural gas – is 0.2014 $tCO2$ / MWh, other variable costs amount to 3 EUR/MWh and the power plant has an installed capacity of 100MW, hence a maximum daily capacity – i.e. maximum amount of electric energy generated per day – of 2400 MWh.

The initial values for the spread calculated from the initial prices as per the technological parameters were:

$$spread_{off\text{-}peak} = 38.8167 - 23.4700/0.38 - 6.2600 * 0.2014 / 0.38 - 3 = - 29.2643$$

$$spread_{peak} = 67.6667 - 23.4700/0.38 - 6.2600 * 0.2014 / 0.38 - 3 = - 0.4143 \qquad 44.$$

As it can be seen from the initial prices, should the power plant for some reason decide to produce energy during the off-peak period, it would make a loss of about EUR 29 on each MWh. The spread is negative for the peak period, as well, production yields a loss, though close to break-even.

The picture is roughly similar if we substitute the long-term means into the formula, production in the off-peak hours makes a loss, while operation in the peak period yields a small profit:

$\text{spread}_{\text{off-peak}}$ = 46.5685 - 21.7829/0.38 - 6.8357 * 0.2014 / 0.38 - 3 = -17.3778

$\text{spread}_{\text{peak}}$ = 68.0518 - 21.7829/0.38 - 6.8357 * 0.2014 / 0.38 - 3 = 4.1055 45.

The lower 5-percentile and the upper 95-percentile values of the spread values simulated by the stochastic price models over a period of 1 year are depicted on the curves below:

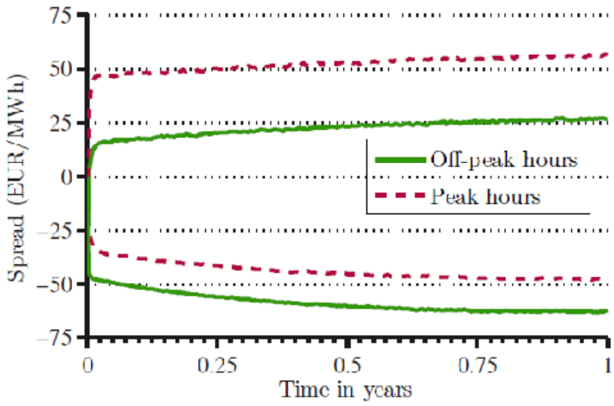

Figure 11: Simulated 5% and 95% percentiles of the spread.

As it can be seen from the figure, the behavior that corresponds to the long-term means is adopted by the spread model in two steps: the fast mean-reverting electricity reaches the state when it only fluctuates around its long-term mean rather quickly, while the more "sluggish" inputs only reach this state slower. Accordingly, we wittness a relatively fast initial correction in possible spread values, while the confidence intervals continue to widen at a decreasing rate in the longer run.

Having reviewed the parameters of the simulation model, we now proceed with an overview of the valuation methods for the spread options.

III.2. Pricing Spread Options

Spread options are a widely-used type of derivative instrument, used for risk management and asset valuation just as well as for speculative purposes. Derivatives traded in large volumes on overseas commodity exchanges are the spread options on the difference of the prices of soybean oil vs. defatted soybean meal, and crude oil vs. gasoline. On the energy market, it is the options for the *spark spread* (price difference of electricity and gas) the trading of which has become particularly widespread.

The most simple form of spread options is written on the price difference of two underlying assets. The option, if exercised, gives its owner the right to receive the difference between the prices of the two assets in exchange for paying the exercise price. In other words, the owner of the option has the right to exchange the two underlyings at a predetermined exercise price.

Considering multi-(more than two)asset spread options, there is usually only one asset with a positive weight and the rest with negative weights, that is, the price difference is calculated between one asset and the sum of the others. Similar to spread options are basket type options, which represent the right to buy or sell a basket of multiple (possibly differently weighted) assets at a predetermined exercise price. The spread option is a special type of basket option, where the weight is positive for the first asset and negative for the remaining assets.

The prices of the four underlying assets (peak and off-peak electricity, gas, emission units) of the real option model employed in this dissertation are, according to the assumptions, log-normally distributed. Contrary to the sum of normally distributed variables, the sum of log-normally distributed variables cannot, to our present knowledge, be expressed in a closed analytic form. This is the case in spite of the fact that the topic (the sum of log-normal variables) has been in the forefront of mathematicians', engineers' and finance professionals' attention for more than five decades now, ever since the time of Fenton (1960). In engineering sciences, an immense variety of problems are described by log-normal models (for instance, the shadowing effect and the distance-proportional radiation intensity of antenna towers). In finance, one of the most frequently used risk management measure is portfolio Value at Risk (VaR), which is calculated from the value distribution of a portfolio comprising log-normal assets. Many a time, this unsolved problem (the distribution of the sum of log-normal variables) gets simplified and a normal distribution is assumed for the resultant distribution. Modern Portfolio Theory, a work of Harry Markowitz (1952), the 1990 Nobel Laureate for Economics, also presumes the portfolio's value to be normally distributed. The normality assumption of the resultant distribution may, depending on the correlation structure and the weights applied, lead to a bias of significant proportions.

It is partly "thanks" to this problem that the pricing of spread options is one of the most challenging areas in stochastic finance, as it requires us to solve far more complex problems than for European vanilla call options (for details on those see (Hull, 1999, pp. 301-303.), (Benedek, 1999) and (Száz – Király, 2005)).

As of now, a closed formula has only been found for two assets and an exercise price of zero (Margrabe, 1978); there is no analytic solution for more general cases. Lacking an analytic solution, researchers resort to numerical integration techniques, simulations and various analytic approaches for pricing spread options.

What the numerical integration procedure is based on is that the value of a European option is, in the general case, the discounted expected value of the option payoff calculated according to the risk-neutral measure (Harrison and Pliska, 1981):

$$V_t^b = E^Q\left[\exp\left(-\int_t^T r_v dv\right)\cdot \max\left(B_T - K_B, 0\right)\middle| F_t\right] \tag{46.}$$

where *E[.]* denotes the expected value, calculated according to the risk-neutral (martingale equivalent) measure *Q*, and *B* stands for the price of the underlying asset.

In order to arrive at the formula for the multi-asset case, we need to be able to write the joint probability density function of the risk-neutral measure in an analytic form. The subsequent numerical integration of the pdf can be done in several ways. In one of my prior works (2011), I relied on a special convolution integral to express the joint probability density function of *N* correlated variables in terms of an integral of dimension *N-1*. The problem with numerical integration methods is that an increase in the number of the option's underlying assets increases the number of dimensions (of the joint probability density function), and hence the time required for the calculations (*curse of dimensionality*).

The method probably most widely used for pricing exotic derivatives is Monte Carlo simulation. A multi-dimensional Monte Carlo simulation essentially means the generation of multi-dimensional normal distributions of a given correlation structure. Between independent standard normal and dependent normal distributions, a bi-directional relationship can be established (for details see Nagy, 2011). The set of multi-dimensional normal distributions is closed under linear transformation: what we obtain by adding a vector to a multi-dimensional normal distribution and/or right-multiplying it by a matrix is always a normal distribution (Glasserman, 2003, p. 65.). A multi-dimensional normal distribution of given expected value vector and covariance matrix can be generated from a multi-dimensional standard normal distribution. Let us consider a multi-dimensional standard normal (N($\mathbf{0}$,$\mathbf{1}$)) distribution Z, left-multiply it by matrix $\boldsymbol{L}$ and add vector $\boldsymbol{\mu}$:

$$\boldsymbol{L} \cdot Z + \boldsymbol{\mu} = N(\boldsymbol{\mu}, \boldsymbol{L} \cdot \boldsymbol{L}^{\boldsymbol{T}}) = N(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \qquad \text{where } \boldsymbol{\Sigma} = \boldsymbol{L} \cdot \boldsymbol{L}^{\boldsymbol{T}} \qquad 47.$$

The resulting equality can be used in a wide range of applications, as any real (symmetric and positive definite) covariance matrix can be factorized, using Cholesky decomposition, as the product of a lower triangular matrix and its transpose. Thus utilizing the above, we can easily generate correlated yields and, hence, prices for underlying assets from a given covariance matrix and expected value vector.

From amongst numerical methods, discrete price trees are a frequently applied. While the binomial tree method was addressed by a relatively large number of authors (and even more practicing professionals), numerical tree techniques meant to model more than one asset are far more rarely encountered. Korn and Müller (2009) developed a procedure based on multi-dimensional trees for the pricing of derivative contracts on a set of correlated assets. The procedure makes use of another correspondence derived from the previous equation:

$$\boldsymbol{L}^{-1} \cdot [N(\boldsymbol{\mu}, \boldsymbol{\Sigma}) - \boldsymbol{\mu}] = Z \qquad 48.$$

That is, we can use a correlated normal distribution to generate a non-correlated standard normal distribution. Having performed the transformation to a non-correlated process, the multi-dimensional tree becomes easy to build, since the probabilities in the tree can be obtained as the products of the respective marginal probabilities pertaining to the individual assets.

A drawback of using numerical integration, simulation or discrete price trees for pricing is that they cannot provide a closed formula for the option value. Which, however, we would need for the sake of quickness on the one hand, and, on the other hand, because they can be used to derive the various factor sensitivities – the so-called *Greeks* – by way of partial derivation. One of the methods most frequently referenced and most frequently used by traders is Kirk's approximation (1995), which gives an approximate solution for the price of a two-asset spread option. Jarrow and Rudd (1982) also worked to price a two-asset spread option, and approximated the risk-free measure with a *Gram-Charlier A* series. Alexander and Venkatramanan (2009) applied compound exchange option (CEO) approximation for pricing and hedging spread options. Carmona and Durrleman (2003) worked out a relatively accurate pricing method, which requires the user to numerically solve a multi-dimensional nonlinear equation system. Its implementation is relatively difficult; the authors used the Newton-Raphson algorithm.

Another type of procedure opts for approximating the distribution of portfolio value (the problem of the sum of lognormals) by applying a distribution that does have a closed formula. They determine the theoretical distribution by way of momentum fitting, and then apply the closed Black-Scholes formula to the resulting approximate distribution. The majority of these procedures work with positive portfolio weights only, and are thus unsuited for pricing spread options (for the spread may just as well take a negative value). Milevsky and Posner (1995) used an inverse gamma distribution to approximate the probability density function of the portfolio's value. Borovkova, Permana and Weide (2007) employed a negatively shifted log-normal pdf, which allows for negative portfolio values and is, hence, suitable for pricing spread options.

The primary problem with early approximation methods is that they either provide a solution for two-asset spread options only or yield rather inaccurate results for baskets that comprise a small number of assets. Deng et al. (2008) derived a solution for multi-asset spread options that is relatively quick and delivers an accurate result. This is the method that I will use for pricing the options and calculating the greeks throughout the dissertation. The authors elaborated solutions for two types of stochastic underlying asset processes (geometrc Brownian motion and mean-reverting geometric Ornstein-Uhlenbeck process).

If the geometric Ornstein-Uhlenbeck process is given by the following stochastic differential equation:

$$dS_k = -\lambda_k(logS_k - \eta_k)S_k dt + \sigma_k S_k dW_k \qquad 49.$$

Then, according to the procedure outlined by Deng et al. (2008), the price of the option can be written as:

$$\Pi = e^{-rT+\mu_0+\frac{1}{2}\upsilon_0^2}\mathrm{I}_0 - \sum_{k=1}^{N} e^{-rT+\mu_k+\frac{1}{2}\upsilon_k^2}\mathrm{I}_k - Ke^{-rT}\mathrm{I}_{N+1}$$

$$\mu_k = \eta_k - \frac{\sigma_k^2}{2\lambda_k} + e^{-\lambda_k T}\left(logS_k - \eta_k + \frac{\sigma_k^2}{2\lambda_k}\right) \qquad \upsilon_k = \sigma_k\sqrt{\frac{1-exp(-2\lambda_k T)}{2\lambda_k}} \qquad 50.$$

The values of the integrals I_i included in the formula can be approximated using the method presented by the authors.

The authors report that, considering a three-asset spread option, the order of magnitude of the calculation error is 10^{-4}, while calculation time amounts to 2×10^{-4} seconds per option (i.e. even with 100 dimensions, it does not exceed 10^{-2} seconds).

Because of the multiplication by the weights pertaining to the underlying assets, I had to modify the formula presented by the authors by performing the following substitutions: $logS_k \longrightarrow logS_k + log\, w_k \quad \eta_k \longrightarrow \eta_k + log\, w_k$

This way, the procedure has become suitable for pricing spread options on weighted underlying assets.

IV. THE REAL OPTION MODEL OF CARBON DIOXIDE EMISSIONS

In this chapter, I will give a prediction for the carbon dioxide emissions of our power plant, relying on the real option model. Besides the development of the expected value, I will also calculate the probability density function of emissions, which shows the probability of a given volume of emission occuring. I will furthermore determine the amount of emission units the company has to purchase in order to comply, that is, the maximum cost of compliance at a given confidence level. In a later part of the chapter, I will examine how the emission level of the power plant depends on changes in various technological and market factors. Changes in the individual factors will be evaluated under the *ceteris paribus* assumption, that is, all other parameters were considered unchanged (i.e. exogenous parameters). The power plant is assumed to be of negligible size compared to the market and its behavior has no influence on the market. In the sensitivity analyses, I relied on simulation results, and in order to speed up the evaluation process, I also made use of the variant of the analytic option pricing formula of Deng et al. (2008) that had been adjusted for the use of weights.

As seen in section II.3, daily and peak-period emissions correspond to two three-asset European-style binary (payoff either 0 or 1) spread options. If the value of the spread for a given peak or off-peak period is positive, the company is generating power (and polluting), if however the weighted price difference is negative, the turbine remains turned off. The expected value of the emissions for a future day τ can be obtained, using the following formula, from the price of the binary options as calculated under the physical measure:

$$\mathrm{E}[\mathrm{Q}(\tau)] =$$

$$\frac{\Gamma}{2} \cdot \delta/\eta \cdot e^{r^{*}\tau} \cdot \left[bno^{Pr}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right) + bno^{Pr}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right)\right] \qquad 51.$$

The expected cumulative volume of emissions $E[Q_c(0,T)]$ for the period from the present to date T can be derived as follows:

$$E[Q_c(0,T)] =$$
$$\frac{\Gamma}{2} \cdot \delta/\eta \cdot \sum_{\tau=0}^{T} e^{r^*\tau} \cdot \left[bno^{Pr}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right) + bno^{Pr}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right)\right] \quad 52.$$

The Probability of Production, Expected Carbon-Dioxide Emissions

As we saw earlier, the profit maximizing power plant only generates power if the value of the spread is positive for the time-of-day interval in question. Based on the simulated spreads, the realizations of the binary production decision variable Λ can be determined. The average of the realizations of variable Λ (i.e. its expected value) pertaining to a given future day is equal to the probability of production for that day. The expected daily volume of the power plant's emissions can, then again, easily be expressed in terms of the probability of production: the resulting value needs to be multiplied by the maximum volume of emissions for the period in question, which is given by the formula $\Gamma \cdot \delta/\eta$ if it is an entire day, while it is half of that for a single, half day long, time-of-day interval.

The simulation was performed using the parameters of the model that was fitted to available market data; the last set of market prices were used as initial prices for the simulation. Based on the simulation runs, the following graphs can be plotted for the probability of production and for the expected emissions:

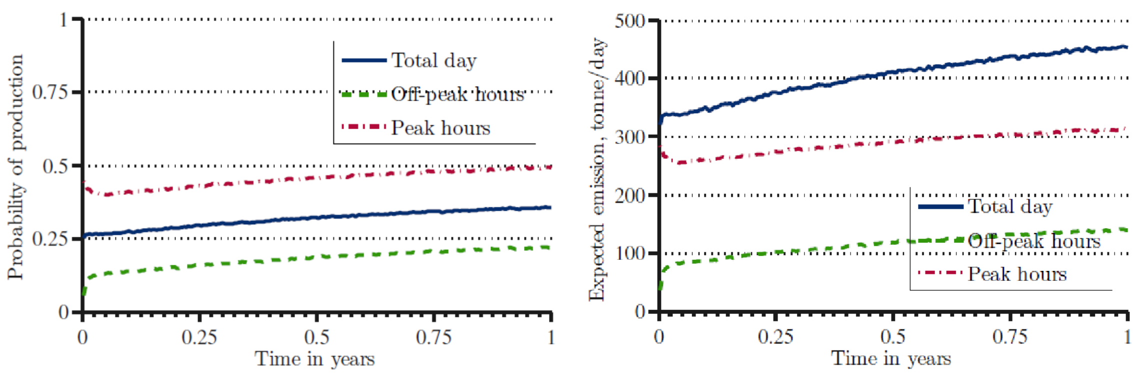

Figure 12: Probability of production (left) and expected daily emissions (right).

As the figure shows, the probability that the company generates power in the peak period is about 50 percent. For the off-peak period, the probability is much lower (15%-25%). The reason is that the price of electricity is much lower during off-peak hours (as compared to the peak period), therefore the chance of the spread being above zero is smaller than it is in peak hours. Since I decided to divide the day into two equal parts, the probability of production for the entire day is the arithmetic mean of the two values. The entire-day value gives the percentage share of the 24 hours within a day during which the power plant is expected to operate and thus emit carbon dioxide. The figure also suggests that there occurs a "correction" in the probabilities during the first couple of simulation runs. This is due to the electricity price (the price with the highest mean-reversion rate λ) quickly reaching the state of fluctuating around its long-term mean from its initial value. The modest long-term increase in the probabilities is, however, "thanks" to the slower mean-reversion rate of gas price, which is therefore slower to adapt to its long-term mean.

The expected value of the emissions level develops in parallel with the probability of production. During the peak period, the power plant is expected to emit nearly 300 tonnes of carbon dioxide into the atmosphere each day, while expected daily emissions for the off-peak period are around 100 – 150 tonnes. The expected volume of emissions for the entire day is obtained as the sum of the peak and off-peak values. Its value rises from 320 tonnes in the beginning of the year to 450 tonnes per day in the end of the year.

Having determined the expected volume of emissions, let us now proceed with a factor that has a significant role in risk management: the probability density function of emissions, which shows the probability that the power plant will emit a given amount of carbon dioxide.

The Probability Density Function of Cumulative Emissions

The cumulative value of the daily decision variables (Λ) is the cumulative production decision variable (Ω). Its value gives the whole-production-day equivalent of the total up time of the power plant until a given date. The day having been divided into two equal parts, the resulting Λ values need to be multiplied by 0.5 before the cumulation. The value of Ω pertaining to the entire first production day may be 0, 0.5 or 1, any one number from the series {0, 0.5, 1, 1.5, 2} on the second day, and any increment of 0.5 between 0 and

252 for the entire simulation period of 252 days. By multiplying the cumulative production decision variable by the maximum volume of emissions we arrive at the cumulative volume of carbon dioxide emissions (Q_c). It is important to underline that while expected values may as well be calculated using analytic formulae, the probability density function can hardly be determined in any other way but through simulation. In the appendix, I will outline a numerical method developed by myself that makes use of recombining binomial trees to approximate the probability density function. What follows below is an overview of the pdf determined through Monte Carlo simulation. To the 50,000 cumulative emissions values the simulation runs yielded, a histogram – or more precisely, the normalized and continuous approximation of that: a Kernel density function (Rosenblatt, 1956) – was fitted. The figure below shows the pdf of the cumulative emissions volume pertaining to the 10th, 100th and 252nd days:

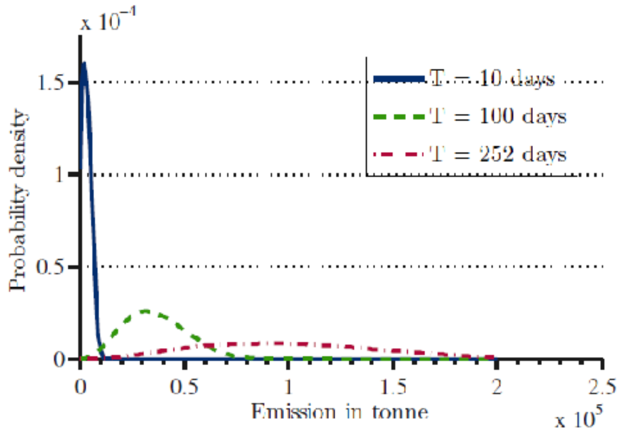

Figure 13: Probability density function of cumulated emissions for time periods of different lengths.

As evinced by the figure, resulting probability density function for the short, 10-day period is extremely right-skewed: the zero emission volume is the most probable to occur. The reason for that lies in the relationship between the consecutive days' emissions. If on a given day the spread is negative and the turbine rests, then its value is very likely to be below zero the following day, as well, so the turbine is likely to remain turned off. Therefore a zero-emission day is very likely to be followed by another zero-emission day(s). Given that under the initial prices of the simulation, the spread values are smaller than zero for both time-of-day intervals, zero-emission days will certainly dominate the first couple of runs, that is, the frequency function will have its peak around zero. As regards longer intervals, the probability of positive-emission days grows with time, thus the distribution becomes more symmetric (less skewed) and flatter.

The pdf of the individual time-of-day intervals are remarkable, as well. Let us have a look at how the 1-year emission probability density functions pertaining to the off-peak period, peak hours and the entire day relate to each other:

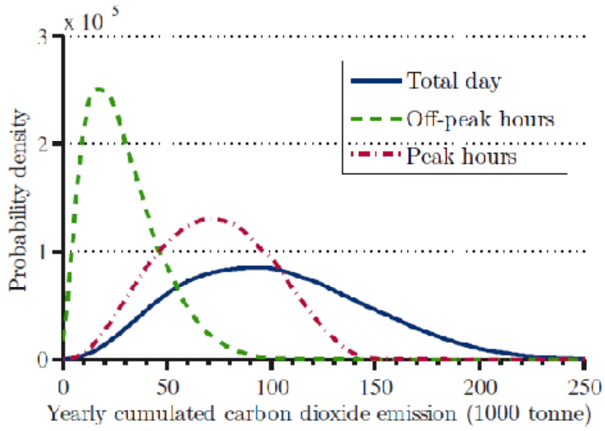

Figure 14: Probability density function of yearly cumulated emissions of off-peak, peak hours and for the total day.

Due to the smaller spread values, the probability of production is lower during off-peak hours, the resulting probability density function is right-skewed, and the probability of a low cumulative annual emission level is high. In the peak period, the probability of production is higher because of the higher electricity price induced by the growing demand, the resulting distribution will be less skewed, and hence higher emission levels will become more probable.

Cost of Compliance, Value at Risk of the Emission Position

As we have already mentioned in the introduction, the power plant has an obligation to comply. It is required to surrender to the authorities an amount of emission units that corresponds to its annual emissions volume. It seems reasonable to ask how much cost the company should expect to incur for its compliance with this regulation.

The probability density function not only facilitates a more accurate prediction of future emission levels, but it can be used as a starting point for approximating the maximum cost of compliance at a given confidence level, as well.

An index widely-used in the field of risk-management is Value at Risk (VaR) (Jorion, 1999, p. 97.), which refers to the maximum loss an investor might suffer on their position at a given confidence level (c):

$$P(|Loss\ potentially\ realized\ on\ portfolio| < VaR) < 1 - c \qquad 53.$$

If for example, we assume the value of a portfolio to be normally distributed with an expected value of EUR 1 million and a standard deviation of EUR 200 thousand, then there is a 95% chance that in 1 year's time we will not incur a loss greater than: $|\Phi^{-1}(1 - 0.95, 0, 200000)| = 328{,}971\ EUR$, where $\Phi^{-1}(P, \mu, \sigma)$ is the value of the inverse cumulated distribution function of the normal distribution with expected value μ and standard deviation σ for a probability P.

The Value at Risk concept can also be applied to compliance: in our case, it will provide us with the maximum cost of compliance at a given confidence level.

From amongst the various costs and expenses associated with EU ETS compliance, the value of the necessary amount of quotas is the most obvious item. Besides, there is a lost profit component, as well: a high quota price acts to reduce the probability of production. There are periods when the quota cost is the only factor that keeps the power plant from generating electricity (the value of the spread would be positive in the absence of quota cost). During these days, "foregone" production results in a deadweight loss. Carbon dioxide emission does, however, also have external costs, and therefore foregone production and emissions lower the damages on the social level and hence increase welfare. The direction of the net effect of pollution is uncertain (for details see Kocsis, 1998). For the sake of simplicity, cost of compliance will hereinafter refer to the value of the necessary amount of carbon credits only, and lost profit will not be considered.

The cost of compliance depends on the emission volume and the quota price: a large volume of emissions coupled with a high carbon credit price result in a significant burden on the company; whereas if the emission level and/or the quota price is low, compliance can be realized at a low cost. The cost of covering the emissions with quotas can be obtained by adding up the products of the emission amounts Q and the respective price realizations S_{EUA} pertaining to the future days τ, with the remark that it is the future values (at the end of the year) of the individual products pertaining to the different dates that need to be added up:

$$\frac{\Gamma}{2} \cdot \delta/\eta \cdot \sum_{\tau=0}^{252} \left[\left(\Lambda_{\text{off-peak}}(\tau/252) + \Lambda_{\text{peak}}(\tau/252) \right) \cdot S_{EUA}(\tau/252) \cdot e^{r(1-\tau/252)} \right] \quad 54.$$

Future values were calculated under the assumption of a constant risk-free interest rate (r), equated to 0.928% based on the 6-month Euribor rate[8].

There are a couple of important conclusions with respect to the calculation of the maximum cost of compliance at a given confidence level. As already mentioned in the introduction, the compliance obligation in the EU ETS always pertains to the total emission volume of the previous year and needs to be met by April 30 the next year (by submitting the corresponding amount of emission units). At the end of the year, the total volume of emissions is already known, thus it seems a reasonable first approximation to state that by the end of the year latest, the risk minimizing power plant needs to possess the amount of quotas (or its equivalent in derivative contracts) they need to cover their obligation on April 30. Otherwise they will have an open position (i.e. exposed to price changes, not covered by quotas) and thus engage in speculation (win if the price goes up and lose if it falls). Narrowing down the condition, we can also conclude that as time passes by in the course of the year, an ever growing volume of emissions will become known for a fact. Consequently, at any point in time during the year, the company should have an emission unit position that corresponds to its total volume of past emissions during that year. Finally, further narrowing the condition, we can argue that even its future emissions are partially known (from the stochastic model). In order to minimize the risks associated with covering its future emissions, the company should possess an amount of emission units that corresponds to (by putting it in not-so-accurate terms) its *expected emissions*. Or more accurately: at any one point in time, it needs to possess an amount of carbon credits that corresponds to the sum of the emission *delta parameters* of the spread options that represent future gross margins in order to cover its future emissions and it should hold an amount of quotas that covers its past – factual – emissions for that same year. In order to completely hedge its future gross margin, the power plant would need to equate the cumulative deltas for the four underlying assets (peak and off-peak electricity, gas, emission units) to zero, which can be realized via hedging transactions for those four assets. The hedging procedure and the calculation of the

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[8] Source of the data for 2 July 2012: euribor-rates.eu

amount of emission units needed to cover future emissions will be discussed in detail in the following chapter.

Subsequently, the Value at Risk associated with the cost of compliance will pertain to the quota position without the hedging contracts. Based on the simulation, the distribution of the future value of the expected annual quota position – required for determining Value at Risk – will be:

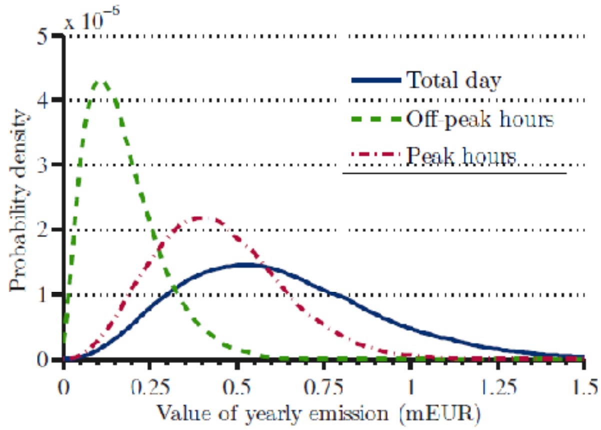

Figure 15: Probability density function of future value of yearly emissions.

The shapes of the resulting functions is very similar to those of the histograms of cumulative emissions; the lack of exact shape equivalence can be ascribed to the quantities having been multiplied by the future values (as of year end) of future carbon credit prices. Considering off-peak hours, lower-emission events are more probable to occur, thus the probability density function is right-skewed. The higher expected electricity price in the peak period implies a higher spread value, which is why the peak of the probability density function is shifted to the right. The pdf of the entire-day value is flatter. The cost incurred by the company because of the carbon dioxide they emit in the course of one year will, with a great probability, be in the range from EUR 0.25 to 1 million.

The cumulated distribution function (cdf) can be derived from the probability density function by way of integration. The Value at Risk pertaining to the maximum cost of compliance at a given confidence level can then be easily read from the inverse cumulated distribution function:

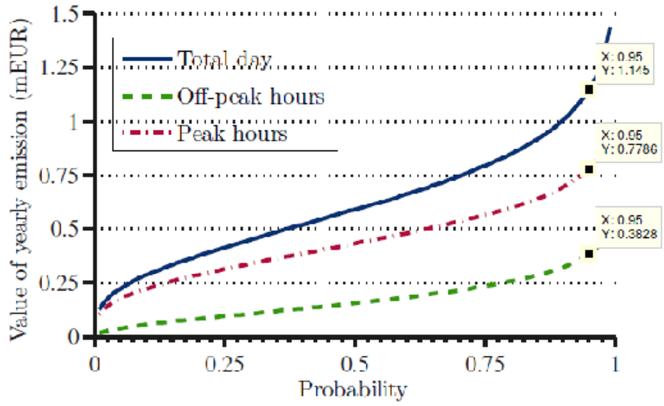

Figure 16: Inverse cumulated distribution function of yearly emissions.

With a probability of 95%, the cost of compliance for this power plant with a 2400 MWh daily capacity will remain below EUR 781 thousand for the peak period and below EUR 383 thousand for off-peak hours, both given in future value terms. Given that the emissions of the two intervals are not independent from each other, the entire-day value is not equal to the sum of the two values; with a 95% reliability, compliance will cost the power plant an amount less than EUR 1.15 million.

Having reviewed the calculation of Value at Risk, let us now examine how changes in the model's various parameters affect our results. The sensitivity analysis was completed for three scenarios:

- Effect of efficiency improvement projects (the role of thermal efficiency)
- Effect of fuel price changes (the role of long-term gas price)
- Significant price changes in the carbon credit market (the role of long-term EUA price)

The Effect of a Change in Thermal Efficiency

The technological parameters of the power plant have a significant role in the determination of the spread and, hence, production and emission volumes. From amongst these, it is thermal efficiency (η) – which shows the percentage share of total energy input utilized in the form of the electric power generated – the potential changes in which I will elaborate on in detail. By manipulating parameter η in the model, we can, on the one hand, compare the expected carbon dioxide emissions of power plants that use the same fuel, incur an equal amount of other variable costs, but are characterized by differing thermal efficiencies and, on the other hand, it may provide some sort of starting point for estimating the environmental impact of a potential efficiency improvement project.

The facility's thermal efficiency parameter gives the amount of resources required to generate one unit of energy, and also, the power plant's maximum possible volume of carbon dioxide emissions. If it has a fixed capacity Γ, the maximum daily volume of its emissions ($\Gamma \cdot \delta/\eta$) decreases if its efficiency increases. Given a fixed daily production capacity, we arrive at the following daily maximum emissions curve by varying parameter η in a somewhat wider (i.e. less realistic) interval:

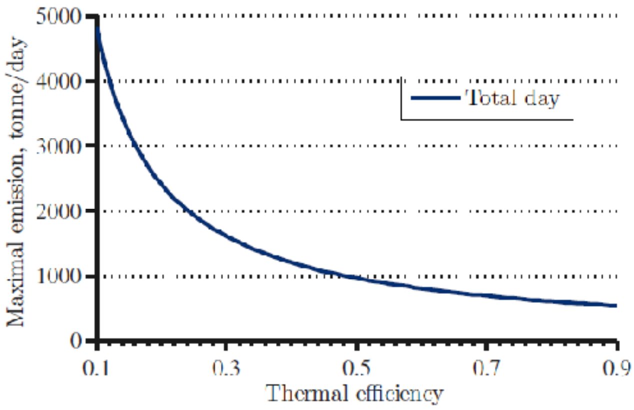

Figure 17: Daily theoretical maximal emissions as a function of thermal efficiency (with fixed output capacity).

Considering the default 38% value and assuming constant production, the daily maximum amount of carbon dioxide potentially emitted by the power plant is 1272 tonnes. Under an efficiency of 50%, the possible daily maximum emission volume drops to 967 tonnes, while a 30% figure would bring about a significant growth, to 1611 tonnes.

An improvement in thermal efficiency, thus, seems to reduce the emissions of the company. In the real option model, however, we can observe the opposite, as well: an improvement in efficiency reduces the required amount of resources, thus the value of the spread to be realized grows larger. Therefore, the probability of production becomes higher which, ceteris paribus, results in higher emissions.

Varying thermal efficiency in a wider-than-realistic interval, the probability of production (the expected value of the daily decision variable Λ) on the day one year from now can be characterized as follows:

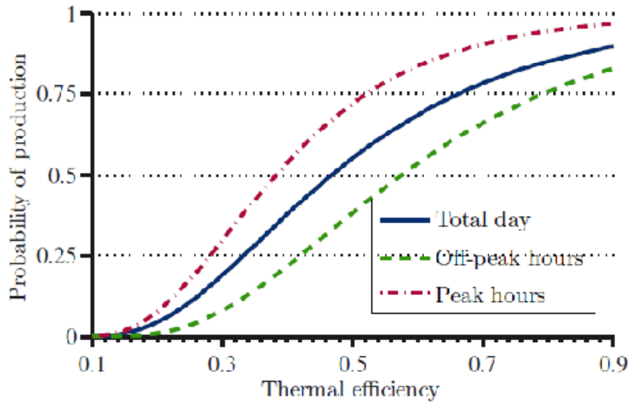

Figure 18: Probability of production at a future day (one year from now) as a function of thermal efficiency.

As seen on the figure, the probability of production grows with increasing η, at a rate that is first increasing, then decreasing. The expected volume of emissions as a function of thermal efficiency is obtained as the product of these two opposite effects (decreasing maximum daily emissions, increasing probability of production):

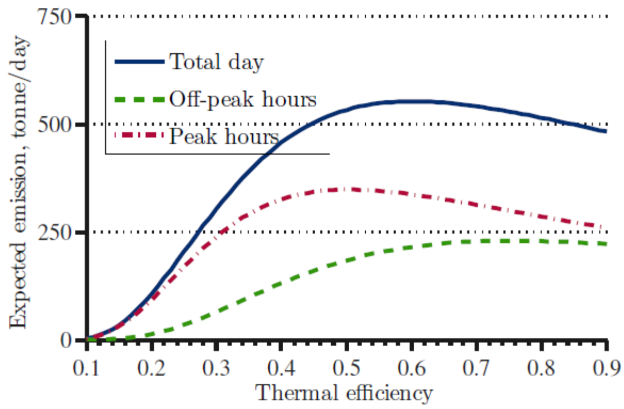

Figure 19: Expected daily emissions at a future day (T=1 year) as a function of thermal efficiency.

The increase in the probability of a positive spread induced by an improvement in efficiency is quite significant with respect to the off-peak period, therefore the expected volume of emissions grows with increasing η; the opposite can only be observed for particularly high (hardly realistic) efficiency figures. As regards the peak period, the higher spread value implies a high probability of production, which the improvement in efficiency can only offset to a lesser degree. The impact of the fall in maximum emissions, during the peak hours, becomes dominant above approximately 47%, and thus results in a decrease in the expected volume of emissions for the peak period. The entire-day volume of emissions, i.e. the sum of the two time-of-day periods, basically grows with increasing thermal efficiency, which only turns into a decrease for very high values of η (around 60%).

Let us examine how changes in thermal efficiency affect the maximum cost of compliance (Value at Risk). The probability density function of the cumulative annual emission volume for three different efficiency levels (30%, 38%, 50%) can be plotted as follows:

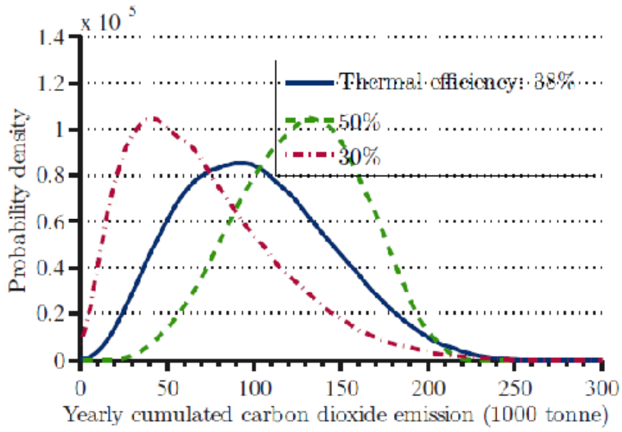

Figure 20: Probability density function of annual emissions in terms of different thermal efficiencies.

A higher-efficiency power plant is expected to produce more electricity due to the increase in its spread, thus the probability density function will be left-skewed (compared to the "default" case). Whereas for lower efficiencies, expected production falls, and therefore the pdf becomes right-skewed. The value of the carbon credit position (quantity multiplied by quota price) exhibits a similar pattern. A more efficient power-plant will, under the values examined, emit more carbon dioxide, therefore the cost of compliance is expected to become higher:

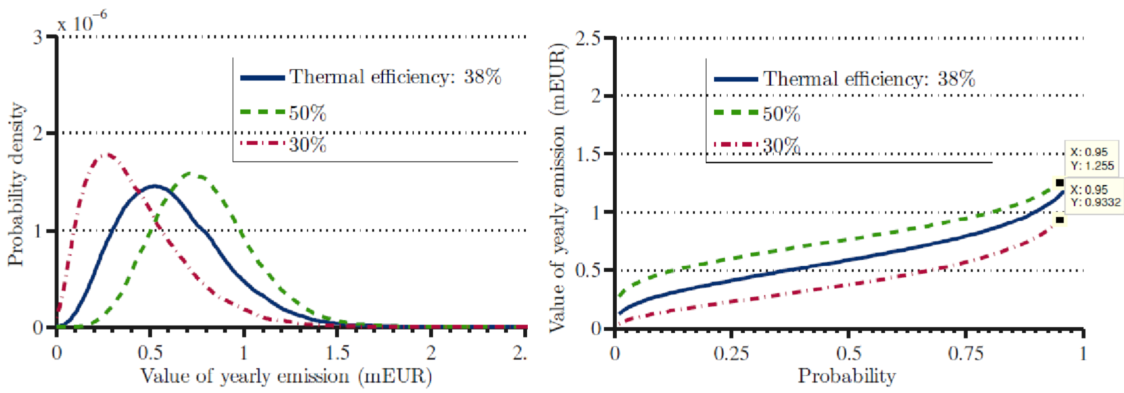

Figure 21: Probability density function (left) and inverse cumulated distribution function (right) of future value of annual emission in terms of different thermal efficiencies.

An efficiency-improvement to 50% acts to raise the Value at Risk to EUR 1.26 million, while a deterioration to a η value of 30% would induce a fall to EUR 930 thousand. With thermal efficiency being varied in a wider interval, the development of the 95% value of *VaR* is shown on the following figure:

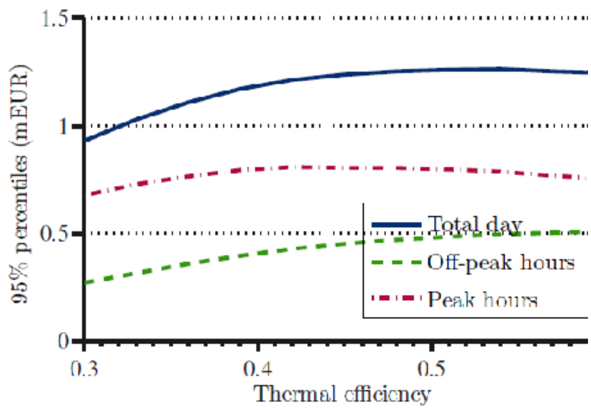

Figure 22: 95% Value at Risk of emission position as a function of thermal efficiency.

The off-peak 95% Value at Risk increases with increasing thermal efficiency. The peak-period *VaR* reaches its maximum around 42%, above that, further improvements in efficiency act to decrease the Value at Risk (for this time-of-day period). Considering its entire-day value, the *VaR* grows with increasing efficiency at lower efficiency levels, yet does not significantly change in higher ranges of parameter η.

The Effect of Gas Price Changes on the Carbon Dioxide Emissions of the Power Plant

Energy markets are characterized by rather significant price fluctuations. A drop in the supply of gas (delivery hindrances, for instance) effects a significant hike in its price, whereas the introduction of new exploitation technologies may allow for the economically feasible exploitation of large, previously unavailable reserves, which might well result in a price drop. As far as our model is concerned, the effect of gas price was tested by varying the long-term mean parameter of the respective mean-reverting price model.

The expected volume of emissions, as a function of long-term mean gas price, can be plotted as follows:

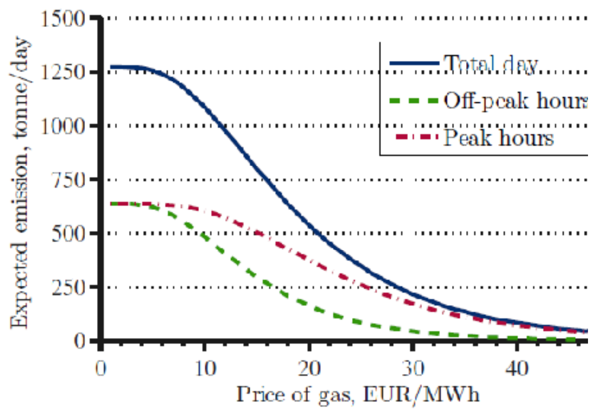

Figure 23: Expected emissions at a future day (T = 1 year) as a function of gas price.

We can see that as the gas price approaches zero, the volume of emissions reaches its maximum, as the spread will, in such a case, almost always be above zero, and thus the turbine will be up and running nearly all the time. The spread and, hence, expected emissions decrease with increasing gas price. Off-peak emissions do so at a faster pace because the lower electricity price associated with it implies a higher probability for a negative spread value. During peak hours, on the other hand, chances are that higher electricity prices can compensate for a higher gas price, and thus the negative impact the fuel price increase has on emissions will be weaker. With the gas price at extreme high levels, the volume of emissions – for both time-of-day periods – converges to zero.

The pdf and the inverse cumulated distribution functions of the value of the power plant's annual emissions volume for different gas price levels are:

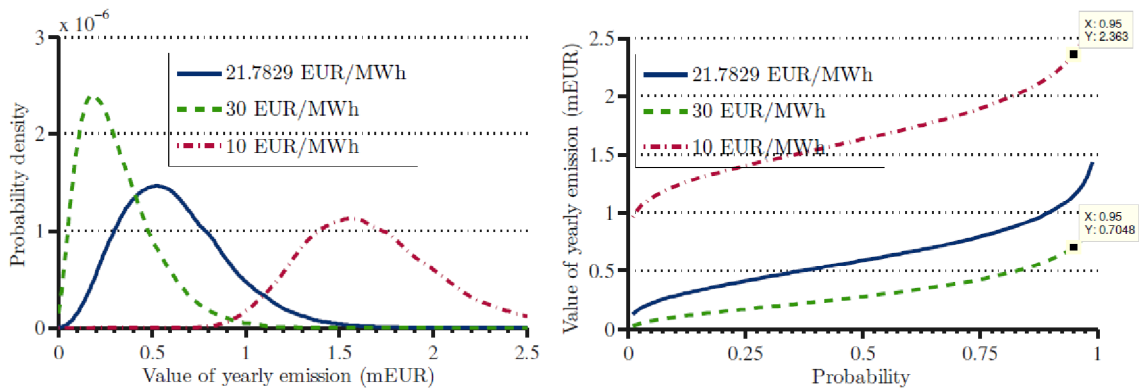

Figure 24: Probability density function of future value of annual emissions (left) and its inverse cumulated distribution function (right) in terms of different long term gas prices.

A higher gas price of EUR 30 acts to reduce expected emissions and to make the value of the quota position right-skewed (as compared to the "default" case), and the Value at Risk of the cost of compliance sinks to EUR 700 thousand. A lower gas price of 10 EUR/MWh distorts the distribution of the quota positions's value to the right, and the Value at Risk of the cost of compliance increases to EUR 2.4 million.

Varying the gas price in a wider range, the 95% Value at Risk of the cost of compliance follows the pattern shown on the figure below:

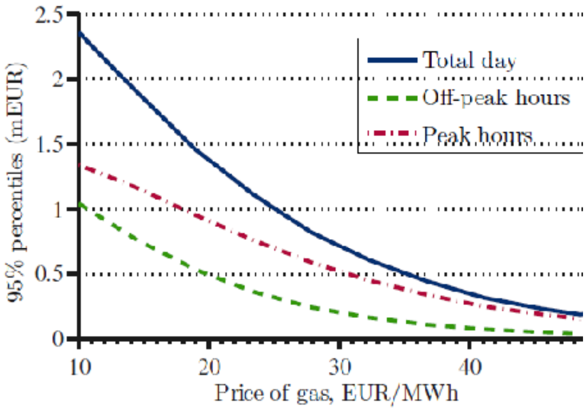

Figure 25: 95% VaR as a function of long term gas price.

It is apparent from the figure that the Value at Risk of the cost of compliance, for both the peak and the off-peak period, decreases with increasing long-term gas price, the curves of the functions for the two time-of-day periods being nearly parallel, but at a certain distance from each other.

The Effect of Emission Unit Price Changes on the Emissions of the Power Plant

During both trading phases of the emissions market, there were certain significant price changes with long-term effects. As the over-allocation at the start of the First Phase became obvious, the quotas turned practically worthless. For the Second Phase, the policymaker reduced the amount of credits to be issued, as a consequence of which the price for 1 tonne of carbon dioxide emissions was, in the beginning, above EUR 20 on the quota market. As the financial and economic crisis unfolded, expected emissions began to fall as a results of faltering growth prospects, which put a downward pressure on the quota price. Now, we will examine how a significant change in the price of emission units affects the power plant in our model. Given a mean-reverting model, the simplest way to perform such a test is to shift the value of the long-term mean.

The relationship between expected emissions and long-term quota price can be illustrated by the figure below:

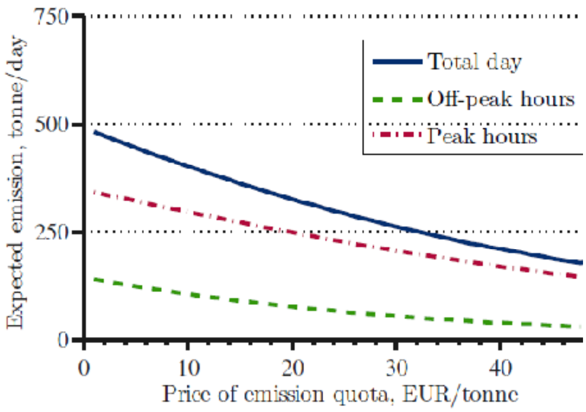

Figure 26: Expected emissions at a future day (T = 1 year) as a function of long term EUA price.

As it can be read from the curves, the expected volume of emissions is not extremely sensitive to changes in long-term quota price. The cause of this apparently surprising result resides in the formula of the spread: while in the case of gas, the resource price only needs to be multiplied by the reciprocal of the thermal efficiency (which is equivalent to a factor of 2.63 if η=38%), in the case of the quota price, there is one more term – the carbon intensity of the fuel – that needs to be included in the multiplication (which yields a resultant multiplier of δ/η). Which then again means that for a δ=0.2014 $tCO2/MWh_{in}$, the cost-increasing effect of the quota price is only about one fifth of that of the price of gas, the effective multiplier being 0.53.

Given that the effect a shift in the long-term mean price of emission units has on the value of the spread and, hence, on the volume of emissions is only one fifth in extent in comparison to that of the gas price, the probability density function of the emission volume will not be significantly changed, either. Such a change in the long-term quota price will, however, significantly influence the cost of compliance. The pdf and the inverse cumulated distribution function of the value of the quota position under three different long term quota prices can be plotted as follows:

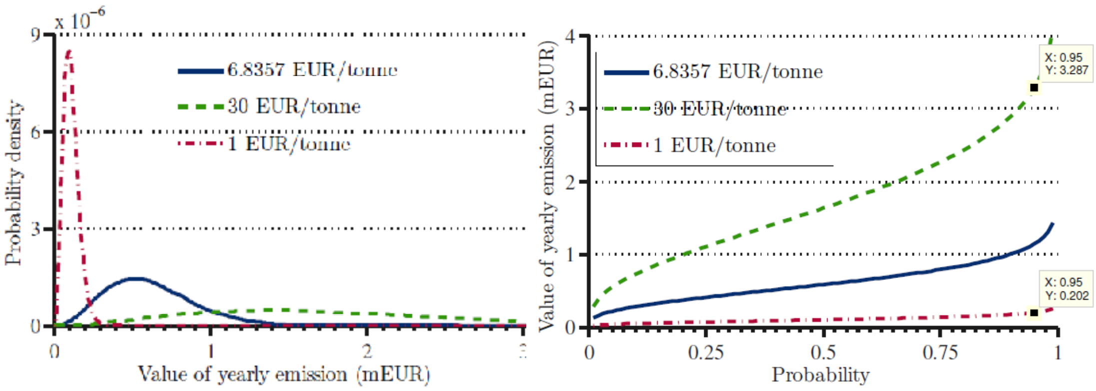

Figure 27: Probability density function of future value of annual emissions (left) and its inverse cumulated distribution function (right) in terms of long term EUA price.

Given a carbon credit price of 1 EUR/ton, the maximum cost of compliance at the given confidence level is EUR 200 thousand, while for a long-term quota price of 30 EUR/ton, it is EUR 3.3 million. Varying the quota price in a wider range, the 95% Value at Risk will be:

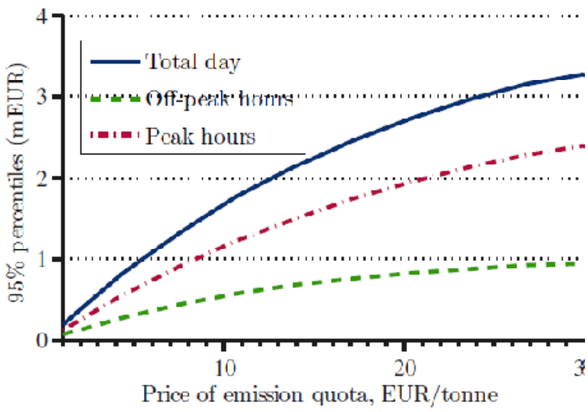

Figure 28: 95% VaR of compliance cost as a function of long term EUA price.

The Value at Risk grows with increasing quota price, albeit at a diminishing rate.

As I have shown in this chapter, the expected volume of emissions and its probability density function can indeed be derived from the real option model. The emission volumes of consecutive days are highly autocorrelated, therefore the probability density function of cumulative emissions is extremely skewed (to the right, in our case) for shorter intervals. With time, the function becomes more symmetric and flattens. Based on the probability density function, we can obtain the total value of the emission units required to cover the emissions (i.e. to comply), which then again can be used to determine the Value at Risk with respect to the cost of compliance. Having examined the development of the results for different parameter values, it can be concluded that thermal efficiency has a significant influence on the volume of emissions. Given a fixed daily production capacity, an increase in the value of η reduces the maximum volume of emissions and, inducing a simultaneous yet opposite effect, increases the number of days with a positive spread and the probability of production. For lower efficiency levels, it is the emission-increasing effect, while for higher efficiency ranges, the maximum-emission-decreasing effect that outweighs the other. From amongst the inputs, it is the price of gas that has a significant impact on expected emissions. A high gas price deteriorates the probability of a positive spread and, hence, that of production, the probability density function of emission volume becomes skewed to the right, the expected cost of compliance drops. A change in the price of emission units has less of an influence on the probability of production and the probability density function of emission volume, it does, however, significantly affect the cost of compliance: high carbon credit prices yield high levels of cost of compliance.

V. THE REAL OPTION VALUATION MODEL OF THE POWER PLANT

As we have seen in the previous chapter, the real option model is suitable for the estimation of the power plant's carbon dioxide emissions and the related costs of compliance. The results do not only pertain to the expected values, but, relying on the probability density functions, the risk characteristics of the quantities can also be estimated.

In this chapter, I will perform the valuation of the power plant using the real option model. The questions we will seek to answer are:

- How does the value of the power plant depend on various technological and market factors?
- What sort of hedging strategy should the company follow in its efforts to manage the financial risk associated with the power plant's operation? What is the (optimal) amount of emission units to be held at any given moment?
- How much is a thermal efficiency improvement project worth for the power plant, as a function of various factors?
- How much would be lost in profits if the power plant would not be operated in a profit-maximizing fashion, i.e. based on the relevant daily prices, but on a fixed schedule, by entering long-term production agreements?

As we have already seen when reviewing the real option model, the value of the power plant's income generating capacity corresponds to the present value of the incomes expected to be generated during the period in question, which can be expressed in terms of spread options:

$$V = \frac{\Gamma}{2} \cdot \sum_{\tau=0}^{T} \left[spo^{Pr}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right) + spo^{Pr}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right) \right] \qquad 55.$$

The value of the power plant, according to the model, equals the product of the sum – for the relevant period – of the daily averages of the European-style options on the daily peak and off-peak spread values multiplied by the facility's maximum daily capacity. For the purposes of this valuation, a 30-year life span and a zero residual value are assumed; furthermore, all maintenance costs and replacement investments are assumed to be included in the other variable costs (v, which in our case corresponds to the exercise price of the spread options) term; and tax effects are ignored. For the modeling of the price movements of the four underlying assets, we employ the geometric Ornstein-Uhlenbeck process – as presented earlier –, with parameters that have been fitted to market data. The valuation of the spread options involved in the calculation is performed using the modified form of the analytic pricing formula of Deng-Li-Zhou (2008). Incomes are discounted by using the 12-month EURIBOR rate of 1.213% for 2 July 2012[9], with a risk premium of 2 percentage points.

Let us first examine the expected payoffs and values of the spread options as a function of their maturity. The results for the two time-of-day periods and their averages pertaining to the entire day have been plotted on the following graphs:

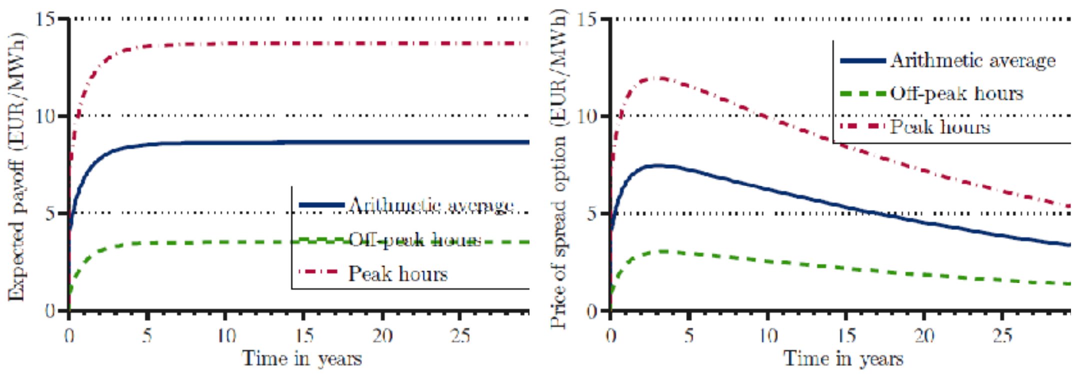

Figure 29: Expected payoff (left) and value (right) of spread options as a function of maturity.

The expected payoff of the spread option gets stable with time, the reason for which is that the realizations of the mean-reverting model are, in the longer run, determined by those constant long term mean values. Under the model's parameters, the expected payoff of the spread option is, in the long run, 3.5 EUR/MWh for the off-peak period and 13.8 EUR/MWh for peak hours. The values of the options are lower than these because of the discounting, and they reach their maximum values for a maturity of about 3 years (3 EUR/MWh and 12 EUR/MWh, respectively).

The sum total for the 30-year period corresponds to the product of the area under the curve of the options' average value function multiplied by the maximum daily capacity,

[9] http://www.euribor-rates.eu/euribor-rate-12-months.asp

with the restriction that it is the discrete value for each day that needs to be considered in the calculation of the area under the curve.

The power plant's value is, given the model's assumptions, EUR 97.3 million. Let us examine how sensitive this value is to changes in a number of different factors. The starting point for this analysis will be the sensitivities of the spread options to the various parameters.

The price of the spread options depends on a variety of technological and market parameters. From amongst these, the following will be examined in detail for their impact on the options' value:

- Thermal efficiency
- Spot prices of the underlying assets
- Long-term mean prices of the underlying assets
- Volatilities
- Value of the correlation coefficient between the prices of gas and emission units

In stochastic finance, the so-called *Greeks* indicate the change in the value of a derivative generated by a unit increase in the parameter examined. Their value equals the partial derivative of the option's price function with respect to the factor in question.

The Role of Thermal Efficiency

Thermal efficiency has a central role in the calculation of the spread option's value. A more efficient power plant requires less inputs – less gas and fewer emission units – to produce a unit of electric energy. In the formula of the spread, thermal efficiency modifies the impact of input prices: a 1 percent improvement in thermal efficiency increases the value of the spread by the amount given by the following formula:

$$\left(\frac{S_{gas}}{\eta}+\frac{S_{eua}\cdot\delta}{\eta}\right)-\left(\frac{S_{gas}}{\eta+1\%}+\frac{S_{eua}\cdot\delta}{\eta+1\%}\right)=\frac{S_{gas}+S_{eua}\cdot\delta}{100\cdot\eta\cdot(\eta+1\%)} \qquad 56.$$

The extent to which a change in thermal efficiency modifies the option's value is different for the peak and the off-peak spreads, because the probability of exercise changes, as well. The following figure shows the sensitivities of the options to changes in parameter η, for varying maturities:

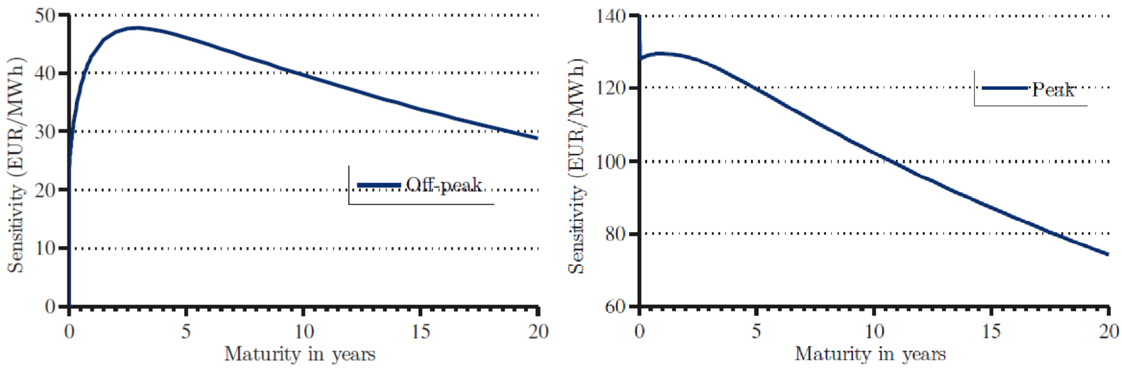

Figure 30: Thermal efficiency sensitivity of spread options for off-peak (left) and peak hours (right) as a function of maturity.

The resulting sensitivity values are positive for all maturities, that is, an improvement in thermal efficiency always induces an increase in the spread options' value. Concerning the values taken by the function, it is important to note that a unit change in parameter η would mean an efficiency improvement of 100 percentage points, which is obviously impossible in reality. For the ease of understanding, it is practicable to divide the resulting function values by 100 and interpret the sensitivity values as pertaining to percentage point changes in thermal efficiency.

Considering off-peak spread options, sensitivity sets off from zero, the reason for which is that the initial value of the spread is negative, thus the power plant is not generating electricity and, hence, it cannot realize a margin, either. In other words, the option is OTM (out of the money), the cashflow from the immediate exercise of the option would be zero. Sensitivity rises sharply with increasing maturity, as the reversion of the underlyings' initial prices to their long-term means acts to increase the spread's value. Having reached the peak, the derivative with respect to thermal efficiency starts to drop, the reason for which is that discounting assigns a lower present value to longer-maturity options, which renders the impact of an efficiency improvement smaller, as well.

The immediate exercise of the peak-period spread option would be a near break-even transaction (the option is approximately ATM (at the money)), the sensitivity parameter takes high values even for short maturities, because even a small improvement in thermal efficiency yields a significant change in the expected payoff, and thus in the value of the option. The function value drops for longer maturities, which is a consequence of the discounting, just like it was the case with off-peak options.

The following figure shows how the value of the power plant changes with improving thermal efficiency:

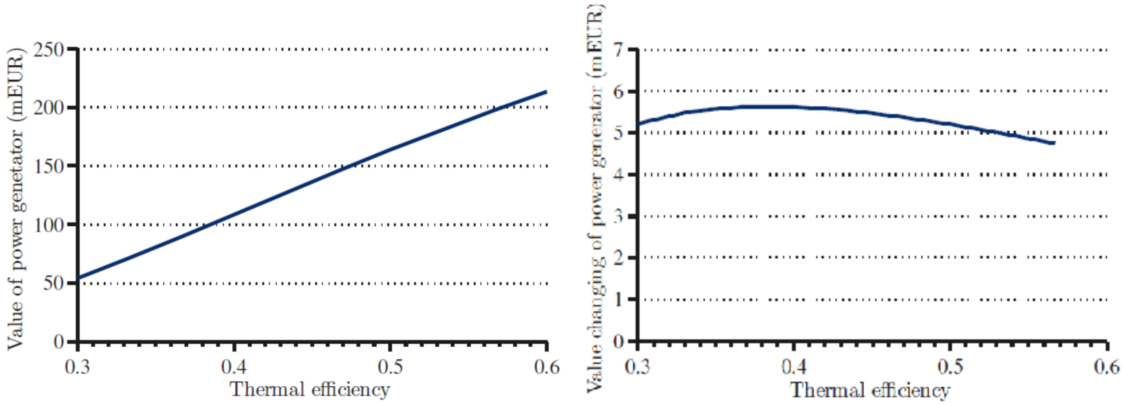

Figure 31: Value of power generator (left) and value increasing effect of 1 percentage point efficiency improvement (right) as a function of thermal efficiency.

An improvement in thermal efficiency increases the value of the power plant, the effect being nearly linear. By plotting the change in value as a function of the percentage point change in parameter η, we can formulate the relationship in more exact terms: a 1 percentage point improvement adds nearly EUR 5 million to the value of the power plant; the resulting function is concave, and it is the facilities with an efficiency between 37-40% the value of which increases most with improving thermal efficiency.

The Role of the Prices of the Underlying Assets

As we have witnessed in Chapter III, the prices of the four underlying assets all exhibit a significant degree of volatility. This is particularly true for the price of electric energy, the annualized volatility of which amounts to 533% in the off-peak period and to 410% during peak hours. One of the things worth examining is how much the values of the spread options would be modified by a change in the initial price used in the valuation process. One of the specialties of mean-reverting models is that the role the initial price used in the simulation has in the determination of future prices is getting smaller and smaller with time; for long maturities, it is the long-term mean the effect of which dominates. One of the consequences is that, as far as the model is concerned, short-term price changing has no significant influence on the value of real assets.

Concerning derivatives, the *delta* parameter shows how the price changes as a function of the initial prices of the underlying assets. The model includes 3 underlying assets for both the peak and the off-peak period. The following figure shows the sensitivities of the spread options to changes in the initial prices of the underlying assets, for varying maturities:

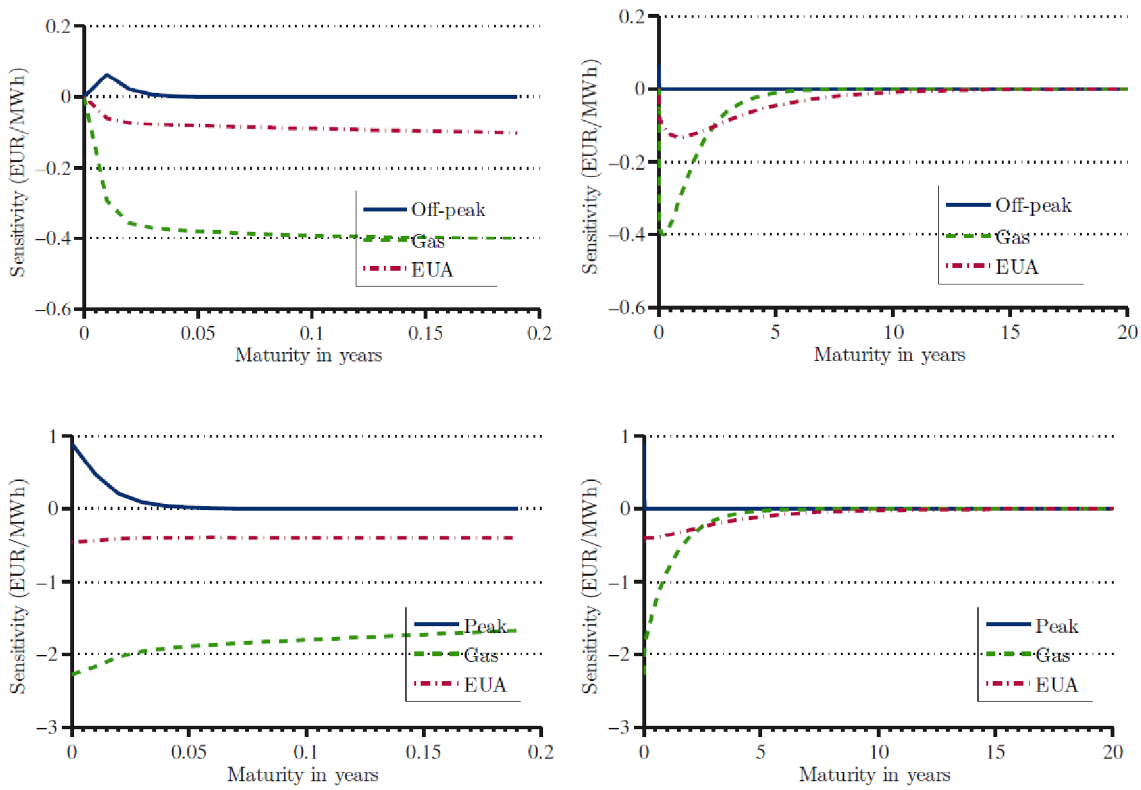

Figure 32: Sensitivity of spread options on initial prices of off-peak (top) and peak (bottom) hours with shorter (left) and longer (right) maturities.

As regards the off-peak period, the option is OTM under the initial prices, its intrinsic value is zero, as would be the cashflow from its immediate exercise. The delta values for very short maturities are, therefore, zero: small changes in the initial prices have no influence whatsoever on the option's value. As a consequence, the delta functions pertaining to off-peak power, gas and emission units all start at the origin. The delta parameter takes a value of zero for longer maturities, as well, since the effect of the initial prices fades away with time and the long-term mean prevails, as it is usual with mean-reverting models. Considering peak hours, the value of the spread as calculated using the initial prices is close to zero (the option is near-ATM). The initial value of the delta parameter for near-zero maturities is determined by the spread's formula: 1 for peak-period electricity, $-1/\eta$ for gas and $-\delta/\eta$ for emission units. With respect to longer maturities, the deltas will be zero for this time-of-day period, as well.

The following figure shows the sensitivity of the power plant's value to changes in the initial prices:

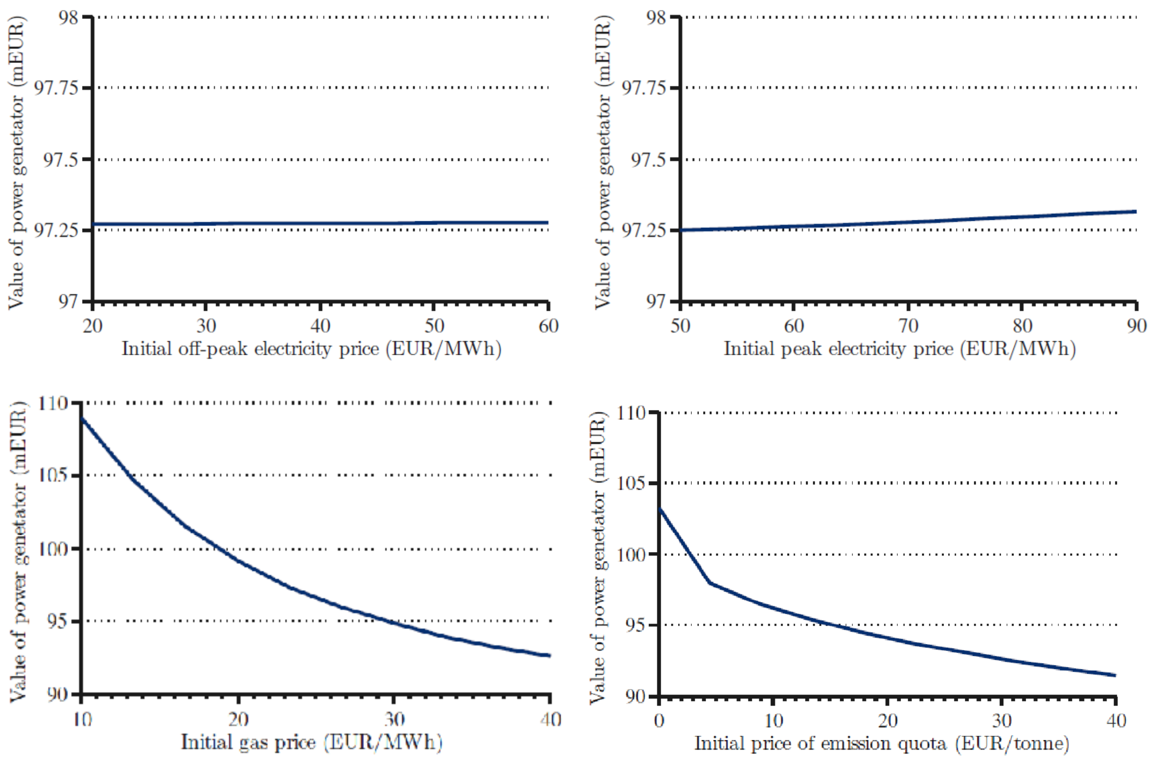

Figure 33: The value of power generator as a function of initial prices, top: off-peak (left) and peak (right) electricity, bottom: gas (left), eua (right).

It can be read from the figure that the spot prices used as a starting point for the simulation have a rather limited impact. The factor that the power plant's value is the least sensitive to is the initial price of electricity, for the high volatility and the high mean-reversion rate make the effect of the initial price fade away relatively fast. The impact is stronger for both gas and emission units: a 40 EUR/MWh gas price gives 93 million as the power plant's value, while the lower price of 10 EUR/MWh yields EUR 109 million. Under a near-zero quota price, the power plant is worth EUR 103 million, while a 40 EUR/tonne price results in EUR 91 million.

To demonstrate the relationship with initial prices, I also calculated – relying on the model and on past price data – historical figures for the value of the power plant. The rest of the parameters (long-term mean, mean-reversion rate, volatility, correlation) remained unchanged, that is, the stochastic model was not re-fitted again and again at each step. The results are shown on the figure below:

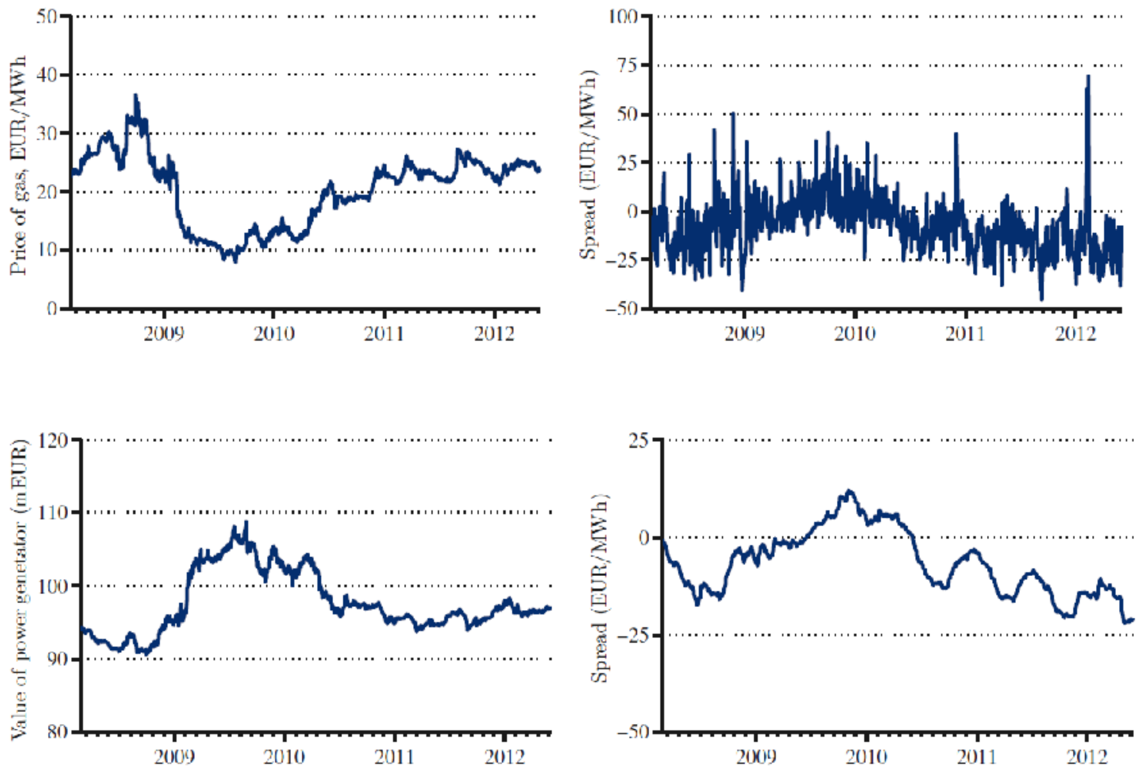

Figure 34: Modeled historical value of power plant. Top-left figure shows the historical prices of gas, top-right shows the daily spread. Bottom-left figure shows the model value of generator, the bottom-right shows the 21 day moving average of spread.

As we can see by contrasting the two graphs on the left-hand side, the price of the power plant, as given by the model, and gas price move in almost exactly opposite directions. The daily value of the spread, which fluctuates wildly because of the high volatility of the price of electricity, is less significant in effect as compared to fuel price, yet the graph of its 42-day moving average looks similar to that of the power plant's value. The historical prices of the power plant having been determined using the model, the relationships between the power plant's value and the historical factors were not analyzed using statistical methods.

The Role of Long-Term Means

In the geometric Ornstein-Uhlenbeck process, prices revert to their long-term means. For longer maturities, the spread options' value is far more sensitive to changes in the means than to changes in the initial prices. The options' sensitivities to the long-term means are illustrated by the following figure:

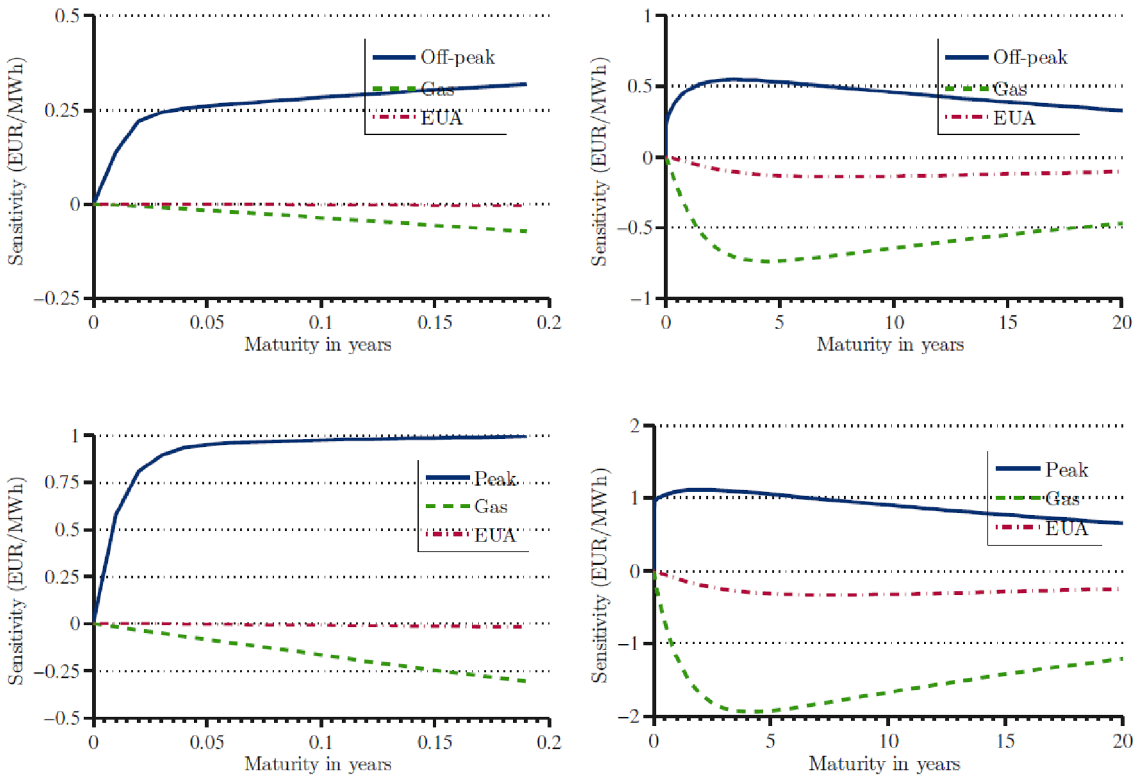

Figure 35: Spread options' sensitivity (EUR/MWh) on long term prices of underlying assets (off-peak hours (top), peak hours (bottom), shorter (left), longer (right) maturities).

The graphs clearly evince that in the mean-reverting model, the future gross margin realized by the power plant is not very sensitive to the initial prices, whereas the role of the long-term means is indeed significant. This conclusion is particularly valid for the fast mean-reverting electricity price. The mean reversion rate and the volatility of gas and emission unit prices are lower, thus here the spot prices have a remarkable impact, as well. For near-zero maturities, the options' sensitivities are zero, that is, the price of the option is, as far as short maturities are concerned, insensitive to changes in the long-term means. For longer maturities, the low sensitivities to the initial prices are accompanied by high sensitivities to the mean prices. Similar to what has been said with respect to the initial prices, the technological parameters incorporated in the spread's formula set an absolute upper limit for the sensitivities: 1 EUR/MWh for electricity, $-1/\eta \approx -2.63$ EUR/MWh for gas and $-\delta/\eta \approx -0.53$ EUR/MWh for emission units. The sensitivities decrease with increasing maturity (because of the discounting).

The power plant's value reacts to changes in the long-term means in the following ways:

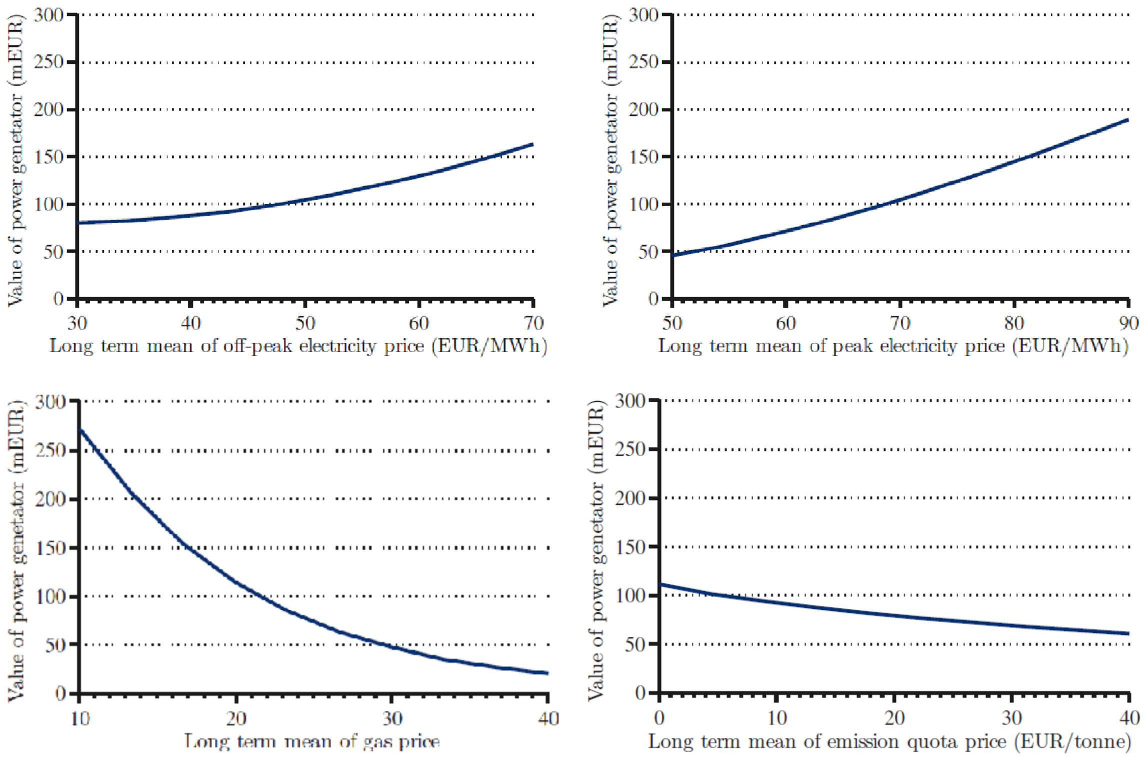

Figure 36: Value of power generator as a function of long term underlying prices. Top: off-peak (left) and peak (right) electricity, bottom: gas (left), eua (right).

In contrast to spot prices, long-term means have a significant impact on the value of the power plant. A rise in the mean price of electricity makes the power plant more valuable. Varying the off-peak figure, a mean price of 30 EUR/MWh yields EUR 80 million as the power plant's value, while at a mean price of 70 EUR/MWh, the power plant is worth EUR 170 million. The price of the power plant is somewhat more sensitive to variations in the long-term mean price of peak-period electricity: for a 50 EUR/MWh price, the approximate value of the power plant is EUR 50 million, while a figure of 90 EUR/MWh implies a value of EUR 185 million.

An increase in the long-term price of either gas or emission unit acts to decrease the power plant's value. A 40 EUR/MWh gas price yields EUR 21 million, while a 10 EUR/MWh figure leads to EUR 272 million. As regards the quotas, the effect is weaker: a 40 EUR/tonne price reduces the power plant's value to EUR 60 million, while a near-zero carbon credit price boosts it to EUR 111 million.

It is important to underline, again, that these effect were examined under the ceteris paribus assumption, which in this case means that the size of the power plant is negligible in comparison to the entire market and that changes in resource prices do not cascade through to the price of electricity.

The Role of Volatility

In capital markets, the measure of the variation of prices is the volatility (the standard deviation of returns). In the Ornstein-Uhlenbeck model, it is the parameter σ that scales the stochastic Wiener (dW) term, which stands for randomness in the formula. In real option scenarios, the effects of escalating/diminishing market uncertainty can be modeled by varying volatility. Crises and other types of market shock tend to increase the standard deviation of returns, while long calm periods typically decrease it. Let us now examine how a change in the degree of uncertainty influences the value of our spread options.

From amongst the Greeks, it is the *vega* parameter that indicates how the value of the option changes upon a unit change in volatility. By calculating the *vega*s based on the model's parameters and plotting them against maturity, we arrive at the following graphs:

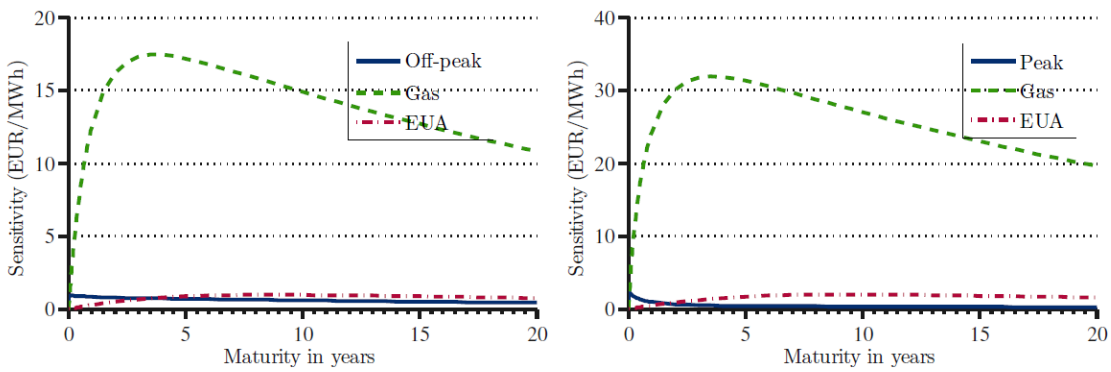

Figure 37: Spread options' sensitivity on volatilities of underlying instruments, left: off-peak, right: peak.

The resulting functions take positive values for all maturities, which might make us recall the general relationship that with respect to European-style call options, a higher standard deviation leads to a higher option price. The reason is that an increase in volatility acts to increase the probability of outstandingly high and outstandingly low prices occurring. For very low prices, the payoff function of the call option, which is bounded from below, will be zero, whereas high prices act to increase future cash flows. The resultant effect will be positive, both the expected payoff and the option's value increase with increasing volatility. From amongst the underlying assets examined, it is the volatility of gas the changes of which induces a significant increase in the spread option's price. The impact is stronger for the peak period than for off-peak hours.

Varying the price volatility of electricity between 100% – 800%, and that of gas and emission units between 10% – 100%, we arrive at the following prices for the power plant:

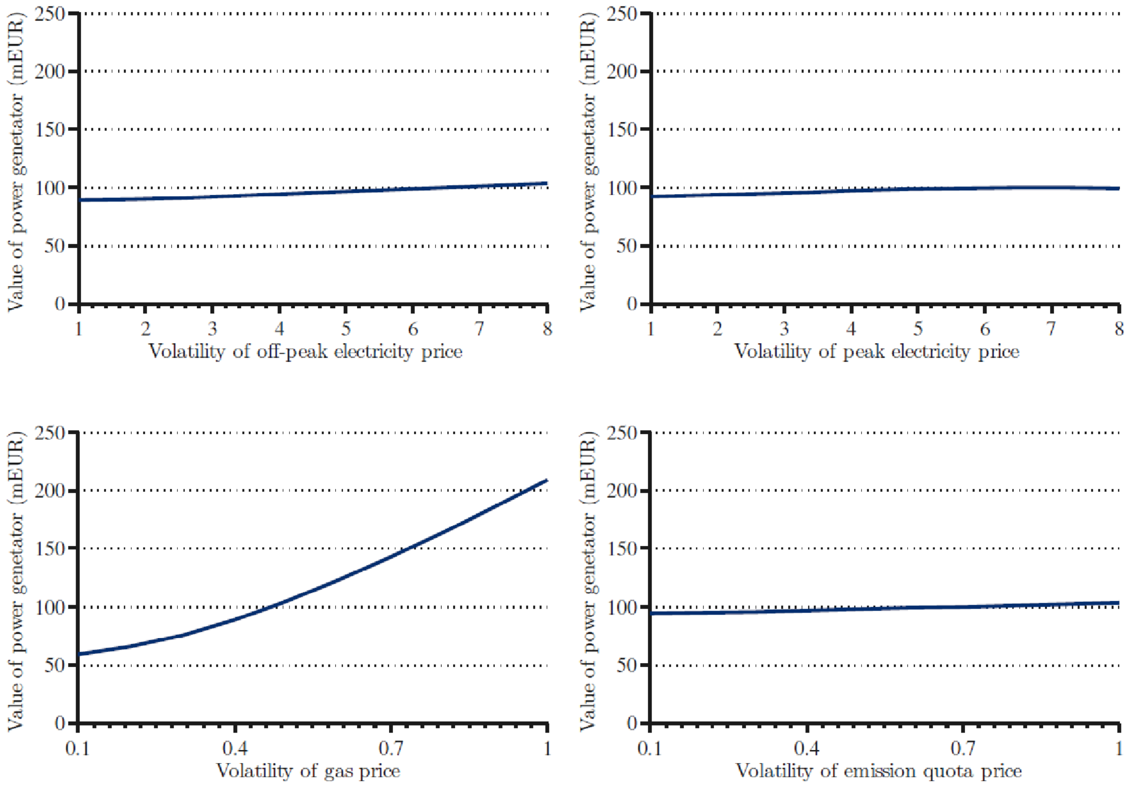

Figure 38: Value of generator as a function of volatilities. Top: off-peak (left) and peak electricity (right), bottom: gas (left) and eua (right).

As suggested by the figure, an increase in the volatility of electricity does not significantly affect the value of the power plant, even though the value of the parameter was varied across an extremely wide range. The reason lies in the mean-reversion rate: because of the high value of parameter λ, even a significant change in the volatility of electricity can only have a moderate impact on the expected value of the future spread (and, hence, on the prices of the options). It is the volatility of the gas price that has the strongest effect: if the initial figure of 45% is reduced to 10%, the power plant's value falls to EUR 60 million, while if raised to 100%, the price goes up to EUR 209 million. Regarding the quotas, the effect is weaker, as compared to that of gas: reducing the initial volatility of 44% to 10% yields an EUR 95 million figure for the power plant's value, while a 100% quota volatility boosts it to EUR 104 million.

The Role of Correlation

In the stochastic model, it is the *Wiener term* (dW) that represents uncertainty. Considering the multi-dimensional case, the linear correlation coefficients pertain to the connectedness of the dW terms. In the fitted model, it was the correlation coefficient between peak and off-peak electricity price and that between the prices of gas and emission units that were significant. In the four-asset model, we employ two spread options, each having three underlying assets. The model is centered around the following expected values:

$$E\left[so_{off-peak}^{PO}\left(S_{off-peak}, S_{gas}, S_{eua}\right)\right] + E\left[so_{peak}^{PO}\left(S_{peak}, S_{gas}, S_{eua}\right)\right] \qquad 57.$$

In the model, the two prices of electricity, peak and off-peak, are part of two separate options, and the correlation between the two periods' prices has no influence on the results. However, the connection between changes in the quota price and changes in the price of gas does affect the results. The reason is that the two stochastic variables S_{gas} and S_{eua}, which represent the prices of gas and emission units, respectively, are both factors of the expected value, as the payoff function of the option is a non-linear function of the underlying assets' prices. Therefore I will only analyze sensitivity with respect to the correlation coefficient between the prices of gas and carbon credits. The results are shown in the figure below, for options with varying maturities:

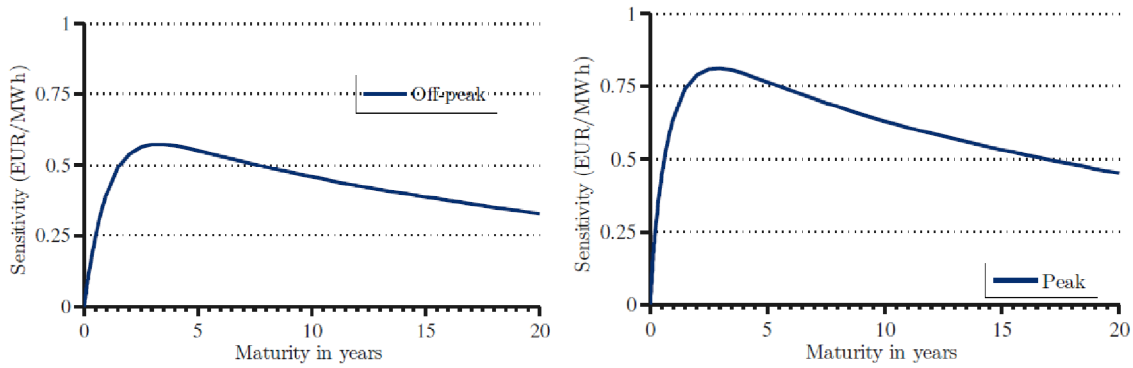

Figure 39: Sensitivity of spread options on correlation coefficient between gas and eua yields (left: off-peak, right: peak).

An increase in the correlation coefficient increases the value of the options; the figure is higher for the peak period than for off-peak hours. Given that correlation, similar to thermal efficiency, is measured in percentages, the consideration mentioned there is also valid in this case: the function will be easier to make sense of if we divide the resulting function values by 100 and interpret the sensitivity values as pertaining to percentage point changes in the correlation coefficient. The power plant's price is approximately hundred times less sensitive to this correlation than it is to thermal efficiency, that is, whatever the value of the correlation coefficient, the price remains practically unchanged:

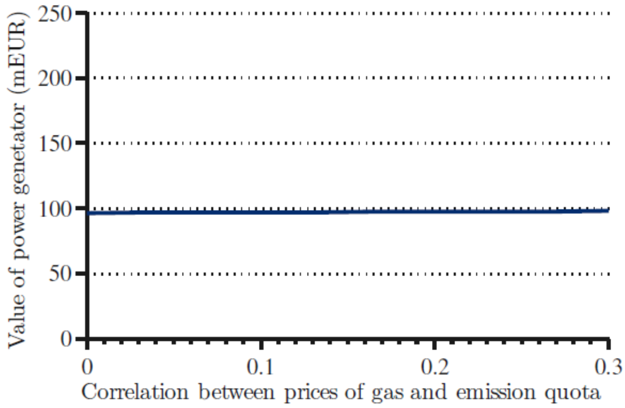

Figure 40: Effect of correlation coefficient between gas and eua on the value of power generator.

Having explored the sensitivities, let us take a look at how the operator of the power plant may mitigate the market risks arising from the fluctuation of prices.

V.1. Dynamic Hedging of the Power Plant

The spread option's sensitivity to the underlying asset is represented by the *delta* parameter, which is the partial derivative of the spread option with respect to the spot price. Considering our two separate three-asset options for the two time-of-day periods, we have four delta parameters in total.

If we do not wish to expose our power plant to unjustifiable risks, the position needs to be hedged against price fluctuations that would induce changes in its value. One of the most widely used techniques for hedging the risks associated with derivatives is the *delta hedge*, by which we aim at eliminating the risk arising from changes in the price of the underlying asset. This type of strategy requires us to enter hedging contracts the delta parameters of which take the same value but opposite sign as the position we intend to hedge. This way we can set the resultant delta parameter of the entire portfolio (the sum of the hedge position and the one to be hedged) close to zero, which renders this combined portfolio virtually insensitive to changes in the price of the underlying asset. Assuming a perfectly hedged position, the loss (profit) we make on the hedge contract in the case of a change in the price of the underlying asset equals the profit (loss) we make on the portfolio to be hedged, i.e. the total effect is approximately zero. Since the delta parameter tends to continuously change with time, the hedge positions need to be adjusted dynamically. The more frequent the adjustment of the hedge contracts, the more precise the hedging will be. With transaction costs taken into account, too frequent adjustments may be very costly, which is why in practice, professionals opt for a "reasonable" adjustment schedule, where the exact meaning of "reasonable" depends on the changeability of the delta parameter and the transaction costs (Benedek, 1999).

The only modification we need to make to the "default" dynamic *delta hedging* strategy to tailor it to our needs is that the power plant's initial portfolio, made up of a derivative the value of which depends on four underlying assets, requires us to calculate a delta parameter for each one of these assets, and that the appropriate hedging contracts will, as well, need to be entered into for all four underlying assets.

The Optimal Quota Position

Upon the launch of the emissions trading system, the participants received a remarkable amount of emission units from the authorities free of charge. However the actors, for the most part, did not really know how many quotas they should keep and how many they should sell on the market. Relying on the dynamic hedging strategy, we can determine the "optimal" amount of quotas to be held. Drawing from the heuristic approach, one might be inclined to think that the amount of emission units held with the purpose of covering our future emissions should correspond (i.e. be equal) to the expected volume of our emissions. This is, however, only an approximation of the amount to be held. The correct solution is to hold an amount of quotas that corresponds to the delta parameter pertaining to the emission units – which quantity will indeed be close to the expected volume of emissions. The amount needed to cover our current, factual past emissions will, of course, need to be added to the optimal quota position so determined.

The relationship between expected emissions and the delta parameter may be illustrated by way of an example including European-style vanilla call options. According to the

Black-Scholes formula – if r is the risk free interest rate, T denotes the maturity, S_0 stands for the initial price and X corresponds to the exercise price – the value of a call option is (Hull, 1999, p. 334.):

$$c = \exp(-r\,T) \cdot [S_0 \cdot \exp(r\,T) \cdot N(d_1) - X \cdot N(d_2)] \qquad 58.$$

The value of the expression $N(d_2)$ in the formula equals the probability of exercise, that is, the probability P that the spot price at maturity S_T will exceed the exercise price X:

$$N(d_2) = P(S_T > X) \qquad 59.$$

The option valuation formula may be expressed in a form that is easier to interpret; this includes the discounted expected value of the option's payoff:

$$c = \exp(-r\,T) \cdot \Big[\big(1 - P(S_T > X)\big) \cdot 0 + P(S_T > X) \cdot [E[S_T|S_T > X] - X]\Big] =$$

$$= \exp(-r\,T) \cdot P(S_T > X) \cdot ([E[S_T|S_T > X] - X]) \qquad 60.$$

That is, we can obtain the price of the option by multiplying the probability of exercise by the difference of the conditional expected value at maturity and the exercise price, and then calculating the present value of the figure so obtained[10].

These formulae can be used to express $N(d_1)$:

$$N(d_1) = \frac{E[S_T|S_T>X]}{S_0 \cdot \exp(r\,T)} \cdot P(S_T > X) = N(d_2) \cdot \frac{E[S_T|S_T>X]}{S_0 \cdot \exp(r\,T)} \qquad 61.$$

That is, we can obtain the value of $N(d_1)$ by adjusting the probability of exercise by the quotient of the conditional expected value of the price at maturity and the forward price.

In this example, $N(d_2)$ corresponds to the probability of production – a simple multiplication of which gives the expected volume of emissions –, while $N(d_1)$ is equivalent to the delta parameter. These two values are very close – but not equal. This is why, for the sake of more precise hedging, we should hold an amount of quotas that corresponds to the delta parameter, and not to the expected volume of emissions.

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[10] In arbitrage-free pricing, expected values are determined using the risk-neutral measure, while for discounting purposes, we are to use the risk-free interest rate.

V.2. Value of Efficiency Improvement Projects

The value of the power plant as an asset depends on its revenue generating capacity, which can be derived, according to our model, from the sum of the cumulative values of the two three-asset spread options for a given interval. If the power plant improves the efficiency of its production, the amount of gas required to produce a unit of electric energy will be smaller, which acts to also reduce carbon dioxide emissions and, hence, the necessary quantity of quotas, as well. The value of the spread will grow, thus the company is likely to realize a higher gross margin, and the number of days when the facility actually generates power and makes a profit will increase.

Let us assume an efficiency improvement project that increases the value of parameter η from the "default" 38% to 43%. Here, the value of the investment will be equal to the change in revenue generating capacity, that is, the difference between the facility's value assuming a thermal efficiency of 43% and that under the original 38% figure:

$$V_{Project} = V_{PowerPlant}(\eta = 43\%) - V_{PowerPlant}(\eta = 38\%) \qquad 62.$$

Given a thermal efficiency of 43%, the model yields a price of EUR 125.5 million for the power plant, which we have to diminish by the EUR 97.3 million figure that corresponds to the default efficiency level of 38% in order to arrive at the value of the project: EUR 28.2 million.

Let us examine the factors that the value of this 5 percentage point efficiency improvement depends on. Changes in the initial prices of the underlying assets affect the value of the investment as follows:

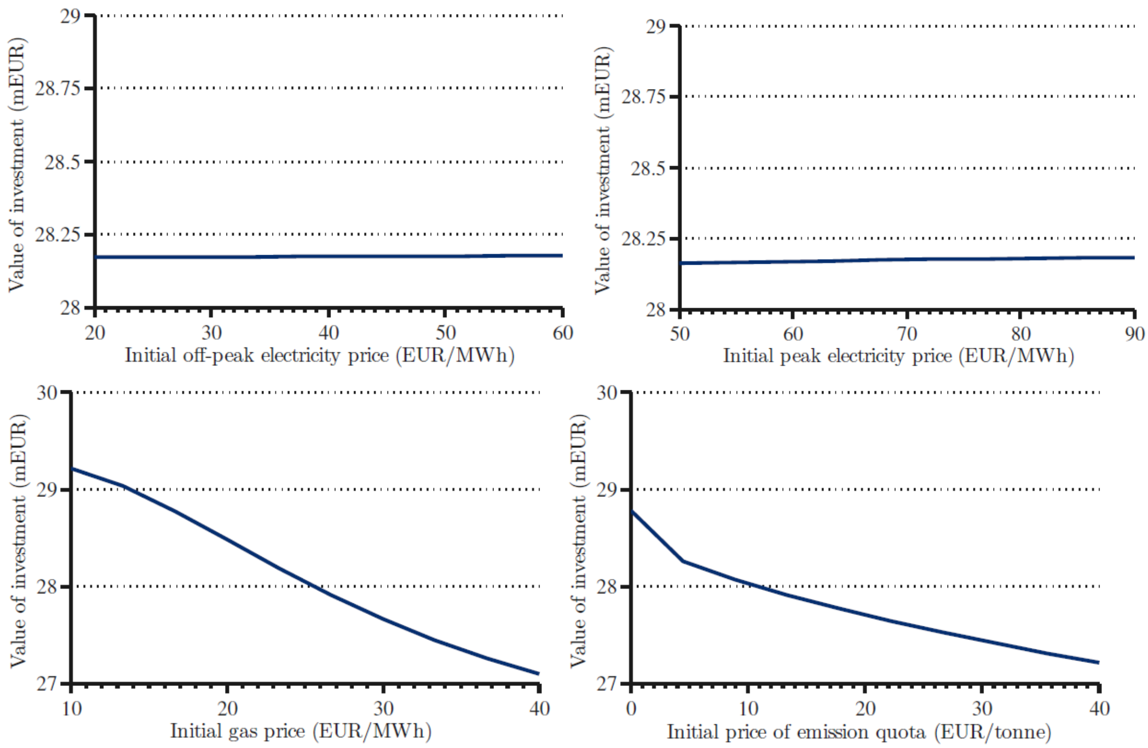

Figure 41: Effect of initial prices on value of thermal efficiency improvement project.

As it is apparent, the initial prices of electricity have no influence on the value of the 5 percentage point efficiency improvement project. An increase in the price of either gas or emission units leads to the savings from more efficient production becoming slightly smaller. The role of the long-term means is illustrated below:

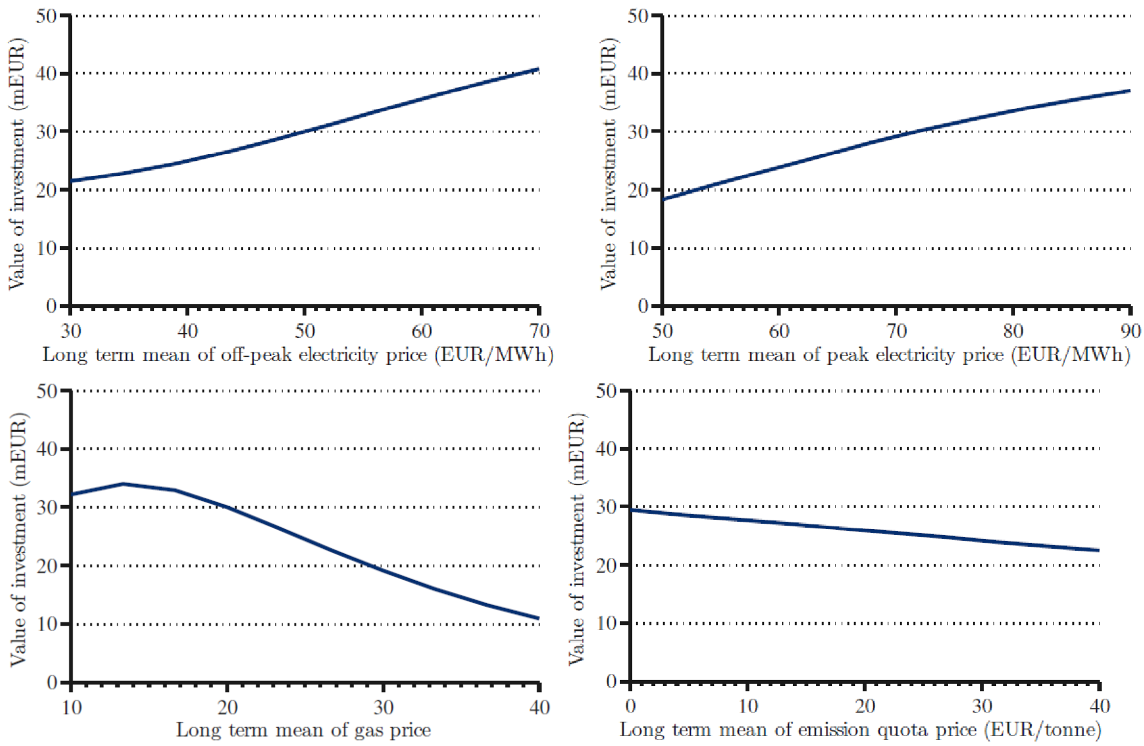

Figure 42: Role of long term means in determining the value of efficiency improvement project.

Long term price means have a rather significant influence on the value of the efficiency improvement project. Considering electricity, a ceteris paribus shift of about +/- 20 EUR/MWh boosts the value of the investment to EUR 40 million or cuts it back to EUR 20 million, respectively. As long as it is in a relatively low range, an increase in the long term mean of gas price induces a slight growth in the savings, while in higher ranges, the value of the investment clearly falls. The reason is that if the gas price is low, the turbine is up and running most of the day, thus an increase in the price of gas means bigger savings. Further gas price hikes, however, act to significantly diminish the possibility of production, and thereby reduce potential savings – that is, the value of the project. An increase in the mean of emission unit price lessens the value of the investment, yet the effect is more moderate than for gas.

The role of volatility can be illustrated as follows:

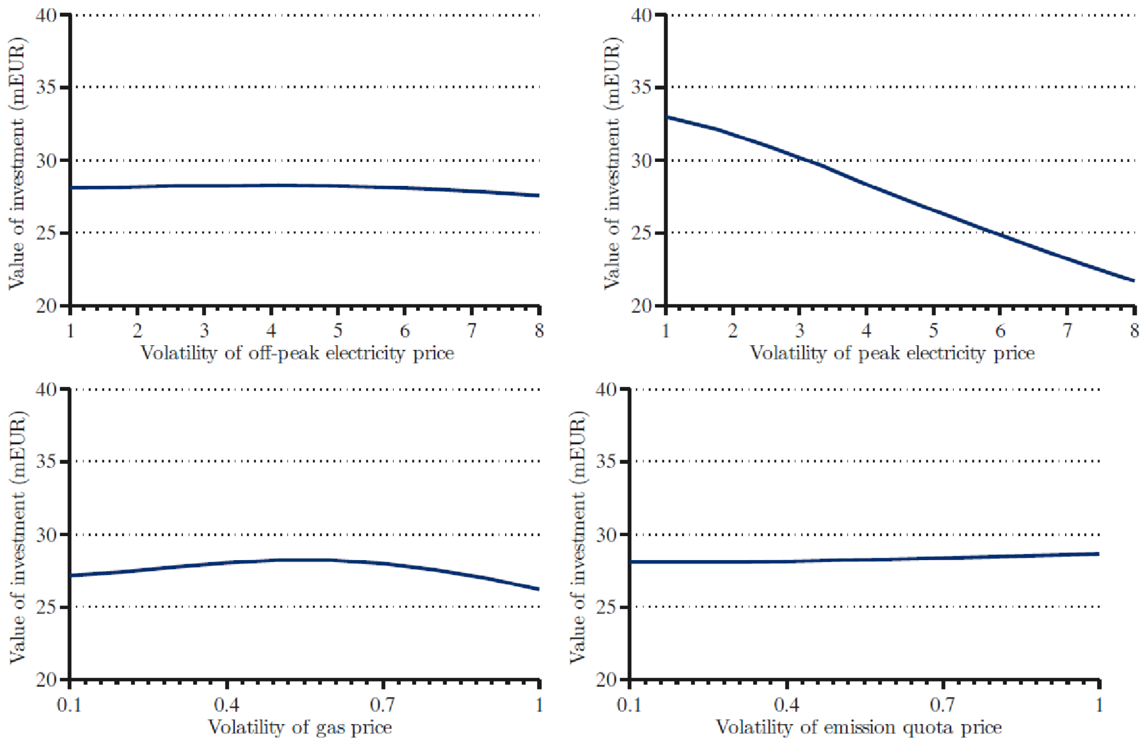

Figure 43: Role of volatilities in determining the value of efficiency improvement project.

Out of the four underlying assets, it is peak-period electricity the volatility of which has the most significant impact: an increase in volatility decreases the value of the project. The effect of price volatility is not significant in the case of off-peak electricity and emission units. An increase in the volatility of the price of gas initially induces a slight increase and then a slight decrease in the value of the project.

I also looked into how the value of the efficiency improvement project changes as a function of the correlation coefficient between the price of gas and that of emission units. According to the model, there is no significant relationship between the two.

V.3. The Production Decision in the Presence of Long-Term Supply Agreements

In the Hungarian electric energy market, long-term supply agreements are quite commonplace, that is, it is typical for power plants to sell a portion of their capacities for a long interval at a pre-determined price, or in other words: to assume fixed-price supply obligations. Many a time, power plant operators are inclined to interpret supply obligations as actual production obligations, that is, they keep on generating power according to the schedule stipulated in the contract irrespective of current market prices. Below we will show that, given a turbine that can be flexibly operated, this approach is false.

With technological constraints not taken into account, a profit maximizing power plant generates electricity only if and when its gross margin per unit of energy generated is positive. Let us examine whether this profit maximization condition changes if the power plant assumes a long-term supply obligation.

Hlouskova et al. (2005) call our attention to the two most obvious consequences of electricity market liberalization: the formerly prevailing fixed price was replaced by a highly volatile market price, and production facilities now have the option to not meet their contractual obligations by generating electric energy themselves but to buy the required amount of electricity in the marketplace. This rendered the production decision independent from any potential supply obligations.

Fixed-Price Supply Agreement

Let us first see what happens if the company sells electricity at a pre-determined (fixed) price (denoted by C) under a long-term agreement.

The supply obligation is equivalent to a forward contract, the value of which on the maturity date τ is:

$$C - S_{pow}(\tau) \qquad 63.$$

If the decision makers at the company interpret the supply obligation as a production obligation, then the company's profit on day τ will be equal to:

$$\pi'(\tau) = C - S_{gas}(\tau)/\eta - S_{eua}(\tau) \cdot \delta/\eta - v \qquad 64.$$

If however they follow the line of thought presented earlier and regard the power plant as a production opportunity, then the profit can be obtained by adding the supply obligation to the payoff function of the relevant real option:

$$\pi''(\tau) = \left[C - S_{pow}(\tau)\right] + max\left(S_{pow}(\tau) - S_{gas}(\tau)/\eta - S_{eua}(\tau) \cdot \delta/\eta - v, 0\right) =$$

$$= max\left(C - S_{gas}(\tau)/\eta - S_{eua}(\tau) \cdot \delta/\eta - v, C - S_{pow}(\tau)\right) \qquad 65.$$

It can be easily shown that $\pi''(\tau) \geq \pi'(\tau)$, since

$$max\left(C - S_{gas}(\tau)/\eta - S_{eua}(\tau) \cdot \delta/\eta - v, C - S_{pow}(\tau)\right) \geq$$

$$\geq C - S_{gas}(\tau)/\eta - S_{eua}(\tau) \cdot \delta/\eta - v \qquad 66.$$

As it is apparent from the above, the conditional (i.e. dependent on the daily value of the spread) operation of the power plant yields higher profits than the approach that equates the supply obligation to a production obligation. The reason for which is that in the case of a negative spread, the company is better off having its equipment idle and meeting its supply obligation with electricity purchased in the market.

Variable-Price Supply Agreement

The above conclusion remain valid even if the supply agreement does not stipulate a fixed price, but one that depends on current production costs. For the sake of simplicity, let us assume that exercise price *C'* is equal to the sum of the production costs increased by the profit per unit of energy generated (denoted by *h*). The future price of the energy generated will be:

$$C'(\tau) = S_{gas}(\tau)/\eta + S_{eua}(\tau) \cdot \delta/\eta + v + h \qquad 67.$$

The agreement corresponds to the forward sale of the spread at an exercise price of $v + h$. In this case, unconditional production will yield the following profit per unit of energy generated:

$$\pi'(\tau) = C' - S_{gas}(\tau)/\eta - S_{eua}(\tau) \cdot \delta/\eta - v = h \qquad 68.$$

Under a fixed production schedule, the power plant realizes a fixed profit. If the facility is, however, operated depending on the daily value of the spread, then the unit profit will be equal to:

$$\pi''(\tau) = [C' - S_{pow}(\tau)] + max(S_{pow}(\tau) - S_{gas}(\tau)/\eta - S_{eua}(\tau) \cdot \delta/\eta - v, 0) = max(h, C' - S_{pow}(\tau)) \qquad 69.$$

It still holds that $\pi''_{pow}(\tau) \geq \pi'_{pow}(\tau)$, since

$$max(h, C' - S_{pow}(\tau)) \geq h \qquad 70.$$

As we have just shown, the company is still better off generating power only if and when the spread is positive. The reason is that in the case of a negative gross margin, the company gains more than the guaranteed profit of h if it meets its supply obligation by way of a market transaction.

If the company is worse off with constant production, it is absolutely reasonable to ask how much the loss suffered amounts to. If and when the spread is positive, the payoffs from the two production strategies (fixed schedule vs. operation depending on daily spread) match. The loss associated with the fixed schedule comes from the potential that the power plant may be generating energy even if and when the spread derived from the daily prices is negative. In such a case, it would be better off stopping production and meeting its supply obligation through the marketplace.

Under constant production, the capacity of the power plant corresponds to a series of forward spread swaps, the exercise price of which is v. If the power plant is in constant operation, then the loss – as compared to following the profit maximization condition – incurred on each unit of energy generated (*lcp*, loss of constant production) on an arbitrary future day τ is equal to the difference between the values of the spread option (*spo*) and the spread swap (spread swap, *ssw*) with the same exercise price:

$$lcp(\boldsymbol{S}(0), \boldsymbol{w}, v, \tau) = spo^{Pr}(\boldsymbol{S}(0), \boldsymbol{w}, v, \tau) - ssw(\boldsymbol{S}(0), \boldsymbol{w}, v, \tau) \qquad 71.$$

The amount of the loss is independent from the price stipulated in any potential long-term agreement. For the supply agreement has a value in itself – irrespective of the power plant –, which can be realized even if the turbine is switched on/off depending on daily market prices.

The cumulative loss of constant production for a given period *(0,T)* equals the product of the average of the losses pertaining to the two time-of-day periods and the maximum daily capacity:

$$\frac{\Gamma}{2} \cdot \sum_{\tau=0}^{T} \left[lcp\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right) + lcp\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right) \right] \qquad 72.$$

The options in the formula can only take positive values, whereas the swap transactions might as well be negative. Therefore the loss of constant production may, in absolute terms, exceed the cumulative option value for the interval in question. Assuming constant production, the loss that the power plant in our model incurs is, for the intervals of 1, 3 and 5 years:

Values in million Euros	**1 yr**	**3 yr**	**5 yr**
Option value (spo)	3.34	12.09	21.04
Value of swaps (ssw)	-7.05	-16.17	-22.79
Loss caused by constant production (lcp)	**10.39**	**28.26**	**43.82**

Table 5: Relative loss of constant production in 3 different cases (the power plant operates constantly for 1, 3 and 5 years).

The longer they keep the power plant constantly running, the bigger the loss they make. Below, we will determine how various technological and market factors affect the loss of constant production.

The dependence of the loss of constant production on thermal efficiency can be illustrated by the following graph:

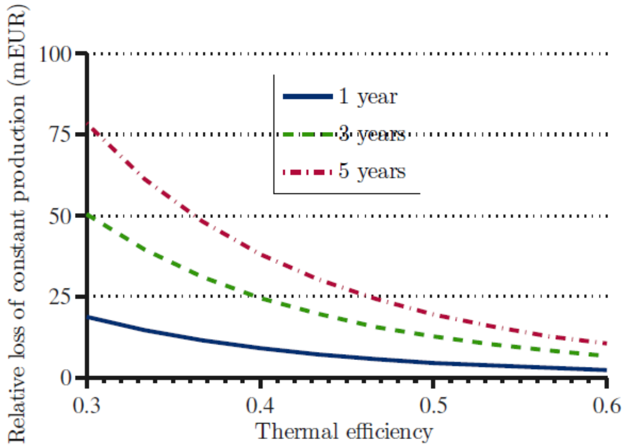

Figure 44: Effect of thermal efficiency on loss of constant production.

As thermal efficiency increases, the loss of constant production gets smaller. The reason for which is that an increased efficiency means that the probability of a negative spread is smaller, thus a situation when it is rational to have the turbine rest and meet one's supply obligation through the market occurs less frequently.

The following figures illustrate the roles of the initial prices:

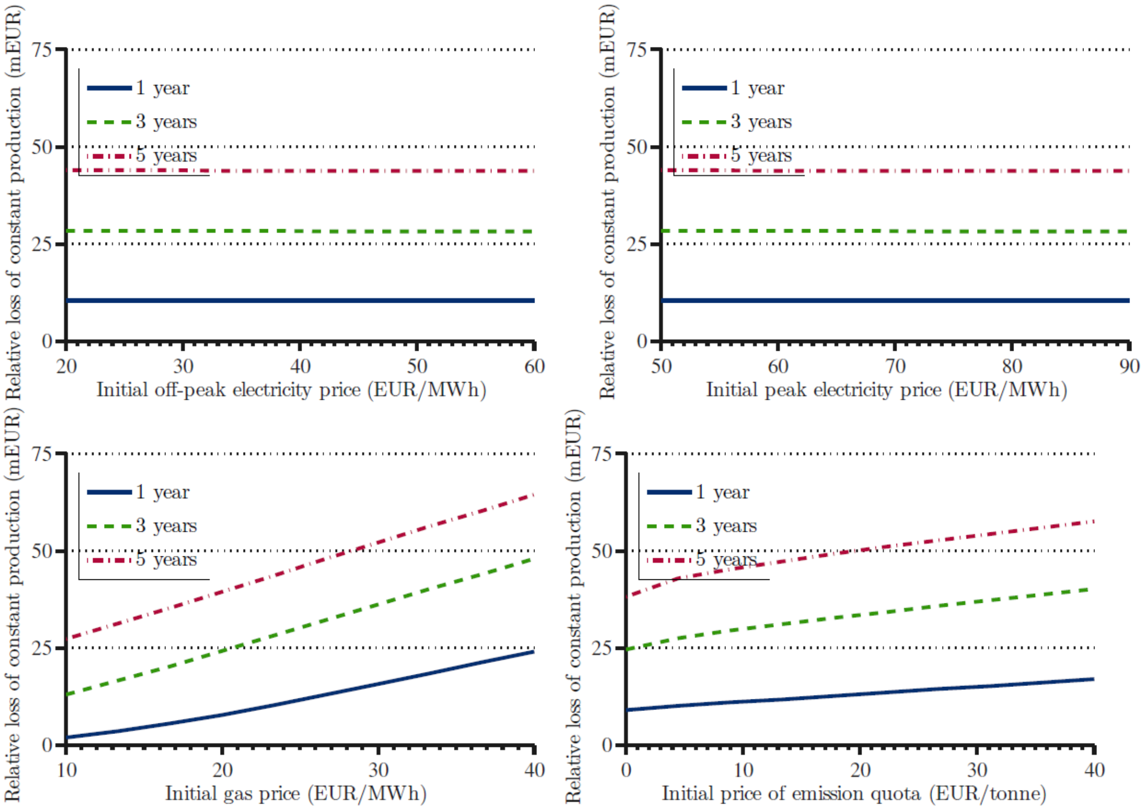

Figure 45: Effect of initial prices on loss of constant production.

Because of the reasons already mentioned in earlier chapters (high mean reversion rate and volatility), the initial prices of electricity have no impact on the loss of constant production. An increase in the prices of the inputs with lower mean reversion rates does, however, induce a significant rise in the amount of the loss. The reason is that the higher resource costs increase the probability of a negative spread and, therefore, the proportion of the days when the power plant would be better off meeting its supply obligations by way of market transactions – instead of constant production – will be higher, as well.

The long-term means have the following impact:

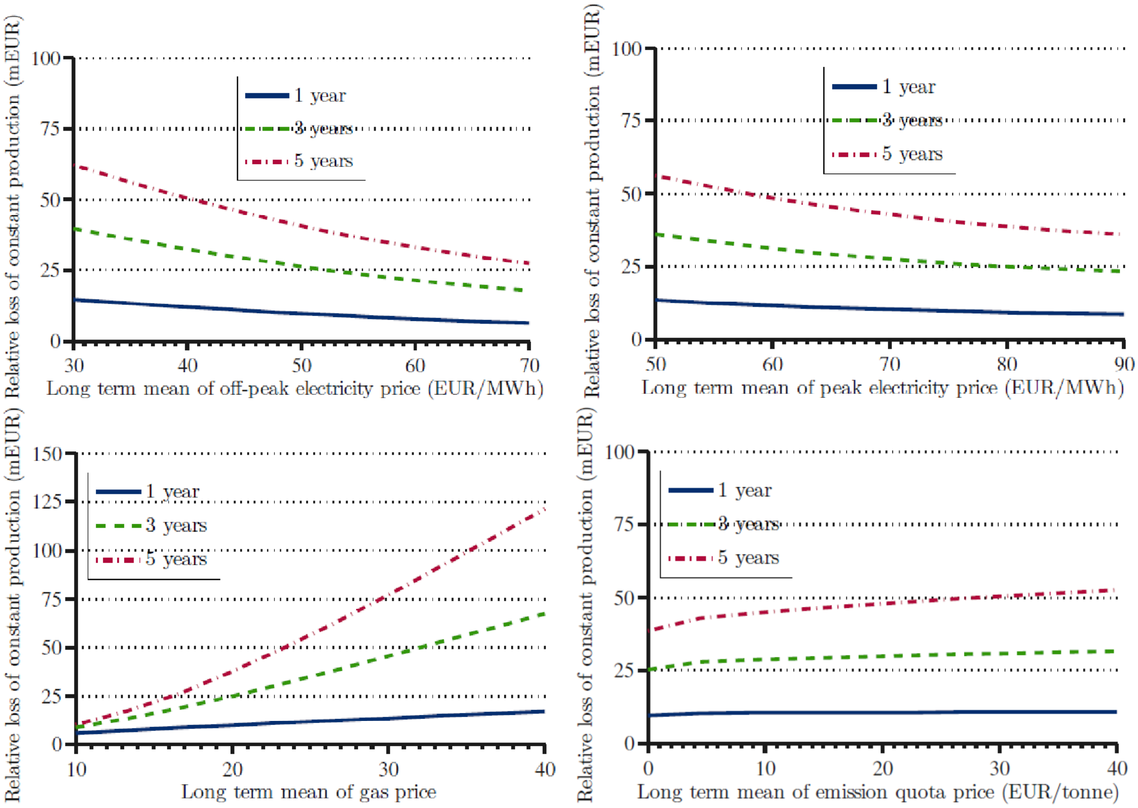

Figure 46: Effect of long term means on loss of constant production.

An increase in the long-term mean of electricity price reduces, while the same change in inputs increases the loss of constant production. The resulting values are particularly sensitive to increases in the mean price of gas.

Changes in the volatilities influence the loss of constant production as follows:

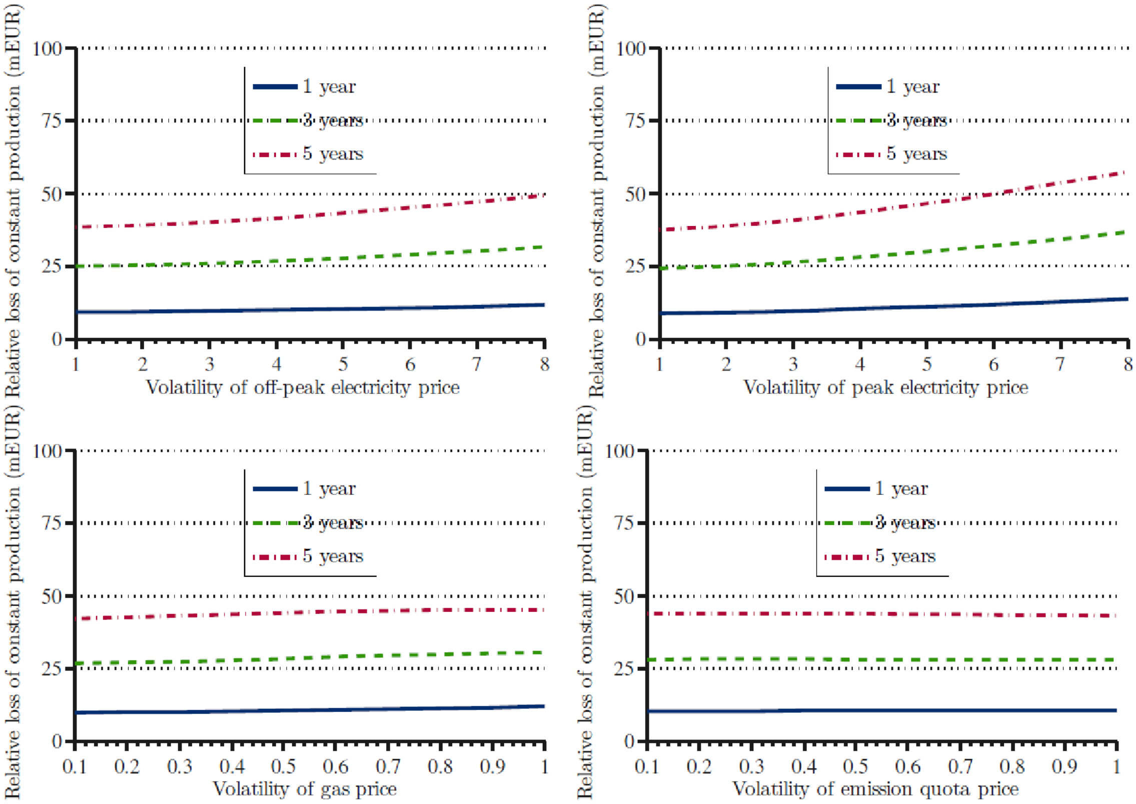

Figure 47: Effect of volatility on loss of constant production.

As regards electricity, an increase in volatility renders the loss of constant production higher, while the value examined was not significantly affected by the variation – in a relatively narrow range – of the volatility of neither gas, nor carbon allowances.

The impact of the correlation coefficient between gas price and allowance price is negligible.

VI. DERIVING THE MNPB AND THE INDIVIDUAL ALLOWANCE DEMAND FUNCTIONS

In the third trading phase (2013-2020), policy makers intend to allocate an increasing share of the emission allowances for the given phase among the participants by way of auctions. In order for the power plant to be able to develop the right strategy for the auctions, it will need to know its own reservation price, that is, the maximum amount that the emission units are worth for them.

For the power plant, the emission allowances cannot be worth more than the amount of profit they can realize on a unit of carbon dioxide emission: the reservation price of the quotas equals the value of the spread that does not include the per unit cost of the allowances. It is the first emission unit that the power plant is willing to pay the most for, as the marginal profit to be realized gets lower and lower with each additional quota.

The MNPB (Marginal Net Private Benefit) function mentioned in the introduction shows how much the company's profits increase upon a unit increase in production (or pollution). The function pertains to the revenue before quota costs and shows the value of the spread (without the cost of emission units) that the company can expect to realize on a unit of pollution. We can obtain the MNPB function by sorting the reservation prices for the individual allowances in descending order.

Assuming a quota market of infinite liquidity, the potential buyers at the auction will not place a bid higher than the market price, after all, they can buy the desired amount at that price in the marketplace. Nonetheless, given the opportunity to sell the units at the market price, it is not worth for the authorities to sell them for less. Is the liquidity of the market low, however, the alternative of selling/buying in the market is available to the participants to a limited extent only. If the authorities opt for an auction-only allocation, the market liquidity of spot transactions will definitely be low in the beginning. In these cases, the market price represents less of an upper limit with respect to large-volume purchases. Hereinafter, I will only examine the case when the company exclusively relies on the MNPB function to arrive at the price to be offered at the auction, without taking the market price of quotas into account, that is, the upper price limit that the current market price might constitute will not be considered.

The *individual allowance demand function* shows the maximum *per unit price* the power plant is willing to pay for a given amount of emission units. The demand function can be obtained by integrating the MNPB function up to the quantity in question and dividing the resulting value by this quantity.

The maximum amount to be paid for a given quantity of allowances is equal to the value of the MNPB function's integral up to that quantity or, deriving it from the individual demand function, the product of that given quantity and the respective price.

Future MNPB values can be derived from the real option model. The price of the option $spo2^{Pr}$ on the weighted price difference between electricity and gas on a given future day gives us the gross margin (without the carbon cost) the power plant can expect ro realize on the energy generated. Each unit of energy generated results in an emission of volume δ/η. The gross margin realized on each unit of emission can be obtained by multiplying the gross margin realized per unit of energy by η/δ. The reservation price P^{eua} for a unit of emission on a future day τ will be:

$$\mathrm{P}^{\mathrm{eua}}_{\mathrm{off-peak}}(\tau) = \eta/\delta \cdot spo2^{Pr}_{off-peak}\left(\boldsymbol{S_{off-peak}}(0), \boldsymbol{w}, v, \tau\right)$$

$$\mathrm{P}^{\mathrm{eua}}_{\mathrm{peak}}(\tau) = \eta/\delta \cdot spo2^{Pr}_{peak}\left(\boldsymbol{S_{peak}}(0), \boldsymbol{w}, v, \tau\right)$$

$$\boldsymbol{w} = [1 \quad -1/\eta] \qquad \boldsymbol{S_{peak}}(0) = \begin{bmatrix} S^{peak}_{pow}(0) \\ S_{gas}(0) \end{bmatrix} \qquad \boldsymbol{S_{off-peak}}(0) = \begin{bmatrix} S^{off-peak}_{pow}(0) \\ S_{gas}(0) \end{bmatrix} \quad 73.$$

The model includes separate reservation prices for peak-period and off-peak-period emissions. The reservation price pertaining to past (already emitted) emissions differs from both. The power plant has no way of undoing its past emissions (no technology to capture atmospheric carbon dioxide). The reservation price for this case, therefore, is to be derived from the conditions of non-compliance: in the Second Phase, they need to pay a 100 euro fine and its quantity of quotas for the next year will be diminished by the appropriate amount. Here, the reservation price can be approximated with the sum of the fine and the expected gross margin for the future period in question. For the ease of understanding, I will only deal with future emissions, that is, I will assume that the power plant intends to determine the reservation price right at the start of the year for its emissions during the year ahead.

The prices of the option on the spread between electricity and gas as a function of maturity are:

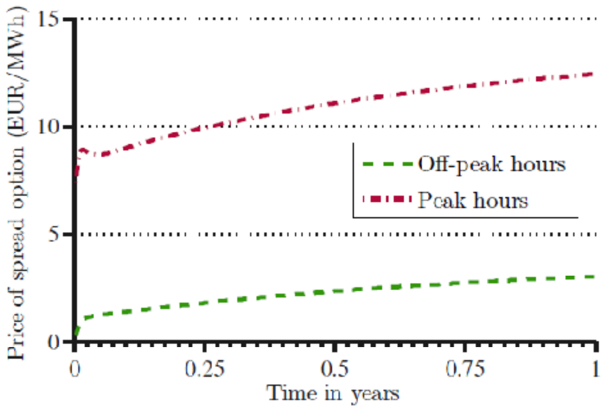

Figure 48: Two-asset spread option values as a function of maturities.

The off-peak option value is close to zero for very short maturities, because during this period, the spread calculated from the initial prices is negative even without the quota cost. The peak-period value starts much higher, indicating that the value of the profits to be realized is much higher during these hours.

By multiplying the resulting option prices by the factor η/δ, we obtain the reservation prices for the emission units, i.e. the MNPB values (without the cost of the allowances). The values pertain to half-day periods, the corresponding quantities for which are $\frac{\Gamma}{2} \cdot \delta/\eta$. By sorting the resulting values in descending order, we arrive at the MNPB function, which, if integrated and divided by the respective quantities, leads us to the individual demand function:

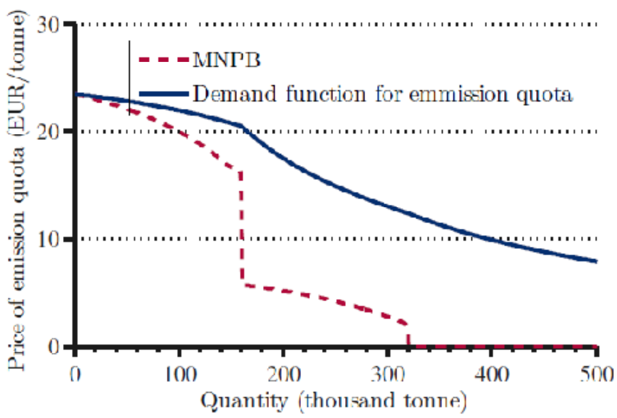

Figure 49: MNPB on emission unit (calculated without emission cost) and the individual demand function for the annual emissions.

The maximum annual volume of emissions is 321 thousand tonnes. Above that value, the marginal benefit to be realized is zero, because the power plant cannot possibly produce more than its maximum emission. The resulting individual demand function sets off from a value of around 23 euro/ton, which belongs to the most valuable peak-period spread option. With increasing quantity, the MNPB function starts to sink, first slowly and then, as we "run out" of peak-period spread options, more steeply.

Let us now examine, as we have done before, how changes in thermal efficiency, input prices and volatilities affect the MNPB and the individual demand functions.

Evaluating the functions for a +/-5 percentage point change in thermal efficiency, we arrive at the following results:

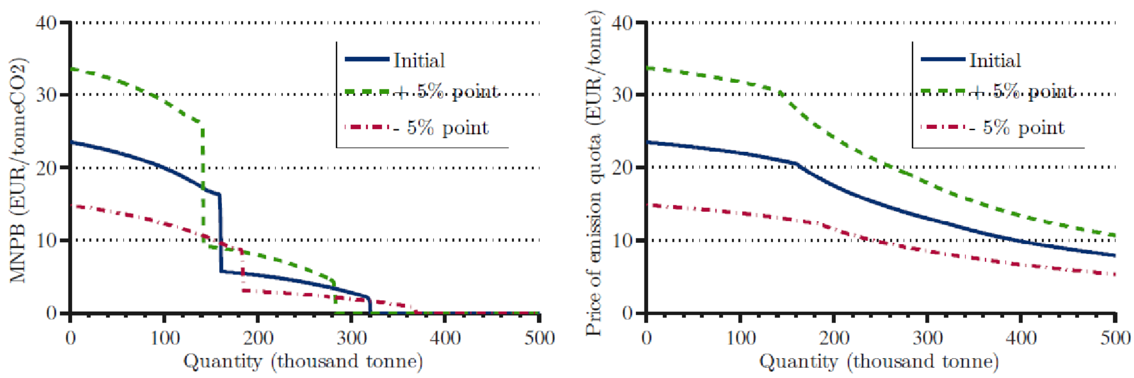

Figure 50: Shifting of MNPB (left) and individual demand function (right) caused by +/- 5 percentage point change in thermal efficiency.

On the one hand, an improvement in thermal efficiency shifts the MNPB function upwards due to the improvement in the spread's value and, on the other hand, shifts the x-axis intersection point to the left due to the reduction in the maximum daily volume of emissions. The quota demand function is shifted upwards. The deterioration of the thermal efficiency has the exact opposite effect.

Let us now see how a change in the price of electricity affects the MNPB and the individual demand functions. Given that electricity is extremely volatile and fast mean-reverting in the model, I did not analyze a shift in the spot price separately, but modified it along with the long-term mean, by +/- 25 euro/MWh. The resulting functions are shown in the figure below:

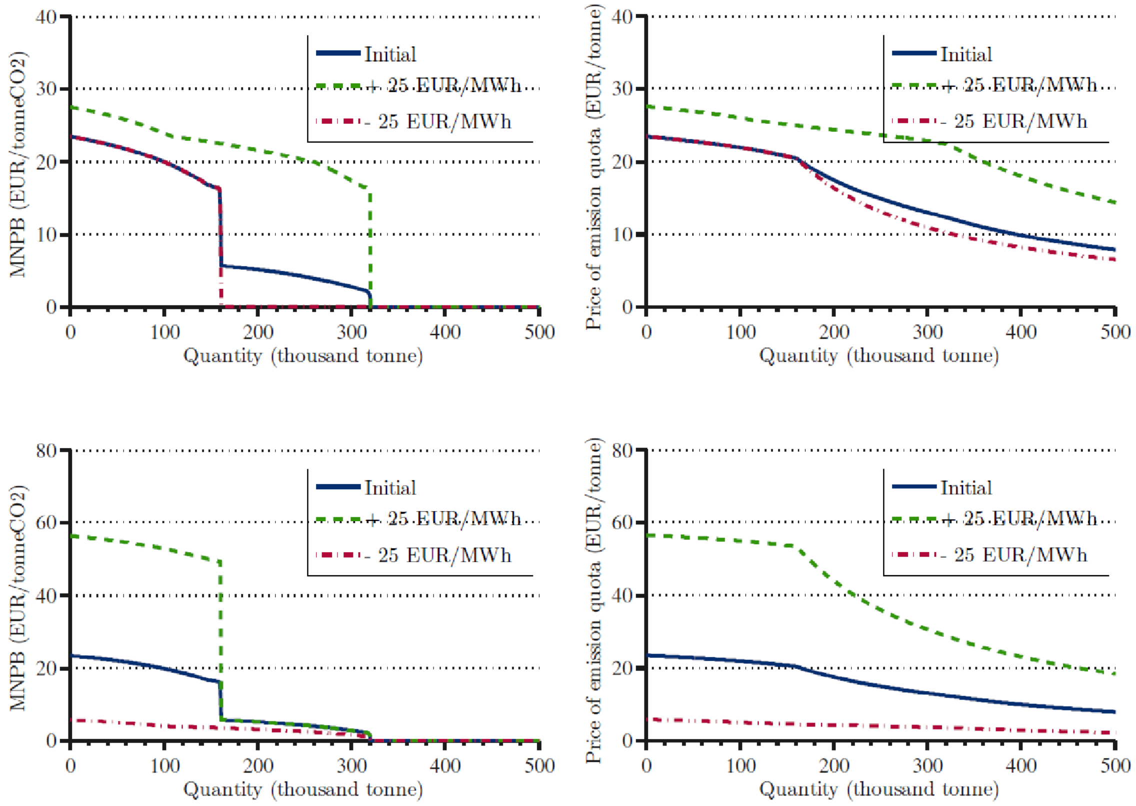

Figure 51: Changing of MNPB (left) and individual demand function (right) caused by the joint shifting of initial and long term mean prices of peak (top) and off-peak (bottom) electricity.

A price hike in electricity shifts both functions upwards, while a price drop has the opposite effect. A change in the off-peak power price shifts the “lower step” of the MNPB curve, as a consequence of the option prices pertaining to the low-demand period. The peak-period price moves the upper section of the curve.

The following figure illustrates the changes in the two functions according to the gas price:

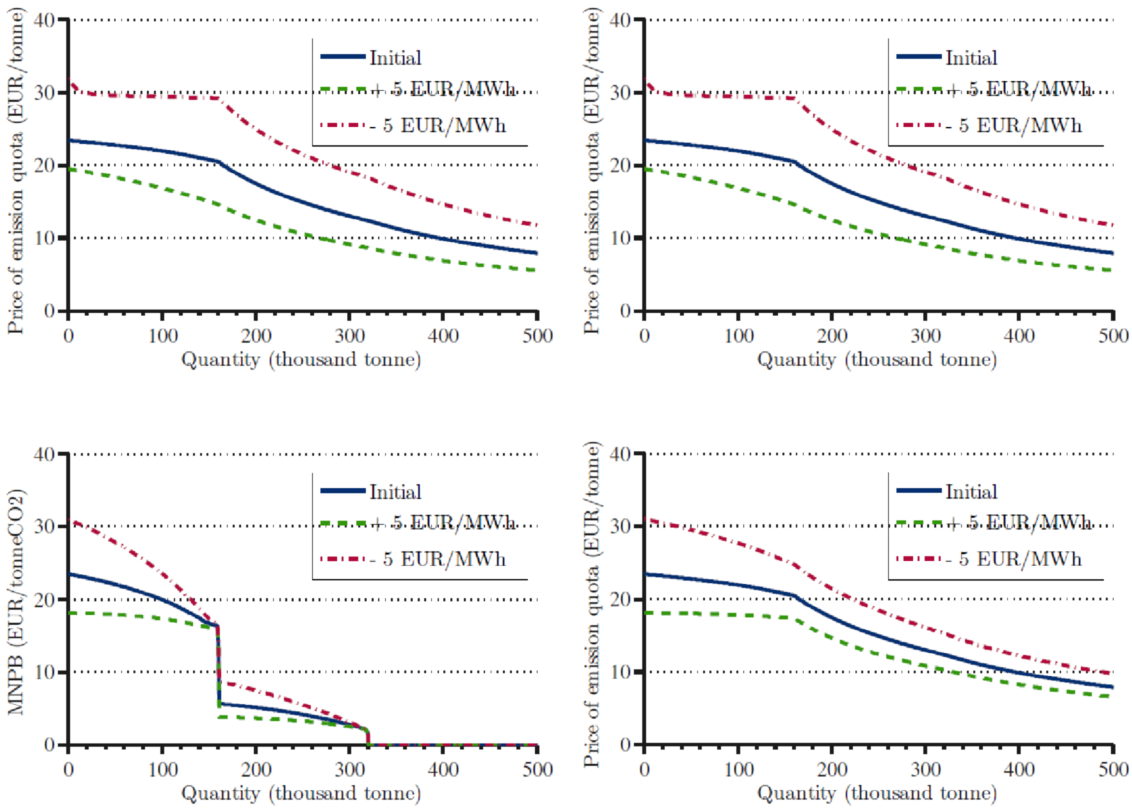

Figure 52: Shifting of MNPB (left) and individual demand function (right) caused by initial (top) and long term (bottom) price change of gas.

An increasing gas price shifts the functions downwards, whereas a price drop does the opposite. An increase/decrease in the spot price of gas increases/decreases the gradient of the MNPB curve. For changes in the long-term means, the nature of the effect remains the same, yet its direction is the opposite: a price increase flattens the MNPB function, while a decrease makes it steeper.

The real option model allows for the impact of volatility to be examined, as well:

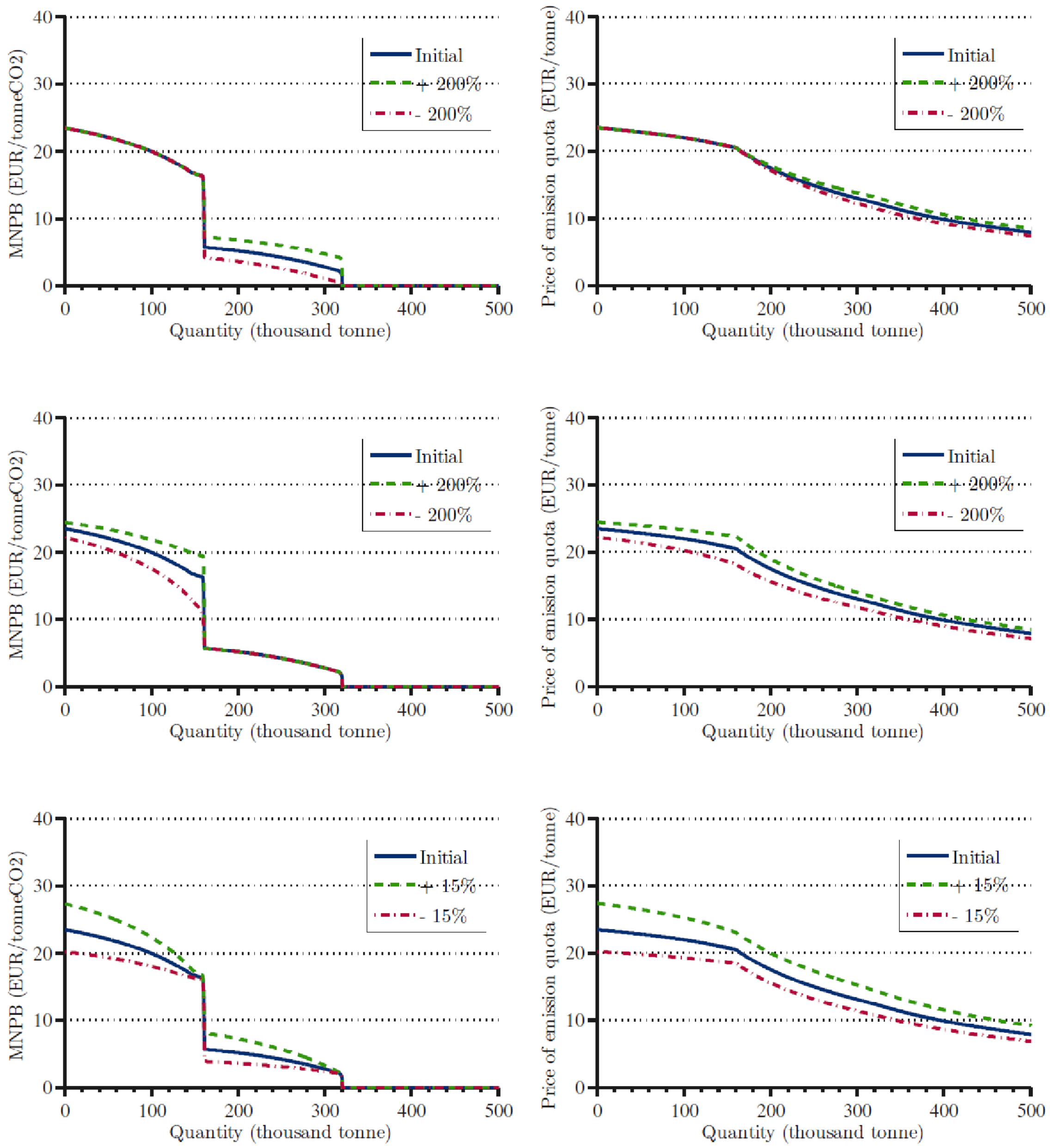

Figure 53: Shifting of MNPB (left) and individual demand function (right) caused by change in volatility (top: off-peak electricity, in the middle: peak electricity, bottom: gas).

An increase in volatility shifts both the MNPB and the individual allowance demand function upwards. Off-peak volatility affects the lower "step" of the MNPB curve, while peak-period volatility is related to the upper section. The asset to the volatility of which the functions in question are the most sensitive to is gas: an increase in the volatility of its price shifts both sections of the MNPB curve and the entire allowance demand function upwards.

VII. CONCLUSIONS

Upon the launch of the EU ETS, companies have to face a new type of compliance requirements: they need to cover their emissions with carbon credits. The new risk factor acted to increase electric power plants' demand for risk management. The compliance requirement (to surrender to the authorities an amount of allowances that corresponds to the volume of their emissions) made it inevitable for them to actively "manage" their quota position and to make forecasts about its expected emissions. As far as the power plant is concerned, it is not only the expected volume of emissions that is of interest, but the "risk profile", i.e. the probability density function of the value, as well.

The prediction of the power plant's decisions is unthinkable without managing the uncertainty arising from the prices of the underlying assets. Relying on real options, the future production decisions of the profit maximizing power plant can be derived from the simulated prices of the underlyings, and we can also find answers to our questions concerning risk management.

At the center of the real option decision model employed in the dissertation is the spread, which is equal to the profit realized on a unit of energy generated. The company operates the turbine whenever the spread is positive, and leaves it off whenever it is negative. Our conclusions from the real option model are primarily valid for power plants with gas turbines, which lend themselves to flexible operation. It is important to note that even though potential technological constraints (minimum up and down times, for example) would alter the concrete figures themselves, the essence of the conclusions would remain the same. These constraints were ignored in my calculations.

The tools of stochastic finance are highly suitable for real option based calculations. Throughout the dissertation, I used a closed-form analytic formula and Monte Carlo simulation for the pricing of the spread options. The model necessitated a stochastic price model, which was chosen to be the geometric Ornstein-Uhlenbeck process. Mean-reverting models are more suitable for modeling commodities, where prices are controlled by forces of demand and supply that are more or less stable in the longer run. The decision model includes four underlying assets: peak and off-peak electricity, gas and emission units. I fitted the four-dimensional stochastic process to prices from the German energy exchange.

The first application of the real option model was to calculate the expected volume of the power plant's emissions and its probability density function from two three-asset binary spread options, by way of simulation. If the spread follows a one-dimensional geometric Brownian motion, then the probability density function of cumulative emissions can also be approximated with recombining binomial trees, using a procedure of my own, as presented in the Appendix. The probability density function of the short-interval emissions, which was derived from the Monte Carlo simulation, is extremely right-skewed. The reason is that the states (turbine on vs. off) of consecutive days are strongly correlated and that the value of the initial spread is negative, that is, the power plant does not generate electricity in the beginning. For longer intervals, the distribution of the emissions becomes more symmetric.

In its lower ranges, an increase in thermal efficiency increases emissions: for the amount of resources needed to produce a unit of energy will be less, which improves the value of the spread and increases the probability of production. In very high efficiency ranges, the effect turns around: an increase in efficiency will not significantly increase the probability of production, as the value of the spread will be high anyway. In these cases, it will be the effect that reduces the maximum daily volume of emissions that will be stronger, and the expected volume of emissions will shrink. A change in the spot price of electricity does not significantly influence the volume of emissions, because according to the fitted parameters of the mean-reverting model, both the volatility and the mean-reversion rate of electricity is high, and therefore the impact a change in the spot price has in the longer run is small. Changes in the gas price do, however, affect emissions significantly: a low gas price will make the volume of emissions reach its theoretical maximum (measured under constant production), while a high gas price induces a significant fall in expected emissions. Changes in the quota price have no significant effect on emissions, the reason for which lies with the formula of the spread: given the parameters of the model, the impact a change in the quota price has on the value of the spread is approximately five times smaller than the impact of a change in the gas price. The volume of emissions is not, but the cost of compliance is largely influenced by the quota price: is the market price high, the cost of compliance becomes quite substantial. The cost of compliance will be particularly high if a high quota price is accompanied by a low gas price.

In the second application of the real option model, I derived the value of the power plant, which is equal to its cumulative income generating capacity for the interval in question. I approximated the value with the sum of the spread options' prices. The value of our imaginary power plant in the model turned out to be EUR 97.3 million. These spread options are sensitive to changes in certain technological and price-related parameters. From amongst these, the ones that I examined under the ceteris paribus assumption were: thermal efficiency, the initial prices and the long-term mean prices of the underlying assets, the volatilities of the prices and the correlation coefficient between the price of gas and that of the allowances.

Improvements in thermal efficiency act to increase the value of the power plant, the effect being nearly linear: a 1 percentage point improvement effectuates a EUR 5 million increase in the facility's value; it is the power plants with an efficiency in the 37-40% range that are the most sensitive to improvements in thermal efficiency.

The impact of spot prices is not significant in the model. From amongst the four underlyings, it is electricity to the initial price of which the value of the power plant is the least sensitive to, because the impact of the initial prices fades away quickly due to the high volatility and the high mean reversion rate. Increases in the spot prices of the inputs, and especially that of gas, have a more substantial impact on the price of the facility. Gas price has an effect that is five times larger than that of the price of emission units. Are we, however, to manipulate the long-term means instead of the spot prices, the value of the power plant will react with significant changes. An increase in the long term mean of electricity price will increase, while higher mean prices for the inputs will reduce the value of the power plant. Increases in price volatilities induce an increase in the facility's value. Out of the four underlying assets, the impact is significant only for gas. From amongst the correlation coefficients I evaluated when fitting the model, the one between peak and off-peak electricity prices was found to be significantly positive. The value of the power plant is not influenced by changes in this coefficient, because it is the sum of the option prices for the two time-of-day periods that we use, which is independent from the correlation between the two electricity prices. The other correlation coefficient that was significant was that between the prices of quotas and gas, which does have an influence on the power plant's value, because the two underlying assets in question are part of the very same option formula. However, the impact of the correlation coefficient between gas price and quota price is, according to the model, not significant.

If we do not wish to expose our power plant to unjustifiable risks, the position needs to be hedged against price fluctuations that would induce changes in its value. We may achieve that through a dynamic delta hedge, that is, by entering hedging contracts that ensure that the resultant delta parameters for all four underlying assets be close to zero. Knowing that electricity is rather volatile and that from amongst the underlyings, gas is the one the price of which has the greatest influence, it appears reasonable to hedge the power plant against changes in the gas price, at a minimum. A dynamic delta hedge delivers the solution to the optimal quota position problem, as well: the delta parameter pertaining to the emission units tells us how many quotas the power plant should hold at any given time in order to cover its future emissions. To arrive at the quantity required to cover the emissions of the entire year, the amount needed to cover the facility's past emissions still needs to be added to this figure.

As a third application for the real option model, I used spread options to derive the value of a 5 percentage point efficiency improvement project. I examined the sensitivity of the resulting value to changes in various factors. In the formula of the spread, thermal efficiency appears in relation to input prices, which is why it is input prices that have the greatest influence on the value of the efficiency improvement project. The only impact electricity has is that a higher electricity price at a higher efficiency level will increase the probability of a positive spread.

Considering spot prices, those of the two inputs act to decrease the value of the investment. From amongst the long-term means, increases in those associated with electricity increase, while increases in those of the inputs decrease the value of the project. As regards volatilities, an increase in that of off-peak electricity reduces the value of the investment. The project's value is insensitive to the correlation between the price of gas and that of emissions units.

Oftentimes, power plants are not operated depending on the market prices, i.e. in a profit maximizing fashion, but kept constantly running during certain periods based on long-term supply agreements. Under the assumptions of the model (ignoring technological constraints, for instance), this means a loss of value for the power plant. Therefore, the fourth application was to estimate the loss of constant production based on the value difference between certain spread options and forward spread swap contracts. The loss of constant production decreases with increasing thermal efficiency. The reason is that an increased efficiency means that the probability of a negative spread is smaller, thus a situation when it is rational to have the turbine rest and meet one's supply obligation through the market occurs less frequently. The real option model also revealed that the lower the mean price of electricity and the higher the initial prices of gas and emissions units, the higher the loss of constant production will be. The reason for which is that in all these cases, the probability of a negative spread grows larger, which makes the power plant incur losses (under constant production, that is).

The fifth application of the real option model is intended to facilitate the development of an efficient auction strategy. The MNPB (Marginal Net Private Benefit) function, a central part of a number of environmental-economic models, shows the marginal revenue the company can realize on an additional unit of pollution. The function, which does not include the cost of the quotas, can be obtained by sorting in descending order the reservation prices derived from the two-asset spread options on the weighted price difference of electricity and gas. By integrating the resulting MNPB function and dividing it by the volume of emissions, we can calculate the individual allowance demand function, which informs the power plant about the maximum unit price worth paying for a given quantity of emission units. Relying on the real option model, we could conclude that an improvement in thermal efficiency increases the gradient of the MNPB function, shifts the x-axis intersection to the left, and shifts the entire allowance demand function upwards. An increase in the long-term mean of electricity price shifts both functions upwards, whereas a price drop has the opposite effect. A change in the mean price of off-peak power shifts the lower section of the MNPB curve upwards, because of the option prices pertaining to the low-demand period; changes in the long-term mean price for peak hours move the upper section of the curve.

Considering the wide range of applications presented in the dissertation, it can be clearly seen that a power plant operating in a deregulated market must not and cannot manage without modern risk management techniques, and especially not without the real option model to base its profit maximizing decisions upon.

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IX. MOST IMPORTANT NOTATIONS USED IN THE TEXT

$\mathrm{E}[.]$, $PV[.]$ Expected value, present value

η Thermal efficiency of the power plant (in percentage)

δ Carbon intensity of the fuel (tCO2/GWh)

v Other variable costs

$\pi_{pow}(\tau), \pi(\tau)$ Profit per unit of energy generated on a future day τ

$\pi_{eua}(\tau)$ Profit per unit of carbon dioxide emitted on a future day τ

$S_{pow}(\tau), S_{gas}(\tau), S_{eua}(\tau)$ Spot price of power, gas and allowances on a future day τ

Γ Maximum daily production

$\Pi(\tau)$ Total profit realized during the entire future day τ

$\Lambda(\tau)$ Value of the binary production variable on a future day τ

$\mathrm{Q}(\tau)$ Volume of emissions on a future day τ

$\mathrm{Q}(0, T)$ Cumulative emissions from the present until date T

bno^{PO}, bno^{Pr} Payoff (PO) and price (Pr) of the three-asset (electricity, gas, emission units) European-style binary spread option

spo^{PO}, spo^{Pr} Payoff (PO) and price (Pr) of the three-asset (electricity, gas, emission units) European-style spread option

$spo2^{PO}, spo2^{Pr}$ Payoff (PO) and price (Pr) of the two-asset (electricity, gas) European-style spread option

ssw Value of the three-asset (electricity, gas, emission units) forward swap

lcp Loss of constant production per unit of energy generated

V Value of the power plant's revenue generating capacity

X. APPENDIX: Numerical Approximation of the Emissions' Probability Density Function via Recombining Binomial Trees

Below, I will present a method of my own that employs recombining binomial trees to approximate the density function of cumulative emissions. The binomial tree model that has been traditionally used to price derivatives typically consists of three trees. The first describes the price movements of the underlying asset (the values that the discrete model can take), the second contains the probabilities pertaining to the given states and the third tree represents the price movements of the derivative.

In this model of recombining binomial trees, the probability tree of the traditional model is divided up into several trees, according to which cumulative production decision level (Ω) they belong to. Each element of the resulting probability trees shows the percentage chance that the underlying asset takes that given price at the given cumulative decision level.

If the interval examined is divided into N parts, we will have $N+1$ dates – including the starting date ($t=0$) – and $N+1$ possible cumulative emission levels, and thus we will be required to use $N+1$ probability trees. In the probability trees that represent higher values of Ω cumulative decision variable, non-zero probabilities can only be found in later steps: high cumulative production decision levels count as impossible events in the initial period. In the last step, even the elements of the tree associated with the highest cumulative decision and emission levels can take a non-zero value.

If the interval is divided into 3 parts, then we will work with 4 trees; the possible elements of the binomial trees, which have a non-zero probability, are shown in grey in the following figure:

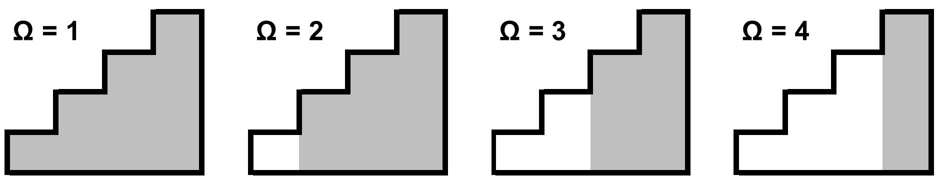

Figure 54: Non-zero probabilities of binomial trees.

An alternative approach may be to substitute the four simple probability trees with a single three-dimensional tree, the horizontal slices of which are the relevant non-zero-probability parts of the individual two-dimensional trees, which contain the probabilities pertaining to the given cumulative decision levels:

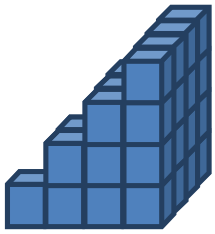

Figure 55: Three dimensional binomial probability tree.

The three-dimensional binomial tree is easier to comprehend, yet more difficult to plot, therefore I will continue to use the two-dimensional price trees (the individual slices) in the figures.

The relationship between the probabilities pertaining to the cumulative decision levels is a special one. There are only two ways to get into a state belonging to a given cumulative decision level: either from one of the states that belong to the cumulative decision level directly below or from one of the states that belong to the same tree. That is, there are two things that can happen with respect to a given price and cumulative decision level: first, if the price is above the critical value (the exercise price), then the company decides to generate power, the value of Ω grows by 1 and, in this case, the probability will need to be derived from the tree associated with the Ω value that is 1 less than the one in question. In the opposite case (i.e. if the price is below the critical value and the turbine remains idle) the emission level remains unchanged, thus the probability will need to be calculated from two of the states that belong to the previous steps of the same tree:

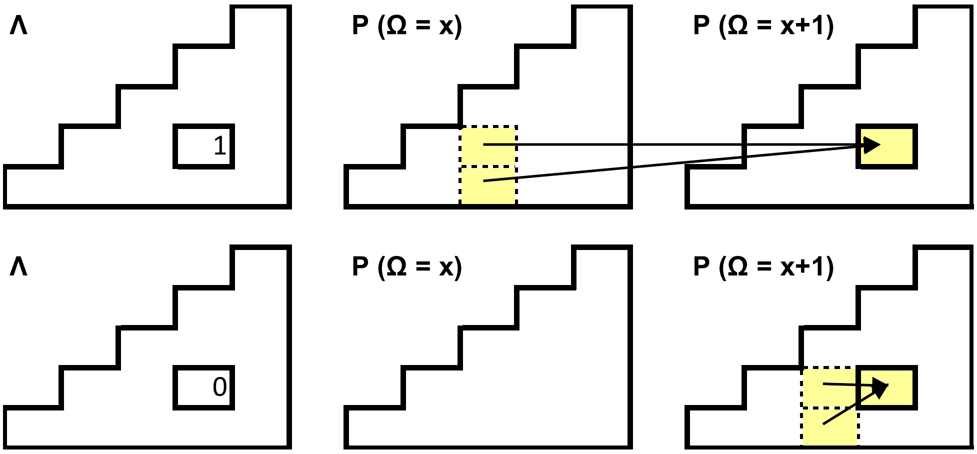

Figure 56: The connection between binomial trees in two different cases (above: Λ=1, bellow: Λ=0).

The relationships pertaining to the probability trees can also be expressed formally. The formula for the probability at the j^{th} price point of the k^{th} step of the l^{th} probability tree is (where S denotes the price, K is the exercise price, p is the probability of an upward movement and $q=1-p$ stands for the probability of a downward movement):

$$P(k,j,l) = \begin{Bmatrix} if\ S(k,j) > K \Rightarrow \Lambda = 1 & O(k-1,j-1,l-1)\cdot p + O(k-1,j,l-1)\cdot q \\ if\ S(k,j) \le K \Rightarrow \Lambda = 0 & O(k-1,j-1,l)\cdot p + O(k-1,j,l)\cdot q \end{Bmatrix}$$

$$O(k,j,l) := \begin{Bmatrix} if\ k \ge j,l\ and\ k,j,l \ge 0, & P(k,j,l) \\ otherwise & 0 \end{Bmatrix} \qquad 74.$$

In order to illustrate the method with an example, let us assume that the spread (without other variable costs) follows a geometric Brownian motion, its initial value is $S_0 = 3.1$, the annual drift equals $\mu = 0.1$, volatility is $\sigma = 0.2$ and the interval examined is 1 year, which we divide up into three parts. The parameters of the price model are: Δt = 0.33, u = 1.224, d = 0.8909, p = 0.6176, q = 0.3824. Let us assumer, furthermore, that the other variable costs term equals 3, that is, the power plant will decide to generate power if the value of the above spread exceeds 3. The possible prices, the respective probabilities P and the values of decision variable Λ are contained in the following trees:

j \ k	S (j,k)			
3				4.4
2			3.9	3.5
1		3.5	3.1	2.8
0	3.1	2.8	2.5	2.2
j, k	0	1	2	3

j \ k	P_S (j,k)			
3				23.6%
2			38.1%	35.4%
1		61.8%	47.2%	35.4%
0	100.0%	38.2%	14.6%	5.6%
j, k	0	1	2	3
	100.0%	100.0%	100.0%	100.0%

j \ k	Λ(j,k): S(j,k)>=X			
3				1
2			1	1
1		1	1	0
0	1	0	0	0
j, k	0	1	2	3

Figure 57: Process of spread and the production decisions modeled by binomial trees.

Having divided the interval into three parts, the process will be represented by four binomial probability trees associated with four cumulative decision levels:

	$P_{\Omega=1}$ (j,k)			
3				0.0%
2			0.0%	0.0%
1		0.0%	0.0%	9.0%
0	100.0%	38.2%	14.6%	5.6%
j, k	0	1	2	3
	100.0%	38.2%	14.6%	**14.6%**

	$P_{\Omega=2}$ (j,k)			
3				0.00%
2			0.00%	0.00%
1		61.76%	23.62%	9.03%
0	0.00%	0.00%	0.00%	0.00%
j, k	0	1	2	3
	0.0%	61.8%	23.6%	**9.0%**

	$P_{\Omega=3}$ (j,k)			
3				0.0%
2			38.1%	14.6%
1		0.0%	23.6%	9.0%
0	0.0%	0.0%	0.0%	0.0%
j, k	0	1	2	3
	0.0%	0.0%	61.8%	**23.6%**

	$P_{\Omega=4}$ (j,k)			
3				23.6%
2			0.0%	29.2%
1		0.0%	0.0%	0.0%
0	0.0%	0.0%	0.0%	0.0%
j, k	0	1	2	3
	0.0%	0.0%	0.0%	**52.7%**

	P_{Ω} (Ω,k)			
4				**52.7%**
3			61.8%	**23.6%**
2		61.8%	23.6%	**9.0%**
1	100.0%	38.2%	14.6%	**14.6%**
Ω, k	0	1	2	3
	100.0%	100.0%	100.0%	100.0%

	P_S (j,k)			
4				*23.6%*
3			*38.1%*	*43.8%*
2		*61.8%*	*47.2%*	*27.1%*
1	*100.0%*	*38.2%*	*14.6%*	*5.6%*
j, k	*0*	*1*	*2*	*3*
	100.0%	*100.0%*	*100.0%*	*100.0%*

Figure 58: Cumulated emissions process modeled by binomial trees.

In the example, the number of possible cumulative decision (emission) levels (*Ω*s) is 4, therefore we will need to use 4 probability trees in the model (two upper rows). The probabilities highlighted with a thick border below the individual probability trees are the sums of the probabilities that pertain to the k^{th} step of the tree that belongs to the given *Ω* value, which sums we "carry forward" to the probability tree of the cumulative emission value (tree on the left-hand side in the bottom row) that we are looking for, and which depends on the values of *Ω* and *k*. The columns of this tree contain the discrete approximation of the continuous density function we are looking for. In the binomial tree on the right-hand side of the bottom row, I added up the probabilities of the trees for the four different Ω values, and the resulting values coincide with the values in the probability tree of the basic process (see the previous figure). That is, the sum of the probability trees associated with the different cumulative emission levels (*Ω*s) is identical with the probability tree of the basic price process. That is because what the method actually does is to separate, in a special way, the price process into sub-processes with the same cumulative emission level. From the values in the last step of the probability tree of cumulative emissions, we can derive the approximation of the density function of the total cumulative volume of emissions at the end of the interval, which in our case is left-skewed and slightly “U”-shaped:

Ω	**1**	**2**	**3**	**4**
P	14.6%	9.0%	23.6%	52.7%

Table 6: The discrete probability density vector of cumulated decision variable Ω.