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LIQUIDITY RISK ON STOCK MARKETS

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Liquidity Risk on Stock Markets

Statistical analysis and possible applications of the

Budapest Liquidity Measure

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Introduction

„Portfolios are usually marked to market at the middle of the bid-offer spread, and many hedge funds used models that incorporated this assumption. In late August, there was only one realistic value for portfolio: the bid price.
Amid such massive sell-offs, only the first seller obtains a reasonable price for its security; the rest lose a fortune by having to pay a liquidity premium if they want to sell. …Models should be revised to include bid-offer behaviour."

Nicholas Dunbar („Meriwether's Meltdown," Risk, October 1998, 32-36)

Liquidity is the essential condition of the normal functioning of financial markets and financial system. Only the appropriately liquid financial markets are able to function effectively, i.e. to transmit the savings to the users, and to aggregate market actors' expectations and available information. The liquidity of markets, more precisely the lack of it affects the whole financial system, and indirectly through it the whole economy, thus inhibiting their normal, operational way of functioning. The financial crisis of 2008 has pointed to the outstanding importance of the liquidity of the financial system, and at the same time it pushed this question to the limelight. The revision and/or supplementation of the standard equilibrium and no-arbitrage models which assume the existence of infinite market liquidity has become a necessity , as there is an evident need to develop new pricing models and risk management techniques.

Although the scientific analysis of market liquidity has a history of a decade, only a relatively few generally accepted and widely spread results can be connected to this field because of the elusive nature of this concept. Another reason is the fact that the concept of liquidity and liquidity risk is used in various senses both in practice and in theoretical studies. Although these various interpretations are connected to each other by a multitude of ways, it is nonetheless important to single out at least the most important ones:

1. in connection with the liquidity of a portfolio and, in a broader sense, that of a company's what we examine is whether it is able to fulfill its cash-flow obligation as they become due;
2. concerning the liquidity of a market, i.e. the market liquidity of a particular financial instrument what we examine is whether we can trade with the given financial instrument at the spot market price in a reasonably big amount quickly with low transaction costs (spread);
3. by the liquidity of the financial system we mean the free, available cash and cash equivalents volume present in the financial system.

The above interpretations are evidently correlated, since for instance the liquidity of a portfolio/company is highly determined by the liquidity of the assets it consists of/has at its disposal, which is closely related to the liquidity of the financial system. In the same way, it is worth to make the following distinctions concerning liquidity risk as well:

1. cash flow risk;
2. risk of trading on an illiquid market, i.e. the price impact risk;
3. risk of the liquidity circulating in the financial system to dry up, i.e. the system risk.

The first interpretation is important to the portfolio managers and for corporate chief financial officers, the second is to the traders active on financial markets (indirectly issuers and investors), and the third is important to central banks and other supervisory institutions safeguarding the stability of the financial system. This also shows the diversity of the involved market participants and of the degree of involvement itself.

In my dissertation I am especially concerned with market liquidity and trading risk from different aspects: both from theoretical and also empirical points of view. Parallel with my research we have made a series of interviews supported by the Budapest Stock Exchange, during which stock-traders and portfolio managers were asked about the practical ways they manage liquidity risk (see Szűcs and Váradi, 2012). The responses I got from the interviews contributed to a large extent to the formation of and refining my research questions and hypotheses. During the interviews a view gradually emerged that dynamic portfolio optimization on illiquid markets is a remarkably complex problem, which cannot be regarded as solved either from the practical or from the theoretical aspect for the time being. Market participants (in the absence of anything else) attempt to simplify the question, e.g. some are only willing to trade on liquid markets exclusively, while others decide on the portfolio they intend to create, then they give orders to traders who are specialized in carrying out the transaction of the requested size within a given time frame in a way that they are able to minimize the price impact of the transaction. Many others yet attempt to decrease liquidity risk during the build-up and/or the liquidation of a portfolio by setting up simple rules of thumb. In my dissertation I do not undertake the task of precisely describing and solving the optimization task, either, instead I attempt to take the first steps towards it by presenting the nature of liquidity risk and the options to manage it.

On illiquid markets trading costs are significantly higher than on liquid markets, i.e. transactions can only be executed with a notably higher cost and time. Therefore it is not surprising that market participants' basic requirement is that each stock's liquidity should be comparable and the transaction costs quantifiable. Measuring liquidity is a complex problem in itself, it is difficult to express all of its aspects with one single indicator, and it is also hard to estimate how much cost illiquidity generates during the trade, since liquidity can be interpreted along different dimensions and thus at any given time one or another of its different attributes can come to the forefront.

During my research I put strong emphasis on a liquidity indicator which quantifies the transaction costs of trading in the hypothetic and considerably extreme case when the buyer/seller is not willing to wait at all i.e. they intend to realize the transaction immediately, without any delay. This index is the so called **Budapest Liquidity Measure** (BLM), which has been created according to the pattern of the liquidity indicator firstly introduced and constantly published by Frankfurt Stock Exchange, the Xetra Liquidity Measure (XLM). The database has been provided to me by the Budapest Stock Exchange.

My **main goal** was to help liquidity as a concept to be incorporated into the daily practice of risk management, i.e. to elaborate solutions which can be easily intruded into the daily practice, but also properly developed from a theoretical point of view. From the series of interviews it evidently turned out that a prerequisite for dynamic portfolio optimization would be to get a clear view on how the return, the volatility and market liquidity of risky assets are correlated, i.e. what are the main attributes of this aggregated stochastic process. Accordingly during my research I have focused on three **main issues**: (1) I have examined the cross- and horizontal sectional statistical attributes of the BLM time series; (2) I have shown how the indicator can be integrated into a VaR-based risk management system; (3) I have deducted the correlation between the BLM and the so called price impact function, which is one of the essential concept of liquidity and is often analyzed in literature. With the help of this latter I have also empirically analyzed how price impact evolved on the Hungarian stock markets between 2007 and 2011. This period is especially interesting because it includes the escalation and the run-down of a major liquidity crisis.

The three research issues also differ in the respect of the applied **methodology**: (1) I make a traditional descriptive statistical analysis on BLM database; (2) I build up a theoretical model which can be used in the field of risk management; (3) I make a time series analysis on the time series of the estimated price impact function.

My dissertation consists of four chapters. In the first chapter I shortly sum up the basic concepts and main contexts, the ensuing three chapters center around the three research questions and my own results concerning them.

In **the first chapter** I outline the operation of the stock markets and the main attributes of quote driven and order driven markets. In addition, I also give a detailed description about the statistical characteristics of the order book which is used on order driven markets based on the results of the earlier empirical research. This is significant because the BLM database is based on the order book, namely it actually condenses the pieces of information in the order book by way of a special transformation.

In **the second chapter** I present the basic concepts concerning market liquidity, the dimensions along which it can be measured and the main indicators which can quantify certain aspects of market liquidity. I give a detailed description about the build-up of the BLM indicator and the process of its calculation. I review the Hungarian research literature made concerning market liquidity on the Budapest Stock Exchange. Subsequently I present my own research results, i.e. the traditional statistical analysis of BLM, the analysis of the relation between BLM and other liquidity indicators (e.g. bid-ask spread and turnover) and the co-movement of BLM and volatility before and after the crisis. The daily BLM time series create an opportunity to have a thorough knowledge on its temporal and cross-sectional behaviour.

In **the third chapter** I introduce a possible model with which the BLM indicator can easily be integrated into VaR (*value at risk*) based systems which support market risk management. Here the underlying idea is that on illiquid markets a value of an asset is not equal to its last market price. Namely, the buy/sell transaction reacts to the price and shifts it into the opposite direction. In this situation it is reasonable to determine the return in a way that we take the expected opposite price impact into consideration. In the second part of the chapter I give a detailed description of liquidity adjusted VaR models (LAVaR) which can be found in the literature, and then about my own theoretical model. I regard this latter as one of my most important innovative achievements.

In **the fourth chapter** I describe one of the central concepts of the topic of market liquidity, the so-called price impact function, which shows the relative price-shift caused by a particular order. The knowledge of the behavioural attributes of the price impact function has particularly great significance for market participants, since with its help they can predict the price impact concerning their orders to be given in the future, i.e. the expected surplus cost caused by a price-shift. In this chapter I describe the difference between the virtual and the empirical price impact functions, and I also present a method of estimation of a price impact function with the help of the Budapest Liquidity Measure. Based on the method I elaborated market participants can simply and quickly estimate a virtual price impact function without knowing the whole order book in detail. Finally, I also conclude the examination of the time variation and the basic statistical attributes of the virtual price impact function estimated from the BLM database. Based on this function I examine whether the time variation of liquidity is predictable, i.e. whether the process has a memory, and if yes, then for how long the impact of shocks prevail. Furthermore, I also examine the nature of the trend and the height of the volatility which characterize the price impact and whether the process of price impact is a mean-reverting process. I summarize these results in the last part of the chapter.

Some studies have been published in the Hungarian literature in which market liquidity was analyzed on different markets. However, only a few of these were concerned specifically with the liquidity of Budapest Stock Exchange. I base my dissertation on the findings of these studies, but in many respect I extend and exceed them inasmuch the circle of research questions, their depths and also the size of the examined database are concerned.

The main achievements of my dissertation, which can be regarded as **my own contribution** to the examined field of finance on Hungarian and on international levels, are the following:

1. Simple liquidity indicators (bid-ask spread, turnover) do not measure the transaction cost-type aspects of illiquidity appropriately; therefore it can be misleading to rank different markets according to them, or to base dynamic portfolio optimization on these indicators. This is especially true in case of a crisis or on illiquid markets.
2. Based on the examination of the relationship between liquidity and volatility, it can be stated that the 2007/2008 crisis can also be regarded as a liquidity crisis, i.e. the increased indirect trading costs cannot exclusively be attributed to increased volatility.
3. I have split the return (net return) into two major parts, namely I quantified the proportion of the transaction costs due to illiquidity (liquidity risk) inside the return, and the proportion of the shift of the mid price (price risk). I incorporated this net return into a VaR-based risk management system (LAVaR).
4. In the LAVaR model I have shown that in the case of stock portfolios liquidity costs can be diversified.
5. I have elaborated a way to estimate the virtual price impact function from the BLM database.
6. I accomplished the time series analysis of the estimated virtual price impact function.

I. The order book

1. Trading on stock exchange

Financial markets can be classified and distinguished based on different attributes. There are a number of market characteristics which influence market microstructure and thus have an effect on market price formation and transaction costs. A vast number of studies prove that different market microstructures have an effect on features like price-formation, liquidity, the returns realized by investors and finally the way these affect the general market efficiency (e.g.: O'Hara, 1995). Before the detailed description of the concept of liquidity I intend to enumerate the types of stock trade and the trading methods that market participants are provided with. The following enumeration gives a broad picture of how the stock market functions, as well as of the features of market microstructure and of the differences between markets.

Characteristics of market microstructure:

1. Participants: Various participants can be present on the markets e.g.: institutional investors (hedge funds, banks, enterprises, etc.), agents with intermediary role (brokers), traders, dealers, private individuals, etc. The number of participants and their market share, namely the market concentration are also important from the point of view of market microstructure.
2. Primary and secondary markets: Security issuance happens on primary markets where it basically takes place via investment banks. However, trading with the issued securities takes place on secondary markets: the stock exchange. Furthermore, there also exist a tertiary and a quarternary market. On the tertiary markets participants trade with listed stocks outside the stock exchange (*OTC – over the counter*), which is less regulated than the trading on stock exchange. But these OTC markets have a more regulated form: the multilateral trading facility (MTF) which is a new legal category created by the Markets in Financial Instruments Directive (MiFID). The crucial difference between the stock exchange and MTF is that while regulated markets can only be operated by

organizations having an exclusive operating license, namely they are market operators, MTF can also be run by authorized investment enterprises, credit institutions or even by regulated markets (Gellén, 2009, p. 214). Apart from MTFs there exists another tertiary market category, which is regulated by MiFID: the „dark pools", whose aim is to enable the big institutional investors to make large ticket transactions via organized trading systems without significant transaction costs (Réz, 2011). On the quarternary markets investors directly trade with listed securities outside the stock exchange without brokers/dealers. This trading method started to improve by leaps and bounds in the last years as a result of the spread of the common electronic platform, called Electronic Communication Network (ECN) (Bodie et al. 2005, p. 91).

3. <u>Characteristics of the product:</u> The attributes of the product have an effect on price-formation, namely there are some products whose prices evolve independently, but there are markets where prices are determined by prices of other markets. An example is the market of derivative products, whose prices are determined by the underlying product. When considering the relationship between a derivative and the underlying product it is important to pay attention to the following attributes:
   a) whether the two products complement or substitute each another.
   b) whether the underlying product is traded or not. For instance in the case of a weather derivative the underlying product is not traded.
   c) whether the underlying product can be delivered at the expiry of the derivative product. Also in the case of a weather derivative the underlying product is not traded and it also cannot be delivered, whilst for instance in the case of buying a stock futures/forward the underlying product is traded and it is also deliverable at the expiry.
   d) whether there is a cost of carry during the holding period of the underlying product, as for example a storing cost of commodities. It is also crucial in this case whether the underlying product is stored at all. For instance, electricity as an underlying product that can be delivered, but it cannot be stored.
4. <u>Order types</u>: Markets can also be characterized by different types of orders, e.g. stop, limit, market, hidden, etc. These orders will be more exhaustively described in Subchapter 1.3.

5. Mechanism of price-determination: There are three significant markets from this respect. The first one is the direct market, where market participants trade directly with each other. The second one is the broker/dealer market where trading is realized through intermediaries. Finally, the third one is the auction market where actors participate in the trade with or without dealers and brokers.
6. Presence of market makers: On quote driven markets a market maker is present on one side of each transaction. The market makers can be divided into two big groups: the Designated Market Makers (DMM-s) who are always obligated to quote a bid and an ask price, and the Supplemental Liquidity Providers (SLP-s) who are obligated to quote only a bid or an ask price in order to provide market liquidity. Another big group of markets are the order driven markets where participants directly trade with each other without the market makers’ presence. The operation of quote and order driven markets will be more exhaustively presented in Subchapters 1.2 and 1.3.
7. Information, transparency: Markets also highly differ to the extent they provide information for e.g. brokers, clients or any other market participants. Transparency means the quantity and quality of information available for the participants. Such information can be for example the publication of the different price levels in the pre-trade phase, the order prices or market depth (Madhavan, 2002). Moreover, there are differences in the speed of information dissemination e.g. whether real-time or delayed data are provided for the market participants. As far as information is concerned another crucial question is anonymity, since if the market participant is aware of the broker’s or dealer's identity then they are able to get extra information, which helps them to more easily filter out the trading strategies based on order splitting (Margitai, 2009, p. 6).
8. Transaction costs: Markets may vary in respect of transaction costs e.g.: brokerage fees, commissions, etc.
9. Level of automatization: markets also differ in this respect. The two great extremes are floor trading and electronic trading. An instance for electronic trade is the SuperDot system, which typically supports programmed trading, block transactions and orders that consists of more than one transaction. It helps orders to be executed quickly, 95% of the transactions are realized within 1 minute.

10. Other regulations, protocols:
    a) Level of standardization: The main difference between stock exchanges and OTC markets lies in their degree of standardization. While during a futures contract on the stock exchange trade can only be realized with pre-determined amounts, expiration, etc., on OTC markets transactions are increasedly personalized.
    b) Centralization: Markets can be divided into two main groups: they can provide trading with financial instruments in a centralized or decentralized way. An example of the decentralized market is the foreign exchange markets, where traders are physically dispersed and they are also market makers at the same time. Centralized trade is the stock exchanges, for example the Budapest Stock Exchange (BSE).
    c) Physical delivery: from the aspect of trade it is important whether the products featuring in the transaction have to be delivered, or is it sufficient to make a financial settlement. For example in case of an index forward contract at expiry there is no delivery obligation, as the underlying product, the index is not traded on the spot market.
    d) Continuity: A vast number of trading systems operate only periodically i.e. a trade can only be executed during definite periods of time, while there are systems in which trades are continuous, i.e. the market is always open (Madhavan, 2002). However, in the case of continuous trading markets are also closed from Friday midnight until Sunday night, because this is the time interval when there is weekend in all time zones of the world. Such continuous trading is typical for foreign exchange markets. An example for periodic system is the Budapest Stock Exchange where orders are collected in the order book until the so called „market clearing" time. According to this, the trading at BSE can be divided into a continuous and an auction phase. Continuous trading lasts from 9:02 A.M. until 5:00 P.M. preceeded by an opening and closed by a closing order collecting phase.
    e) Protocols: Protocols serve as a regulated framework for the trading. They regulate e.g. the minimum amount of trading, the suspension and the pause of the trade, the special rules of opening, closing and reopening, etc. (Madhavan, 2002).

f) Settlement rules: there are different settlement systems, in some of them settlements are T+3- or T+5-day and they can also be different if they have a central settlement house, the so called clearing house, which holds the counterparty risk during the transaction. On OTC markets transactions are directly realized between the seller and the buyer, there is no clearing house, thus the partner risk is more significant than it is during trading on the stock exchange.

g) Permission of short selling: Markets highly differ in respect of short selling i.e. whether there is a possibility to sell a security which is not physically owned at the time of the sale.

Table 1 shows how trading systems differ in some attributes in different markets of the world.

Table 1: Trading systems

Attributes	**Typical ECN**	**NYSE[1] Open Market**	**NYSE Intraday trading**	**Paris Stock Exchange**	**Chicago Board of Trade**	**FX[2] Markets**	**BSE**
Continuity	X		X	X	X	X	
Presence of market maker		X	X		X		
Level of automatization	X			X			X
Anonymity	X	X		X			
Pre-trade order collection	X		X	X	X		X
Post-trade reports	X	X	X	X	X	X	X

Source: Madhavan (2002), p. 34, and my own additions

The literature of market microstructure is more thoroughly concerned with the question of how these different market structures affect prices and the market operation. The main market processes which can be affected by market microstructure:

- Predictability of returns (efficiency, memory);
- Distribution of the returns (expected return, volatility, normal or extreme distributions);

[1] New York Stock Exchange
[2] Foreign Exchange markets

- Correlation between markets;
- The possibility to manipulate returns, emergence of bubbles, shock deceleration, stability/instability, systemic risk;
- Liquidity, trading volumes.

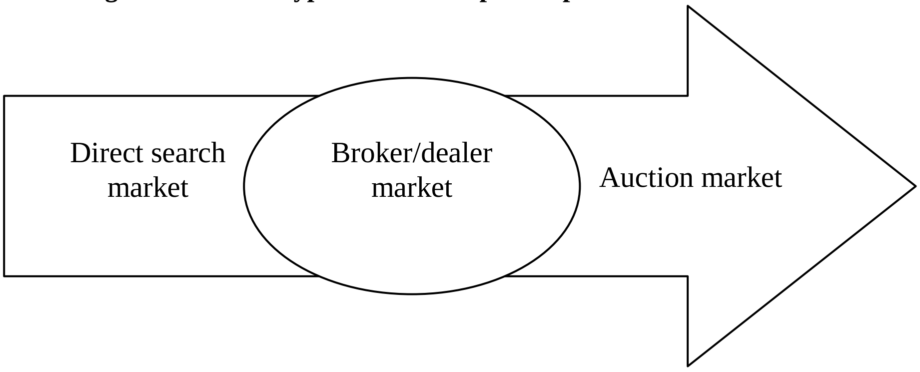
A vast number of studies attempt to find the relation between market microstructure attributes and market processes e.g. how trading volumes and returns were affected by the ban or the reporting obligation of short positions (Boehmer et al., 2010), etc. In this dissertation I examine market liquidity which correlates with the above characteristics. From this point of view the mechanism of price-determination (point 5) is especially significant, and that is why I intend to give a detailed explanation about it in the following point.

1.1. The mechanism of price-determination

As far as market price-determination is concerned the simplest is the direct market (prices are random, transparency and liquidity are low). Dealer/broker markets, where one can trade through dealers and brokers, are slightly more sophisticated. Dealers and brokers realize their profit from the difference of bid and ask price and they provide liquidity in return. Auction markets are the most complex. Auction markets can either be unilateral, e.g. when the issuer invites all the potential buyers interested in the product, gathers their orders and quotes prices accordingly (its mechanism can be manifold). It can also be bilateral, when sellers and buyers are both present and they hand in their orders simultaneously, which are to be matched according to some algorithm.[3] Therefore, on an auction market liquidity is provided not primarily by dealers and brokers, instead the actors directly find each other hence a significant part of the broker fees can be saved. From the point of view of price-determination the sophistication is shown in Figure 1.

[3] On the operation of auctions see a detailed description from Szatmári, 1996.

Figure 1: Market types from the aspect of price-determination

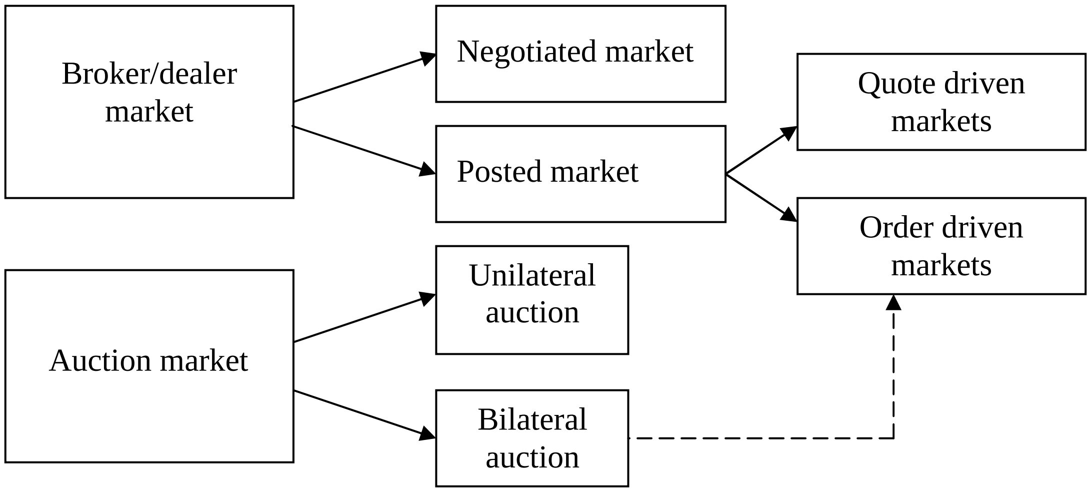


Source: proprietary

Intermediaries can be brokers, dealers, specialists and market makers. The difference between them is that as opposed to dealers brokers are not allowed to trade for their own account. Moreover dealers generally have a significant amount of stock/open positions in the given security or product. Furthermore, the specialist is a market leader of whom there is only one in each market, therefore only one specialist sets the price for all stocks, while market makers are those market leaders, of whom there can be several in the same market.

Broker/dealer markets can either be negotiated or posted. On negotiated markets orders are not visible, but dealers/brokers find each other according to different heuristics (e.g. walking on the floor or phoning, etc.), collect some orders and taking the prices and the partner risk into consideration they pick the most attractive one and then they conclude the transaction. Posted markets can either function in a way that the market maker constantly publishes the order prices in its own order book. They are obligated to trade with a minimum amount at this order price. The other option, which is the way order driven markets work, is that they aggregate the constantly incoming buy/sell limit price orders and make this information available in the so called order book. Figure 2 shows the different variations of broker/dealer markets and their relationship with auction markets.

Figure 2: The relationship of auction and broker/dealer markets



Source: proprietary

The above listed different price-determination mechanisms are not separate, they can also operate simultaneously, e.g. besides a posted option there can be a possibility of a negotiated deal (e.g. NASDAQ[4]); or on order driven markets market makers can work in addition to the order book e.g. by constantly filling up the order book with their own bid and ask prices. For example on the market of Hungarian government bonds the issuance is realized on a unilateral auction market, then the bonds are traded in a broker/dealer system (secondary market) parallel with the primary dealers' continuous price-quotations (Balogh and Kóczán, 2008).

This dichotomy can also be observed at the Budapest Stock Exchange, i.e. it has two different auction systems at the opening/closing phase and during the daytime trading. At the opening and closing phase trading is realized in an auction system in which the market-clearing price (the price at which the most transactions are realized) is the opening and closing price, while during the day the auction system functions continuously in an order-driven way based on the order book. Therefore the order driven mechanism can be equivalent to a constant, bilateral auction (*DCA – double continuous auction*) (Farmer et al., 2002). In the following part I intend to list the characteristics of the auction-, order driven and quote driven markets.

[4] The NASDAQ (*National Association of Securities Dealers Automated Quotations*) is an electronic stock exchange which has the greates turnover in the world.

1.2. Quote driven markets

The main attribute of the quote driven markets is that the so called market makers have an intermediary function between buyers and sellers. Their primary duty is to give a bilateral quotation, thus providing market liquidity. The market makers are always obligated to set a price both on the bid (buy) and ask (sell) sides. This means that there is a market maker on one side of every transaction. They have to execute the transaction either from their own stock of securities or by matching it with another transaction. The market makers' goal is to gain the spread (the difference between the bid and ask prices), independently of the movement of current market prices. Thus for them it is important to have a high turnover and a lot of incoming orders so that they can turn over their stock, thus profiting from the spread.

However, market makers have to quote a price which do not significantly influence market price, i.e. they have to give a price on both bid and ask sides which encompasses the real market price of the given product. It is important to have about the same volume of buy and sell orders therefore market makers should only have their income from spreads, and they should not have an interest in influencing market price. Otherwise market makers would accumulate a short or a long position of a certain financial product and then they would have an interest in shifting prices in their own favor. Inspite of this the hold of a neutral position, i.e. zero stock very rarely occurs (Parlour and Seppi, 2008).

Quote driven markets are widely spread among financial markets. For instance NASDAQ or even LSE (*London Stock Exchange*) function this way.

1.3. Order driven markets

Many stock exchanges around the world function as order driven markets. For example the Paris Bourse (*Paris Stock Exchange*) and even Budapest Stock Exchange belong to this category. My dissertation centers around order driven markets, as my empirical analysis is based on the database provided by BSE.

Markets where there is no assigned market maker but there is a constant flow of bilateral trading and the recording and matching of orders are executed with the help of an electronic trading system are called order driven markets (Bouchaud et al.

2008). As there are no market makers on this market, it can sometimes be extremely illiquid, where transactions cannot be realized, because on one side – e.g. on the buyer side – there are no participants. This generally happens in extreme economic situations, for example during a crisis. In such cases the maintenance of the proper functioning of the market is secured by rules and protocols of the stock exchanges (Madhavan, 2002).

In order driven markets orders are collected in the so called order book, which thus contains all buy and sell orders. The book always contains the price and the volume for each price level for any given moment and it can be seen by market participants (typically the first five or ten rows). Table 2 shows a fictive order book.

Table 2: The order book

Bidsize	Bidprice	Askprice	Asksize
300	8,270	8,275	200
622	8,262	8,276	400
400	8,251	8,280	320
721	8,241	8,290	22
1,200	8,237	8,291	66

Source: proprietary

In the first row sets out the best buy price (*bidprice*) and volume (*bidsize*), and the best sell price (*askprice*) and volume (*asksize*). The second best prices and volumes are in the next row, etc. The prompt bid-ask spread is the difference of the bid and ask price of the best order level.

When a new order arrives to the market, e.g. a bid order, it gets into the book in case it is lower than the best ask order, then it is considered to be a limit order. In case the bid order is equal to or has a higher value than the best ask order in the book, the transaction is immediately realized. Such type of order is called market order (Iori et al. 2003).

On the whole the order book contains only the limit orders. These orders, according to the above, only stay in the order book until they are matched with a market order or another limit order,[5] or until they are withdrawn.

The main difference between the two order types is that market participants who give a limit order are willing to wait in order to have their orders realized at the

[5] In case it is matched with a limit price order, the given order can be regarded as a market order since the transaction is immediately realized and the order did not get into the book.

preferred price; whilst those who give a market order are impatient, and find it important to realize their orders immediately. Thus, participants who give a limit order provide the supply of market liquidity (*liquidity providers*), whilst those giving a market order are the demand for liquidity (*liquidity takers*). For liquidity providers the most interesting thing are the time and the number of transactions their offers take to realize, while for liquidity takers the most important is to know how much their transactions are likely to shift the market price (Bookstaber, 1999).

Therefore, on the whole on order driven markets liquidity is provided by limit orders, whilst those who give a market order are the users of this liquidity. Therefore market liquidity depends exclusively on the supply and demand for such liquidity.

In addition to limit orders and market orders there are numerous order types at the market participants' disposal, which can be regarded as variations of these two order types. They typically differ from the above described two orders in their validity period (e.g. day order, good till cancel, etc.), or market participants may incidentally subject the order execution to some other conditions (e.g. stop-loss order, „iceberg order", etc.).[6]

The sequence of different orders is called the orderflow on which the order book is based. Figure 3 demonstrates the way an order book builds up from different order types. It shows that as soon as a market order arrives, it is fulfilled on the best bid or ask level. The priority of the fulfillment of the incoming orders is first based on the price, and then on the time. If the volume of the market order is bigger than the available amount at the best price level then the orders at the following order levels are realized until the total volume of the market order is executed. Nevertheless, in reality this means that the total volume of the market order submitted by the given trader will be realized at a worse average price than the price available at the best price level, because not only the first, but several price levels could be eventually deleted from the book. On the whole it can be regarded as a cost of an immediate purchase due to its higher volume that is currently available at the best price level. However, such market order leads to a modification in the bid-ask spread and it also changes the mid price, which is exactly halfway between the best bid and ask orders.

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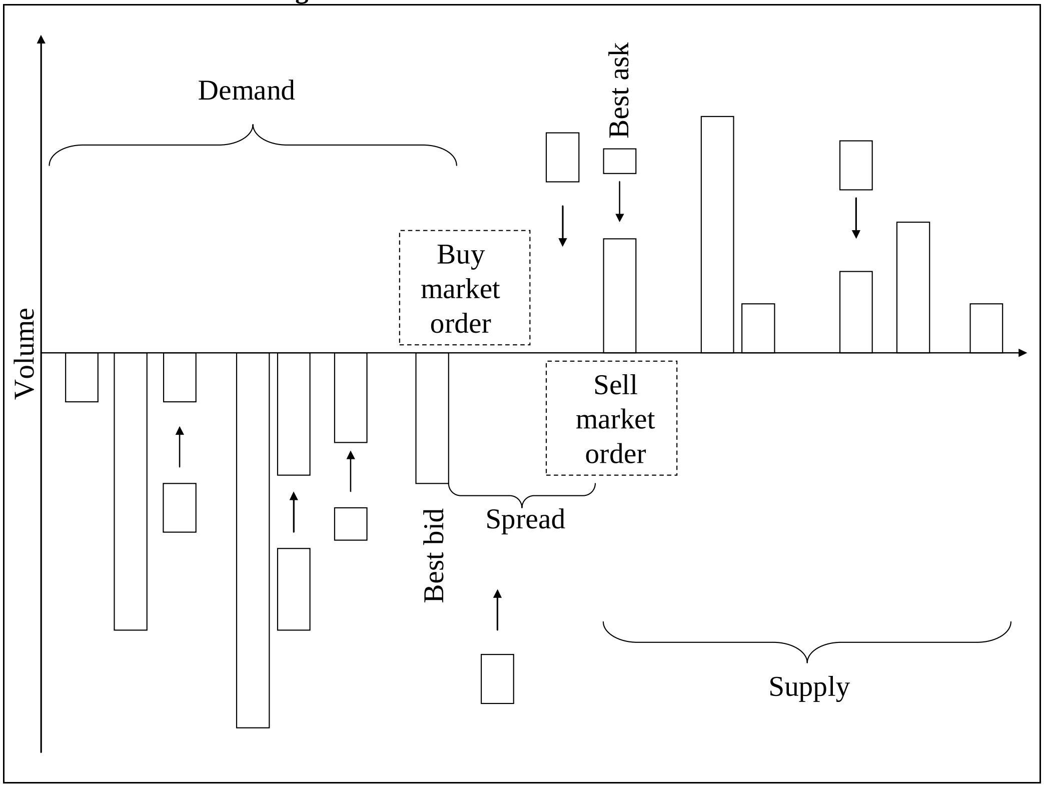
[6] About order types see: Budapest Stock Exchange's homepage: http://bet.hu/topmenu/befektetok/tozsde_lepesrol_lepesre/azonnali_piacismeretek/hogyan_kereskedjunk_a_tozsden/tozsdei_megbizasok or New York Stock Exchange's homepage: http://www.nyse.com/pdfs/fact_sheet_nyse_orders.pdf

Figure 3: The order book and the orderflow



Source: Farmer et al., 2004, p. 3.

Orderflow is actually the resultant of three stochastic factors[7] such as:

- Price
- Signed volume
- Time

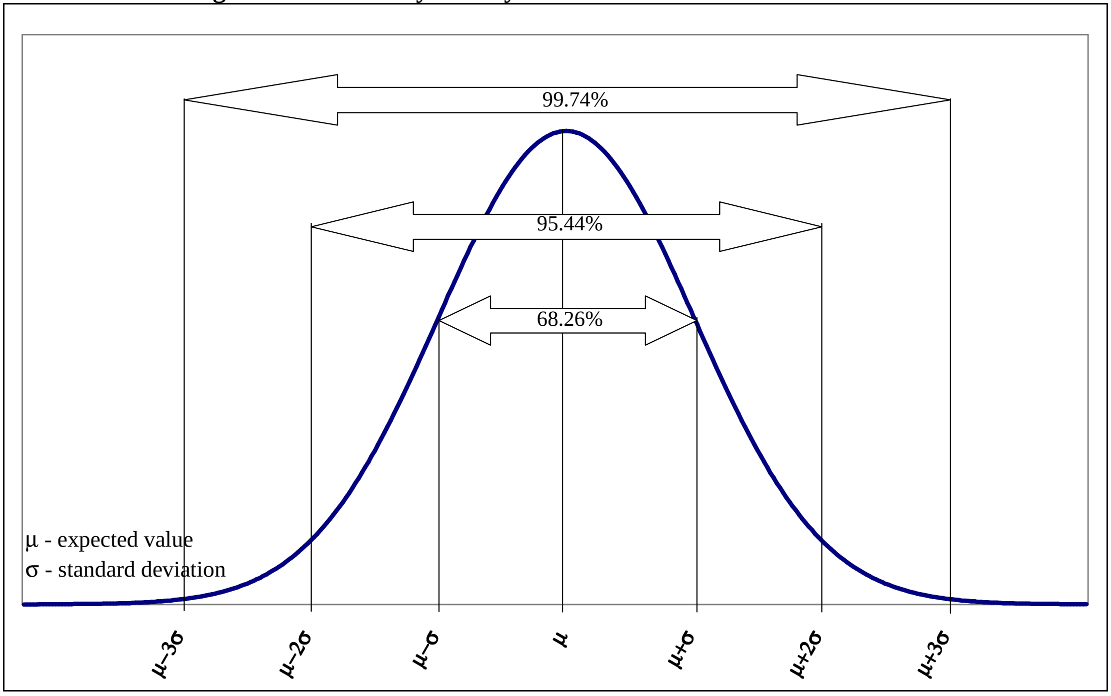
The limit order price, the intended bid and ask volume and the time of order arrivals are stochastic. The current order book evolves according to the constantly incoming orders. Therefore, the distribution of order prices and volumes in the order book reflects the process of all three stochastic factors. To know the nature of this distribution is crucial for market participants.

One of the most important questions from the aspect of risk management is the occurrence probability of extreme values. If for example the particular stochastic variables (order book price or volume) follow normal distribution, then events beyond three sigmas (three times bigger than the standard deviation) practically never occur, therefore it is not necessary to be particularly prepared for such events in the frame of

[7] More detailed on the stochastic processes see Medvegyev and Száz (2010), where one can read about the relevance and applications of the stochastic processes on the field of finance (e.g. Homolya and Benedek, 2007).

risk management. Figure 4 shows the probability density function of the normal distribution and the probability of event occurrence beyond three sigmas.

Figure 4: Probability density function of the normal distribution



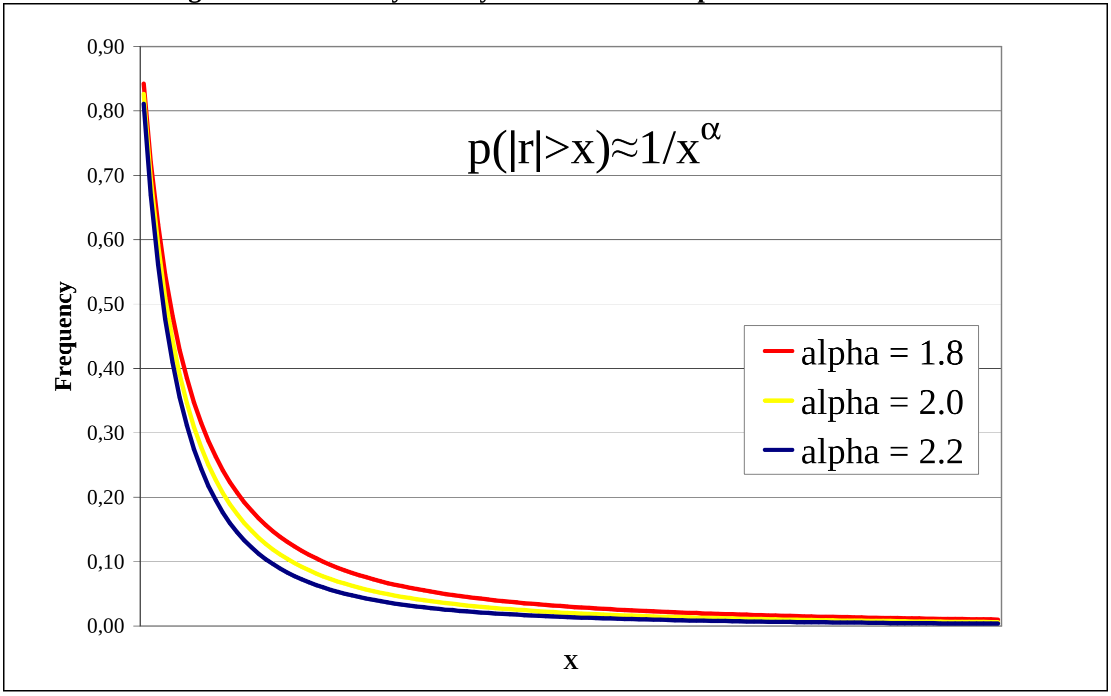
Source: proprietary

As opposed to the above situation if fat-tailed distributions characterize these values, the probability of extreme values is remarkably higher, thus we should pay special attention to such occurrences in the frame of risk management.[8] Thus it is not surprising that the analysis of distribution characteristics is the central theme of a multitude of studies. Empirical examinations mostly show that in the order book prices and volumes – independently from the examined period and market – follow an exponential distribution, their density function is shown in Figure 5.

Compared to normal distribution, the exponential distribution assigns a remarkably higher probability to extreme events; therefore there is no practical barrier for the occurrence of the most extreme cases. It follows from the above that the importance of risk management moves into the forefront and it is not sufficient to prepare for the normal business, but it is also important to have a disaster or contingency-plan.

[8] Probability is an important notion of risk management. The connections, differences of the notions of risk-uncertainty-probablility can be found in detailed at Hitelintézeti Szemle's special edition in 2011. The title is: „Vélekedés a kockázatról és bizonytalanságról". The authors of the articles are: Bélyácz, 2011; Badics, 2011; Dömötör, 2011; Krekó, 2011; Kovács, 2011; Medvegyev, 2011; Száz 2011.

Figure 5: Probability density function of the exponential distribution



Source: proprietary

For instance extreme returns are usually characterized by exponential-like distributions. In a basic case returns are regarded to be normally distributed, but it is a stylized fact that the density function of the empirical distribution of returns are more fat-tailed than it could be justified by the normal distribution, i.e. its drop is less steep at the tails. Empirical research shows that on the sides the drop of the function is exponential-like, i.e. on the sides return (r) can be modeled according to the following formula: $p(|r| \rangle x) \approx 1/x^{\alpha}$, where $\alpha \approx 3$ which is called tailindex (Tulassay, 2009). The lower the tailindex value is, the more fat-tailed the distribution is, but its typical value is cca. between 2 and 3 (Clauset et al., 2009). In the following Subchapter 2, I examine the different distributions characterizing the orderflow and the statistical attributes of the order book.

2. The statistical attributes of the order book

A vast number of studies have been published about the statistical attributes of the financial markets in the recent few decades, and researchers found very similar results whether they examined commodities markets (Mandelbrot, 1963), the foreign exchange or the stock exchange markets (Fama, 1965, Cont, 2001, etc.) in different parts of the world. Researchers have found similar phenomena in every market, which they have summed up under the name of stylized facts. These stylized facts are for instance:

- volatility clustering,
- the fat-tailed, exponential-like drop of returns,
- the low effect fundamental news have on prices,
- leverage effect (The correlation is negative between the price change and volatility. When prices fall, the leverage increases as well, and generally volatility is increasing also.),
- autocorrelation of returns,
- stock prices fluctuate more than it could be justified by the fundamentals, [9]
- benefit/loss asymmetry (i.e. their fluctuations are not symmetric).

The main goal of these studies was to test the efficiency of markets. Their intention was to build models, or find market phenomena which would enable them to forecast returns. The efficient market hypothesis says that market prices „fully reflect” all the available information market participants have (Fama, 1970, p. 383). This means, that all information concerning the appropriate values of securities are reflected in the prices, so no one can earn unusual ex ante profits on a consistent basis using known information set (Pilbeam, 2010). According to the efficient market hypothesis, only new information will change the prices, so at the end the daily returns will be normally distributed and independent, because the arrival of new information at the market is random (Száz, 2009).

[9] For instance Joulin et al. (2008) showed that the volatility after price jumps is too high to be justified by the change in fundamentals.

Since the returns are in the focus of the efficient market hypothesis, the research have focused on them. These studies haven't provided any breakthrough results, they couldn't verify nor deny the efficient market hypothesis.

Research have recently begun to focus much more on analyzing the statistical attributes of the order book, because the change in the order book leads to the price changing on the markets, thus the given orders can be regarded as the most basic part of price-formation. Hence the examination of the order book is important both for market participants and academic professionals, because it provides information about trade and price-formation processes.

One part of scientific articles concerned with the order book attributes, tend to approach their subject mostly from a theoretical point of view. These studies are, among others, the following articles: Bouchaud et al. (2002), Bak et al. (1997), Chan et al. (2001), Luckock (2001), Slanina (2001), Daniels et al. (2002), Challet and Stinchcombe (2001), Willmann et al. (2003), Maslov (2000) and the works of Maslov and Mills (2001).

The other part of the academic articles have statistically analyzed the order book from various perspectives, of which the most important ones are the following:

- the distribution of the distance of limit order prices from the actual market price (Bouchaud and Potters, 2002; Zovko and Farmer, 2002; Bouchaud et al., 2008),
- the examination of the order book shape: the location of its maximum, the distribution of the volumes on the bid and ask sides (Bouchaud and Potters, 2002; Maslov and Mills, 2001; Zovko and Farmer, 2002; Bouchaud et al., 2008; Lillo and Farmer, 2004; Mike and Farmer, 2008),
- attributes of the order volume (Gopikrishnan et al., 2000; Gabaix et al., 2003; Maslov and Mills, 2001; Margitai, 2009; Bouchaud et al., 2008; Lillo and Farmer, 2004),
- the distribution of different order types (Lillo and Farmer, 2004),
- persistence of transaction signs (Lillo and Farmer, 2004; Margitai, 2009; Lillo et al., 2005), and
- the effect of the supply and demand on stock returns (Plerou et al., 2002; Bouchaud et al., 2004; Maslov and Mills, 2001; Chordia and Subrahmanyam, 2002).

2.1. Attributes of order prices

Bouchaud and Potters (2002) have analyzed the statistical attributes of the order book via the database of NASDAQ and Paris Bourse. Among others, they have analyzed the distribution of the prices of the limit orders. They examined the distance between the current price and the incoming limit order. They called this distance delta (Δ). Concerning Paris Bourse they came to the conclusion that delta (Δ) follows a power-law distribution, regardless whether it was a bid or an ask order. They have given the following estimation to the distribution function:

$$P(\Delta) \propto \frac{\Delta_0^{\mu}}{(1+\Delta)^{1+\mu}}, \qquad (1)$$

where the exponent was estimated to be $\mu \approx 0.6$. This result is similar to Zovko and Farmer's (2002), with the difference that the value of their exponent (μ) was 1.5. According to the authors the reason of this difference can be the fact that Zovko and Farmer examined the London Stock Exchange's (LSE) database, where the examined database they were provided with did not contain all of the orders, because only a particular selection of orders get to the electronic system used by LSE; whilst in the case of Paris Bourse the electronic system contains all of the orders.

Bouchaud and Potters (2002) have also examined the distribution of the distance of the prices of the limit orders from the mid price in the case of securities traded on NASDAQ, and they found that the security itself under examination had a high impact on the results.[10] However, the character of the distribution i.e. how slowly the density function of the distribution decreases on the sides, was very similar to that of the French stocks. This phenomenon – market participants give numerous orders far from the mid price – was explained by Zovko and Farmer (2002), Bouchaud and Potters, (2002), and Bouchaud et al. (2008) with the fact that market participants think that a big jump in prices is always possible, and this is why they give orders which are further from the mid price. They do this in order to take advantage of the eventual big price-shifts.

[10] They have examined two investment funds: QQQ and SPY, two indexes: Nasdaq and S&P 500 and one stock: Microsoft.

2.2. Shape of the order book

The studies on the statistical attributes of the order book focused on the shape of the order book. Researchers have counted the number of orders on each price level. Beforehand we could have expected that the orderflow is the biggest around the current market price, and the further we go the orders are fewer. However, we have to take into consideration that an order close to the market price is more likely to get out of the book either because it is matched with a market order or because it is cancelled. This is why the shape of the order book is not evident.

In the case of Paris Bourse Bouchaud and Potters (2002) found that the function was symmetric,[11] therefore its shape was identical on ask and bid side. Taking an average order book into consideration we can observe that the function does not reach its maximum at the current best bid or ask order, but slightly further away from it. The researchers have pointed out, that the further we are from the mid price, the fewer order is in the book. In the case of NASDAQ database it could only be observed with one traded fund (the QQQ) that the function does not reach its maximum at the best order level. In the case of the other examined fund, indexes and Microsoft’s shares the function reached its maximum at the best bid and ask order, and then it gradually decreased. This result is identical to Maslov and Mills’ results (2001), who in connection with the data of NASDAQ Level II also found that the majority of the orders could be found at the best order level in the book. According to researchers the difference between the two function forms is again due to the fact that not all of the traded volumes appear in the database.

Zovko and Farmer (2002) and Bouchaud and Potters (2002) have both explained the order book shape with the fact that on the best price levels orders did not stay in the book long enough, because they can either be executed or cancelled. Zovko and Farmer (2002), and Bouchaud et al. (2008) have also shown that the further the order was from the best price, the more time it stayed in the book. Namely, the market participants who make this sort of order in the book are willing to wait and they do not cancel the order, because they would like to gain from the price shift. In contrast, those who give their orders around the best order level are active market participants, who regularly hand in orders to the book (Bouchaud és Potters, 2002).

[11] They examined the time-averaged size in the function of the distance of the mid price.

The orders of these participants' are either quickly matched with a market order or in case it does not take place and market participants observe that the market price changes in an unfavorable way, they prefer to cancel the order and hand in another one, because they are less willing to wait.

Examining LSE data Lillo and Farmer (2004) have found that 32% of the cancellations were from the best price level, while the other 68% were cancelled inside the book. Mike and Farmer (2008) have also examined the distribution of the lifetime of cancelled orders. They have found that it can also be approached with a power-law distribution.

According to the authors, the cancellation rate, which was measured by the reciprocal value of the lifetime, can depend on more factors, of which I intend to emphasize two significant ones:

1. the further an order is from the best price level, the higher the conditional probability of the cancellation is,
2. if the number of orders on bid and ask sides are highly unbalanced, it also raises the probability of cancellation.

Referring to the order book shape, Maslov and Mills (2001) provide another interesting result. They have found that the bid-ask spread was smaller by 10-20% than the average distance between levels in the order book. Moreover, jumps on the ask side are by 5-10% bigger than the ones on the bid side. However, they could not verify whether it was generally true for the order book, or only a particular attribute of the examined day.

2.3. Attributes of order volumes

Numerous researchers have examined the orders according to the submitted volumes. Some of them have found that the distribution of the volumes of the submitted orders could be described with a power-law distribution, while others have found a gamma distribution[12] for both the bid and the ask sides.

[12] Gamma distribution is a two-parametered (p and λ) continuous distribution whose density function is $f(x)=\frac{\lambda^p x^{p-1} e^{-\lambda x}}{\Gamma(p)}$, where Γ(p) is the gamma function ($\Gamma(p)=\int_0^\infty t^{p-1}e^{-t}dt$) (Spiegel et al. 2000).

Power-law distribution was found by, among others, Gopikrishnan et al. (2000), where the authors got the following result for distribution of the submitted volume (Q) within a certain time interval (Δt):

$$P(Q_{\Delta t}) \approx \frac{1}{Q_{\Delta t}^{1+\lambda}} \tag{2}$$

Concerning one thousand American stocks, the authors have given the following estimation for the exponent: λ=1.7 +/- 0.1. Gabaix et al. (2003) have found the same result when examining the 30 biggest Parisian stocks with the difference that they have given a 1.5 estimation for the exponent (λ). Maslov and Mills (2001) having examined the data of NASDAQ Level II got the result 1.4 +/- 0.1 for the exponent concerning all of the orders, whilst concerning the limit orders only they estimated the exponent to be 1+/- 0.3.

Margitai (2009) has also examined the distribution of the order volume on the Hungarian database: in the case of MOL stocks. His aim was to find out whether the Pareto,[13] or the gamma distributions suits the empirical database better. As a result he found that the distribution of the order volume can be properly approached by Pareto distribution, where he estimated the value for the exponent to be 1.25. The gamma distribution did not fit the empirical data distribution appropriately, which in the author's opinion is the consequence of the fact that the tail of the density function of the empirical distribution is power-law-like, while the tail of the density function of the gamma distribution is exponential-like.

Another part of researchers have estimated a gamma distribution for the distribution of the order volume. Bouchaud et al. (2008) belong to these researchers, who have examined the data of Paris Bourse, and also Lillo and Farmer (2004), who studied the London Stock Exchange data.

A number of researchers have also examined whether there is persistence in the database in the case of the submitted volume. Gopikrishnan et al. (2000), Lillo

[13] Pareto distribution is a special continuous type power-law distribution. This distribution is often referred to as „80/20" rule, because its characteristics is that the 20% of possible events occur with an 80% probability, while 80% of events occur with a probability of 20%. This distribution suits a numerous natural and economical phenomena. I.e. 80% of world wealth accumulates in the hands of the 20% of the population, while the remaining 80% possesses only the 20% of this wealth (Spiegel et al. 2000).

and Farmer (2004), and Margitai (2009) have also found that there was a significant persistence in the time-series, i.e. the autocorrelation function of the volumes have shown that there was a positive autocorrelation between the volumes given in each particular occasion. According to this, the sequence of the given volumes can be considered as a long memory process.

2.4. The distribution of different orders

Lillo and Farmer (2004) have examined order composition in the case of the London Stock Exchange. The authors have sorted the orders into the three categories created by Hopman (2007) which are as follows:

- Market orders: all the orders that are executed immediately.
- Spread orders: orders which are placed between the best bid and ask prices. In these cases transactions are not realized, but the spread is getting narrower.
- Limit orders: orders given inside the book.

Lillo and Farmer (2004) have found that 33% of orders were market orders, 32% were spread orders and 35% were limit orders. According to researchers, limit orders have the smallest price impact, i.e. they do not shift the market price, the spread orders have a more significant impact, whilst the givers of market order are the most impatient. Therefore, market orders have the most significant price impact, because if an order is not realized at the best price level, but it also affects the other rows of the order book, market price will move from its former level.

The result achieved by the authors is interesting because the order, in which different order types arrive, i.e. the orderflow, has a significant effect on the price formation process.

2.5. Persistence of the transaction signs

Lillo and Farmer (2004) and Margitai (2009) have investigated whether there is a persistence in order signs, namely if we know whether it is a buy (positive sign) or a sell (negative sign) order, can we predict the sign of the next order. Both researchers have found the same results, i.e. there is a persistence considering any stock database. It means that similarly to the order volume, the direction of transactions is also a long memory process.

Lillo et al. (2005) justified the long memory with two reasons. One is that investors can be characterized with a herd effect – although it cannot entirely be empirically tested – the other is that there are many institutional investors on the market who trade in a way that they split a big order, and they execute the transaction one by one in order not to have a big influence on the market price. These order types are called hidden orders, as the investors' aim with the order splitting is not to reveal the real size of the transaction they intend to execute. This strategy results in the "sliding" of prices, thus there is no definite trend (Margitai, 2009).

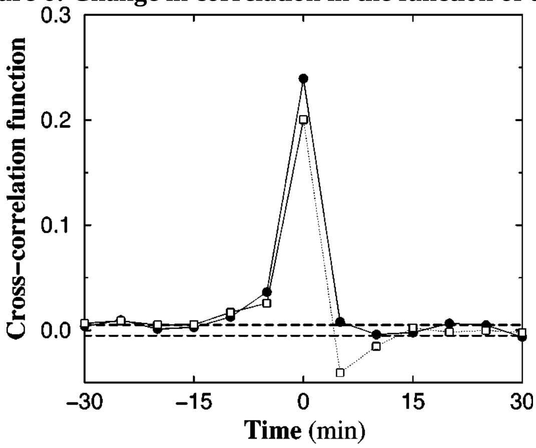
2.6. Effect of the supply and demand on the returns

The basic idea of efficient market hypothesis is that only the newly arrived pieces of information will shift the prices, and thus the price formation process will be unpredictable. However, Bouchaud et al. (2004) state that even though information has a crucial role, it is nonetheless secondary. According to them the really important factor is how the supply and the demand influence price-formation. Bouchaud et al. (2004) think that the price-shift affected by the supply-demand can be caused by the response to new information and also by the change in the demand for liquidity. According to their statement, in both cases there could be a situation when the orderflow becomes predictable. The traders with their buy and sell decisions put a demand or supply pressure on the market, and via this they influence the price-formation process. These supply- or demand-side pressures can easily be identified by the order book, although it is questionable whether it is actually possible to predict the price-shift from this, because it would contradict the efficient market hypothesis. In this subchapter I describe the research which are concerned with the analysis of this phenomenon.

First of all I would like to highlight Plerou et al.'s (2002) work. The authors have examined how a change in demand affects stock prices within a given Δt time interval. The demand change was defined in the following way: Φ measures the difference between the numbers of buyer or seller initiated transactions within a given interval, and Ω means the difference between the numbers of the traded stocks through a buyer or seller initiated transactions. Namely, in the first case they examined the imbalance in the number of transactions while in the second case they observed the imbalance in the volume. Plerou et al. (2002) determined whether a trade is buyer or seller initiated in a way that if during the transaction price is higher than the mid price then the transaction is buyer initiated; if it is smaller, it is seller initiated; and if it is the mid price, then it is indeterminated.[14]

Researchers have primarily examined the correlation of price-change (G) with variables Φ and Ω, and have found that the shorter the time interval was between the price-change and the measured time of Φ and Ω, the higher the correlation was. For most of the stocks the correlation was significant for cca. 15 minutes. Figure 6 shows the change in correlation in the function of time.

Figure 6: Change in correlation in the function of time

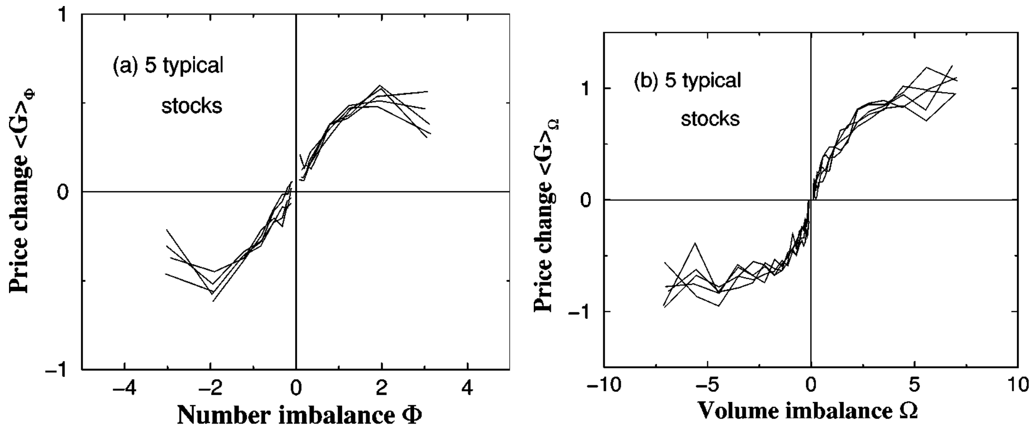


Source: Plerou et al. (2002), p. 3.

Then the researchers have examined how the growth of number imbalance (Φ) and volume imbalance (Ω) in a 15-minute interval affected price-change predictability. The authors have found that the higher the imbalance was, the less it affected the price-change and this relationship could be the most appropriately described with a concave function-shape, as it is shown by Figure 7.

[14] 17% of trades were indeterminated in their database. Lee and Ready (1991) have examined this phenomenon more thoroughly.

Figure 7: The effect of imbalance on price-change



Source: Plerou et al. (2002), p. 3.

Another significant research in this field was accomplished by Maslov and Mills (2001). The authors have got the result that the high imbalance concerning the volume of the orders on the buy and sell sides had made the price-change predictable in a short term, which is the consequence of the law of supply and demand. This was true for the cases in which imbalance was significant and a notable part of orders were close to the current mid price (Maslov and Mills, 2001). The size of imbalance was defined by 10,000 stocks on the examined database, but they suggested as a rule of thumb that this size of imbalance should be proportional to the daily turnover. During their research they did not consider the whole order book, they only picked out the orders at the best order level for consideration. The examined period were the few minutes following the occurrence of the imbalance. The authors have found that the prediction capability lasted only for a few minutes, in the case of some stocks only for 30 seconds at maximum.

Maslov and Mills (2001) used another method to examine the supply-demand effect on price-change. The essence of the method is that they have observed the average price-change in the case of the given supply-demand imbalance levels during a given Δt time interval. Researchers have found that supply-demand had a significant effect on price-change in this case as well. However, the lower the stock's turnover was, the stronger this effect prevailed.

Finally I intend to present Chordia and Subrahmanyam's research (2002), who have examined the relationship between stock returns and order imbalance. The starting point of their research was a model which examined how market makers took the sort of imbalance into consideration which is caused by the fact that big investors do not submit their transactions in one amount, but they split them. The authors have found that there is a positive relationship between the order book imbalance and the stock returns. These statements were tested by empirical data, and they drew the conclusion that the imbalance-based trading strategy resulted in significant returns (Chordia and Subrahmanyam, 2004, p. 485).

After this it is legitimate to ask the question: if the order direction and order volume are a predictable, long memory processes; and we can anticipate the returns from the supply-demand formation in the short term, then how can this be reconciled with the commonly observed fact that returns and thus the price-formation still cannot finally be predicted and we can characterize the latter process as random walk? What is it that still guarantees market efficiency?

The answer to all these questions is market liquidity: this ensures that the market functions efficiently and market prices cannot be predicted. Namely, Farmer et al. (2006) state that buy and sell side imbalance move together with the liquidity imbalance of the two sides, thus a certain amount can be bought or sold with a different price impact on the buy and the sell sides. This statement was based on Bouchaud et al.’s (2004), and Lillo and Farmer’s (2004) findings, who have also come to the same conclusion.

Supply-demand and liquidity imbalance guarantee market efficiency according to the followings: in the case when a buyer initiated order is executed, then the prices should go up. But in the case, when most of the market participants expect to have a buy order more likely, the available volume on the best ask price level will be greater then the buy market order, which will result a smaller price change – if there is a change at all – than is expected. In sum, simultaneously with the expected price-shift, a liquidity imbalance evolves, and the price impact of a buy order soon ceases with the liquidity increase, thus assuring market efficiency and the unpredictability of the directions of the price-shift (Farmer et al. 2006). Therefore the relative size of the orders on bid and ask side and the relative liquidity of the two sides moves in the opposite direction than the imbalance in the order sign (Lillo and Farmer, 2004). Among others, this is why liquidity has a crucial role in market functioning. The concept of market liquidity will be described in the following chapter.

II. The Budapest Liquidity Measure

1. Basic concepts of market liquidity

The concept of liquidity does not have a uniform definition. The different definitions are collected by Michaletzky (2010). However, in the present dissertation I am concerned with the market liquidity of financial assets, accordingly I am going to use the liquidity concept spread on the financial markets, which is a definition also accepted by the Bank for International Settlements since 1999.

BIS (1999, p. 13): "Liquid markets are defined as markets where participants can rapidly execute large-volume transactions with little impact on prices."

Thus in the sense of this definition the larger the volume which can be sold or bought and the smaller the price shift and the shorter the interval, the more liquid the particular market is. It depends on each market participant's utility function to what extent they take these three different factors – time, price impact as transaction cost, volume – into consideration. For instance, in the case of a given volume there are market participants who rather find it important that the transaction is quickly realized, while for other participants it is more important to have the most favourable average price possible, and they are willing to wait more in order to minimize the transaction costs.

Therefore, market liquidity determines how easily and cheaply a particular investment instrument can be traded with. For this reason, the concept of liquidity is very important for all market participants, especially from the investors' point of view. Namely, if the liquidation of a position is only possible with high costs in the future, then the market will built this fact into the current price. Thus the risk caused by low liquidity will appear in the expected returns under normal market circumstances (Csávás – Erhart, 2005).

Amihud and Mandelsen (1991), and Fleming (2003) have shown that the volatility of expected returns will be higher because of the low liquidity, therefore considering two assets which have entirely identical attributes, invertors will expect an extra return (premium) from the one with the lower liquidity. Besides, Amihud and Mandelsen (1986), Amihud (2002), and Pastor and Stambaugh (2003) have also found, that if they filter all risk factors during the estimation of the expected return of a certain asset, then the asset with a lower liquidity has a higher return.

The loss due to the lack of liquidity cannot only be sensed in the price, but it can have a time-value loss as well, namely that the transaction is not executed immediately, and therefore the time value of money is the reason behind the reduction in the value of a financial asset (Major, 2008).

According to the BIS report in 1999, researchers have identified three main stylized facts concerning the dynamics characterizing market liquidity. These stylized facts are the following:

1. <u>Concentration of market liquidity</u>: in the case of substitutable assets liquidity often concentrates in one or only in a few assets. This can be observed on the market of government bonds or even on the market of forward contracts where the most liquid asset is generally the one that expires the soonest (BIS, 1999).
2. <u>Evaporation of market liquidity:</u> Muranaga and Shimizu (BIS, 1999) examined with help of simulation how liquidity affects price discovery during the crisis. During the simulation they got the result that after a market shock the evaporation of liquidity guaranteed for the market that prices would not fall any further and would not drop below a value which is justified by the fundamentals.

   Through simulation the authors have also examined the conditions under which a secondary price-fall also occurs during a shock on the market. They got the result that if after a shock market actors upgraded their views on the market value of an asset, then the secondary price-fall would not occur. However, in case that the expected future market price is low, a secondary shock occurs and entails a further price-fall, which is not justified by the change in fundamentals.
3. <u>Flight to liquidity:</u> a fact can be observed on the market, that in the case of shocks and crises, the investors invest their wealth into assets that can be considered as liquid. During crises investors are willing to pay a premium in order to possess a liquid asset. However, it does not mean that during crises liquidity would rise on the market of these products (BIS, 1999).

1.1. Dimensions of liquidity

Market liquidity is important for numerous market actors. Hence it is indispensable to measure it appropriately. However, the market liquidity concept is too complex to be possible to capture it with a single indicator. A vast number of indicators are at the market actors' disposal, which tends to highlight different aspects of liquidity. Before the thorough analysis of liquidity and the presentation of its possible indicators, it is worth to define the various dimensions of liquidity along which it can be measured. It is important, because each indicator can only measure liquidity in certain dimensions. In the literature an enumeration distinguishes the following dimensions (BIS, 1999) which are completed by Kutas and Végh (2005) with the dimension of diversity:

- Static dimensions:
    - tightness,
    - depth,
        - breadth,
- Dynamic dimensions:
    - resiliency,
    - immediacy.
- Diversity.

Static dimensions and resiliency dimensions are linked to Kyle (1985) who first used these concepts and defined liquidity along these dimensions. The enumeration was completed with the dimension of immediacy by Harris (1990) and with diversity by Kutas and Végh (2005).

There are some indicators which quantify one dimension, these are called one-dimensional indicators. Besides, there are indicators which measure liquidity along more than one dimension (von Wyss, 2004). However, there is no single indicator which would incorporate all dimensions.

During the quantification of liquidity the problem occurs that different measurement methods and indicators do not give the same results, as each dimension highlights different aspects of liquidity (Csávás and Erhart, 2005).

1.1.1. Static dimensions

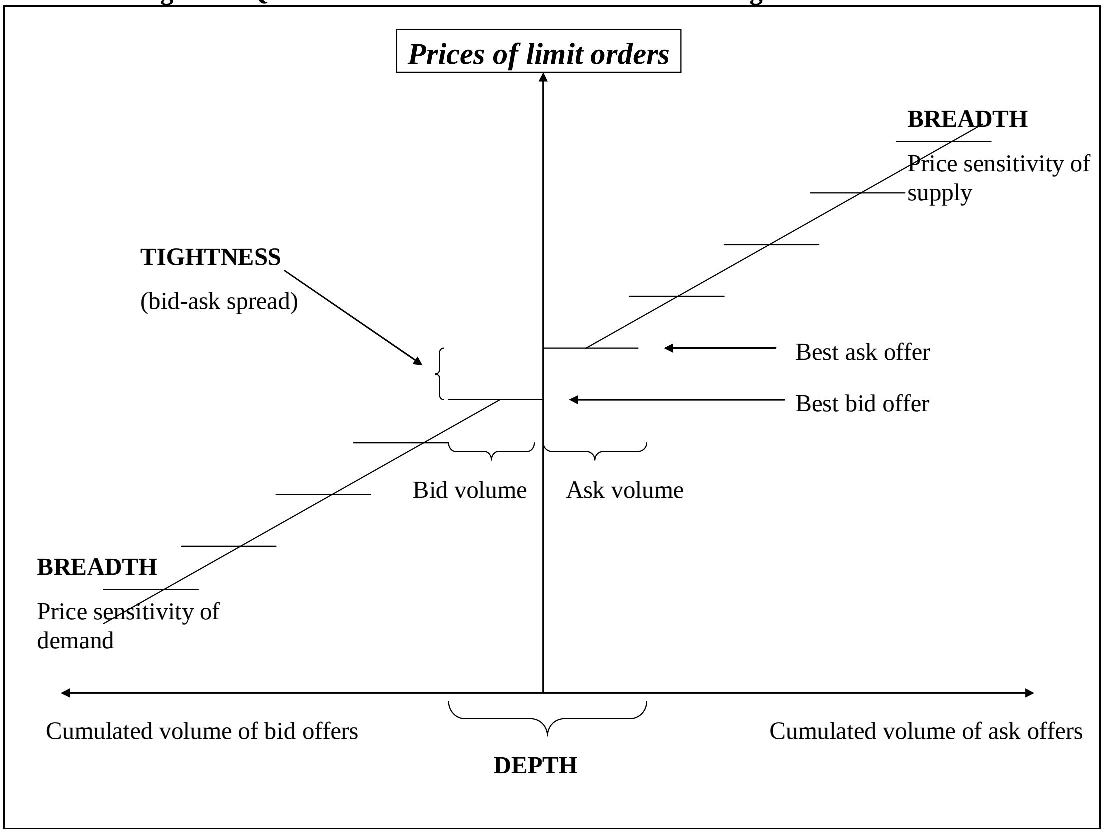
Indicators of the static dimensions of liquidity can be divided into two main groups: one measures tightness, the other measures market depths. Dimension of tightness means the transaction costs of the trading, namely the lowest cost of matching supply and demand. This is generally quantified by the bid-ask spread (Kyle, 1985), which can be determined as the difference of the best buy and sell prices.

Depth of the market means the amount of orders on the bid and ask sides above and below the market price. In a narrower sense depth shows the extent of the order that has the highest volume which can be executed without a price shift in case of selling or buying (BIS, 1999). Depth is generally approached by market turnover.

The concept of market breadth is closely linked to market depths, which can also be regarded as a dimension of liquidity. Csávás and Erhart (2005) determine the concept of breath by modifying Sarr and Lybec's (2002) definition. Breadth is the wider interpretation of depth, i.e. whereas in the case of depth the amount available at the best price was taken into consideration, in the case of breadth we also count the amounts belonging to other market orders. The breadth indicator is generally the price-sensitivity which can be counted as the slopeness of the line determined by aggregated orders and the price as it is shown in Figure 8. The gentler the slope of this line is, the broader the market is. It has a favourable effect on liquidity if the volumes belonging to the same prices grow and if the differences between each order price levels are as low as it is possible. Besides, in the case of breadth dimension it is also important that as many investors as it is possible should appear on the market with their order, because this also has a favourable effect on liquidity (von Wyss, 2004).

The recently mentioned three dimensions can be quantified according to the data in the order book. Therefore, as long as order book data are available on a market, tightness, depth and breadth can easily be determined, as it is shown in Figure 8.

Figure 8: Quantification of static dimensions according to the order book



Source: Ranaldo (2001), p. 312.

These three dimensions are called static dimensions, because they characterize the order book at a given moment. Market liquidity is approached by tightness from the aspect of price, whereas it is measured by depth and breadth from the aspect of volume. However, liquidity is influenced by the change of the order book with the passage of time, thus it is necessary to examine liquidity from dynamic aspects as well.

1.1.2. Dynamic dimensions

Dynamic dimension has two types: resiliency and immediacy. Resiliency refers to the speed with which price-fluctuations originated from trades flatten, i.e. it gives information on how quickly the price returns to an equilibrium level after a shock (Borio, 2000). This equilibrium price can either be a value determined by fundamentals, or even by a state when buy and sell orders were balanced in the order book. In this case liquidity can be measured with the time the bid-ask spread returns to its original value. Besides, liquidity can also be assessed by price impact indicators which quantify how a transaction of a given size changes the price. These indicators are related to the concept of resiliency in the aspect that they can quantify to what extent the trading of different financial assets causes price change. As long as it is low in the case of a certain asset, then it is probable that its resiliency is higher, i.e. its price returns to the equilibrium price quicker.

The dimension of immediacy refers to the time during which a certain size portfolio can be sold or bought in a determined price-range, i.e. it contains the cost connected to the delayed execution of orders (Harris, 1990). It can be measured with the number of transaction realized within a given interval, with the frequency of transactions or even with the number of new orders (von Wyss, 2004).

1.1.3. Diversity

Apart from static and dynamic dimensions, another one exists: the diversity, which shows the market investors' homogenity according to motivation, size, information and home country or foreign residency. The more heterogeneous the composition of the investors is, the more stable the market is in tough market situations. Diversity can be measured with concentration analysis (Kutas and Végh, 2005).

The calculation of concentration serves not only for measuring market participants' homogeneity, but it also can be used to measure the concentration level of market participants doing business with a given market maker. However, in this case we measure market depth with concentration, i.e. the lower this sort of concentration is, the bigger the liquidity is, because the share of large market participants decreases, and therefore the chance that they shift the market price with a bigger transaction size decreases as well.

Besides, concentration can serve as the measurement of market tightness, since the smaller the concentration, the more the volume is distributed among market makers, thus during quotations, market makers can read similar pieces of information from the turnover data, and as a result quotations reflect a more accurate value (Csávás and Erhart, 2005).

1.2. Indicators of liquidity

After visiting the dimensions of liquidity I present its indicators according to Csávás and Erhart's (2005) classification. Von Wyss (2004) provides a more detailed categorization for these indicators. I dedicate a separate subchapter (Subchapter 1.3) to the liquidity measure indicator, since this indicator group is the basis of my empirical analysis. The liquidity indicators can be categorized as follows (Csávás and Erhart, 2005, p. 69):

1. Indicators of transaction costs:
   a. Bid-ask spread: $\mathrm{Spread}_t = P_t^{Ask} - P_t^{Bid}$, where P_t^{Ask} / P_t^{Bid} is the best ask/bid price.

   b. Relative spread: $\mathrm{RSpread}_t = \dfrac{P_t^{Ask} - P_t^{Bid}}{\left(P_t^{Ask} + P_t^{Bid}\right)/2}$

   Analysts generally calculate bid-ask spread and also the relative spread with an actual and an indicative method. The difference between them is that actual spread is counted based on the prices at which a transaction is actually realized, whereas the indicative spread is calculated according to market makers' orders which do not classify as transaction orders. However, the time series of the two different types of spread calculation move tightly together, thus both time series are used for the investigation of liquidity (Chordia et al., 2001).

2. Indicators of volumes:
   a. Frequency of transactions: $n_t = \dfrac{N}{T}$, which gives the number of transactions (N) during a given *T* interval.

   b. Order volume: $Q_t = \dfrac{q_{Ask} + q_{Bid}}{2}$, where q_{Ask} and q_{Bid} mean the average buy and sell volume in the order book within a given *t* interval.

   c. Turnover: $V_t = \sum_{i=1}^{N_t} p_t^i q_t^i$, where *p* denotes the price, *q* the volume of the *i*th trade at time *t*.

   d. Average transaction size: $\mathrm{AvgTrSize}_t = \dfrac{V_t}{N_t}$.

3. Indicators of prices:

   a. Price impact indicator I.: $\gamma_t = \frac{|\Delta p_t|}{TrSize_t}$, where $|\Delta p_t|$ is the price change caused by the *t*th transaction (alternatively the price change in time period *t*), and $TrSize_t$ is the size of the *t*th transaction (the overall transaction size in time *t*).

   b. Price impact indicator II.: $\delta_t = \frac{|\Delta Spread_t|}{AvgTrSize_t}$, where $|\Delta Spread_t|$ is the change of the spread of the *t*th transaction (period), and $AvgTrSize_t$ is the average size of the transactions in the *t*th period.

   c. Spread resiliency indicator: $\varepsilon_t = \frac{|\Delta Spread_t|}{SpreadConTime_t}$, where $SpreadConTime_t$ shows the convergence time of the spread. In other words, if a transaction widens the spread, this is the time needed for the spread to return to the pre-transaction level.

4. Concentration:

Concentration cannot measure liquidity as directly as the bid-ask spread or the turnover can, but indirectly it is a good indicator of market liquidity. Berlinger, Michaletzky and Szenes (2011) examined the uncollateralized interbank HUF market, and found that concentration was closely related to market liquidity and economic cycles. Also Csávás and Erhart (2005) explained the size of the bid-ask spread with concentration, volatility and turnover in a regression model. The authors have found that concentration had a significant explanatory power concerning the bid-ask spread.

Statistics provide a wide range of methods for the measurement of concentration. In the case of market liquidity the Herfindahl-Hirschman index is used most frequently , which is calculated by the $HHI = \sum_{i=1}^{N} Z_i^2$ formula, where *Z* shows a particular market participant's relative market share and *N* is the number of market participants. The value of this index moves between the limits of *1/N* and *1*. In case of lack of concentration, i.e. if all the market actors have the same share of the total value, then *HHI = 1/N*. If all the elements of a statistical population can be found at a certain sub-unit of this statistical population, then $HHI = 1$ (Hunyadi and Vita, 2003).

The above enumerated liquidity indicators – as I have mentioned before – are not suitable to examine liquidity in all dimensions. Table 3 shows which dimensions of liquidity can be measured by the recently presented indicators.

Table 3: Categorization of liquidity indicators

Liquidity dimension	Liquidity indexes
Tightness	Transaction-based indexes
	Concentration of market maker's clientele
	Liquidity measures[15]
Depth	Amounts belonging to the best prices
	Average transaction size
	Turnover
	Concentration of market maker's clientele
	Liquidity measures
Breadth	Supply-demand price sensitivity
	Liquidity measures
Resiliency	Price impact indicators
Immediacy	Frequency of transactions
	Turnover
Diversity	Concentration of market participants

Source: Csávás és Erhart, p. 19, and my own additions

According to Table 3 it can be stated that there are hardly any indicators which could measure liquidity along more than one dimension, which is however an indispensable condition for getting an exact view of market liquidity. Dömötör and Marossy (2010) have accomplished a more detailed analysis and a categorization along several dimensions than Table 3 by using multivariable statistical tools. In the following chapter I present a liquidity indicator, which can measure liquidity along all the static dimensions and thus can give a more complete view of market liquidity.

1.3. Liquidity indicators based on the Xetra Liquidity Measure

The Xetra Liquidity Measure (XLM) has been created by the Deutsche Börse Group in 2002. Based on XLM a few other countries have developed similar indicators. One of them was in Hungary at the Budapest Stock Exchange, the other

[15] I will explain this notion in the subchapter 1.3.

one in Slovenia, at the Ljubljana Borza. The name of the indicator in Hungary became Budapest Liquidity Measure (BLM), while in Slovenia it was named CGT. The only difference between the three indicators – XLM, BLM, CGT – that they are calculated for those securities which are traded in the respecive country.

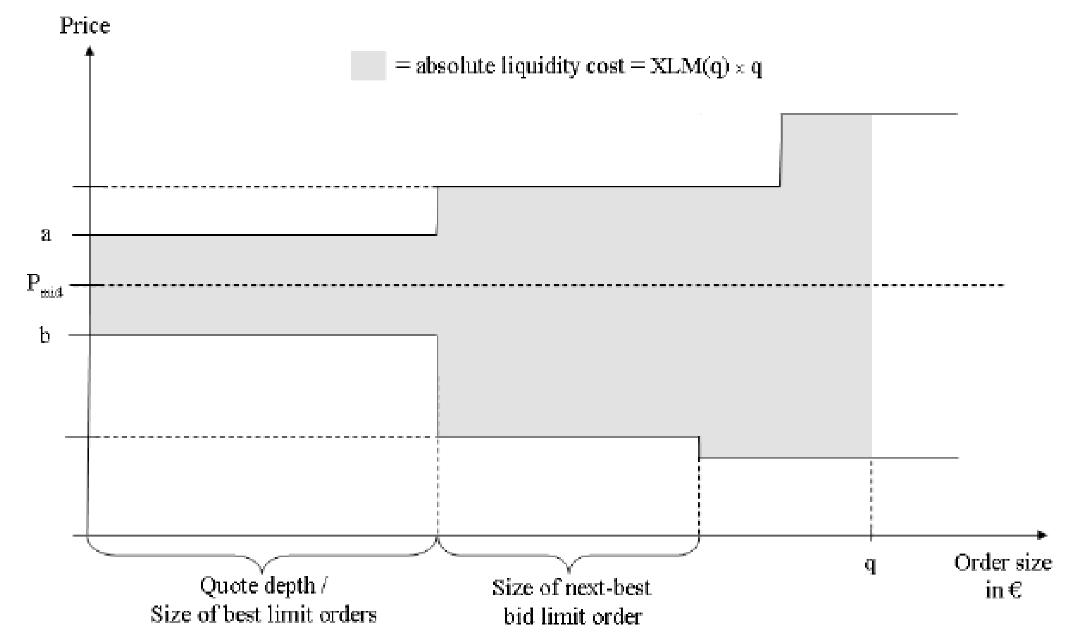
The liquidity measure was created by the Deutsche Börze to provide the market with a simple index which assists market participants in making investment decisions by showing how liquid the individual security and the entire market are at the moment. The liquidity measure quantifies the transaction cost of a certain trade in order to help market participants in their investment decision. Liquidity is calculated as the sum of the adverse price movement (APM) – originated in the transactions of the investors – and the liquidity premium (LP) to be paid for the transaction. The adverse price movement occurs if the total volume of the order cannot be fulfilled on the best price level i.e. on other levels are needed as well. Then the average price of the total order will be worse than the best possible price, while the liquidity premium is the half of the bid-ask spread. These two factors (APM and LP) together are also referred to as the implicit cost or indirect cost of trading (Gomber and Schweikert, 2002). The size of this cost depends on the current state of the order book. Trading also incurs explicit or direct costs, e.g. brokerage fees and commissions, stock exchange fees, taxes, etc. (Kutas and Végh, 2005). These costs are not included in the BLM as these can easily be identified and quantified, and the aim of the BLM is to measure the implicit costs not measured earlier. While calculating the liquidity measure we cannot take the opportunity cost into account and the costs of timing, either. In sum the total cost of a transaction is built up as follows, based on Gomber and Schweikert (2002):

- Implicit costs
    - Market impact costs
        - Liquidity premium
        - Adverse price movement
    - Costs of timing
    - Opportunity costs
- Explicit costs

According to this liquidity measure is one of the transaction based liquidity indicators, but it can interpret liquidity more broadly than the bid-ask spread, since it can measure liquidity not only in the dimension of tightness, but in respect of depth and breadth as well.

The XLM liquidity indicator measures that the percentage of the total order size being paid as a transaction cost. The indicator can be interpreted only at certain order sizes, as it is shown in Figure 9. The figure shows how the liquidity measure quantifies the transaction costs. The grey area shows the total implicit costs. If it is divided by the total order size, then we get the relative cost, the Xetra Liquidity Measure.

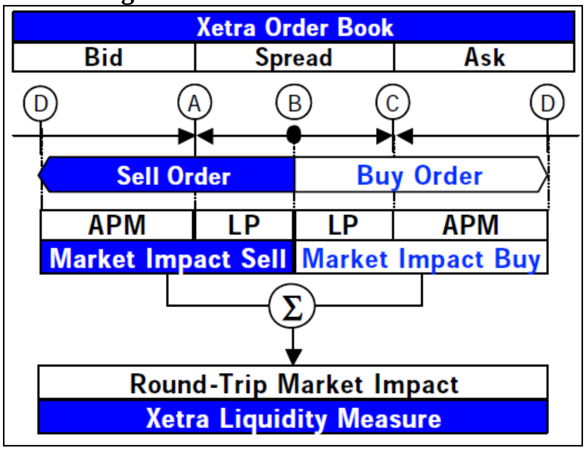
Figure 9: Calculation of the implicit cost



Source: Stange and Kaserer, (2009b), p. 6.

Figure 9 shows the calculation of the Xetra Liquidity Measure, which is used also by the Budapest Stock Exchange to calculate the Budapest Liquidity Measure. Figure 10 shows also the calculation of the liquidity measures from another approach.

Figure10: The calculation of XLM



Source: Gomber and Schweikert (2002), p. 3.

The calculation of the indicator in detail is as follows:

The calculation of the bid-ask spread (*Spread*) and the liquidity premium (*LP*) is based on the following formulae:

$$\text{Spread} = \frac{P_{ask1} - P_{bid1}}{P_{mid}}, \tag{3}$$

$$\text{LP} = \frac{\text{Spread}}{2} \tag{4}$$

where P_{bid1} = the price level of the best bid orders, P_{ask1} = the price level of the best ask orders, and P_{mid} is the mid price, where $P_{mid} = \frac{(P_{bid1} + P_{ask1})}{2}$.

The adverse price movement (*APM*) should be calculated for both the bid and the ask side of the order book, since the two sides can differ substantially from a liquidity perspective. The way the APM is measured:

$$\text{APM\_ask} = \frac{(P_{w\_avg\_ask} - P_{ask1})}{P_{mid}} \tag{5}$$

$$\text{APM\_bid} = \frac{(P_{bid1} - P_{w\_avg\_bid})}{P_{mid}} \tag{6}$$

The software calculating the BLM uses the following formula for $P_{w\_avg\_ask}$, the *weighted average ask price* in Equation 5. The weighted average bid price is similarly determined. For the sake of simplicity let us assume that the order is fulfilled at the three best price levels:

$$P_{w\_avg\_ask} = \frac{P_{ask1} \cdot \text{size1} + P_{ask2} \cdot \text{size2} + P_{ask3} \cdot (\text{transaction size} - \text{size1} - \text{size2})}{\text{transaction size}} \tag{7}$$

where P_{ask1} is the price level of the first best ask order, P_{ask2} is the price level of the second best ask order, P_{ask3} is the price level of the third best ask order, size1, size2 are the quantities transacted at the given price levels. In case the market is not deep enough, and – let's assume – that there isn't any order on the third price level – or if there are orders, but not enough to be able to fulfill the whole order – then the software calculates BLM as if the order book included infinite orders at the last available price level. This distorts the value of BLM, since it shows a higher liquidity on the market than in reality.

The value of the liquidity measure is the sum of the liquidity premium and both sides' adverse price movement:

$$\text{Liquidity Measure} = 2\text{LP} + \text{APM\_bid} + \text{APM\_ask} \tag{8}$$

Based on Equation 8, BLM gives the total implicit cost of turning around a position in basis points (Kutas and Végh, 2005).

For example, if we calculate BLM for an order size of EUR 500,000, and the result is 60 bps, then since the order is not fulfilled at the mid price, the implicit cost of turning around a position of EUR 500,000 is EUR 3,000 (500,000 × 0,006 = 3,000).

The calculation of all three liquidity measures' (XLM, BLM, CGT) is the same than the one I have shown above. The difference is that the three liquidity measures are calculated for different order sizes on each stock exchange.

Deutsche Börse Group provides the market the XLM indicator for standard order sizes. The order sizes at which the XLM is calculated differ from stock to stock. It depends on the turnover of a certain stock (Gomber and Schweikert, 2002). The

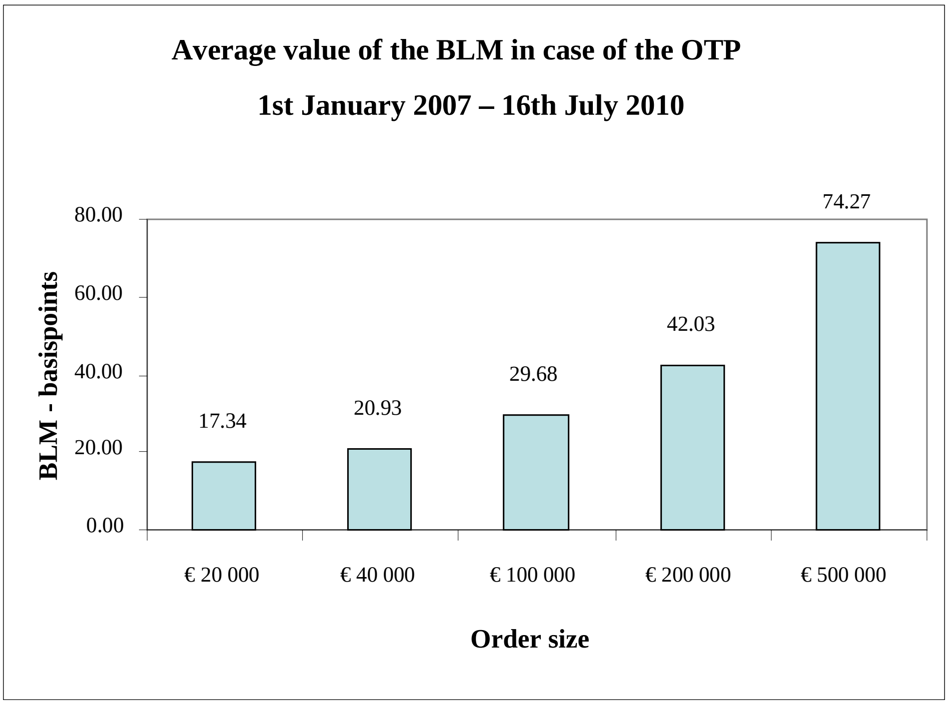
XLM is calculated for the following order sizes in each case: EUR 10 thousand, 25 thousand, 50 thousand. In case of stocks with a higher turnover, the measure is calculated also for the following order sizes: EUR 75 thousand, 100 thousand, 150 thousand, 250 thousand. In a few cases calculation takes place also for much greater sizes, like: EUR 500 thousand, 750 thousand, 1,000 thousand, 2,000 thousand, 4,000 thousand, 5,000 thousand.

The CGT is the liquidity measure of the Ljubljana Borza (LJSE). The liquidity measure on this stock exchange is published twice a day, at 11:00 AM and 12:55 PM for only one order size, to EUR 7,500. The value of the published CGT is the arithmetic average of the CGT values of that certain day (LJSE, 2011).

The BLM database determines the BLM values for 5 different order sizes – therefore I have 5 different BLM figures for each of the shares listed on the BSE – i.e. for transactions worth EUR 20 thousand (BLM1), 40 thousand (BLM2), 100 thousand (BLM3), 200 thousand (BLM4), and 500 thousand (BLM5).

In case of OTP, the average BLM values of the five order sizes between 1st January, 2007 and 16th July, 2010 are shown in Figure 11. It can be seen, that the bigger transaction an investor wants to execute, the higher the BLM value is.

Figure 11: Average BLM values for OTP



Source: own figure, published in Gyarmati et al.(2010a), p. 502.

The previously introduced XLM cannot measure liquidity along the dynamic dimensions (resiliency, immediacy), only along the static dimensions (tightness, depth, breadth). Since the calculation of the measure is based on the actual state of the order book, so its calculation can be carried out only for the given moment. Nevertheless the XLM-type liquidity measures give a more precise picture of the liquidity, since it can measure it along more dimensions.

2. Empirical research: analysis of Budapest Liquidity Measure

The goal of the chapter is to give a detailed description about the database of BLM and its relation to other liquidity indicators in the case of the 13 stocks of which BUX consisted as of April 1, 2009. Apart from this, I also examine the relationship between volatility and liquidity during the crisis, as well as both before and after it. I do this in order to receive a more complete view of the indicator before I present the two possible application opportunities on which my dissertation is based. Namely, how to build a VaR model adjusted with liquidity risk and how to estimate a price impact function with the aid of BLM.

However, before describing the database which is the basis of my analysis, I present my main research questions, the applied methodology, and shortly the Hungarian literature which preceded my examinations and which also analyzed the liquidity of stocks in Budapest Stock Exchange.

2.1. Research on the Budapest Stock Exchange for the time being

On Budapest Stock Exchange Kutas and Végh (2005), Barra (2008), Margitai (2009), and Michaletzky (2010) have made significant research.[16] The starting point of my dissertation was Kutas and Végh's (2005) research, these authors having created the BLM following the pattern of XLM in 2005. The authors have presented the build-up and the calculation method of BLM. Furthermore, they have accomplished an international comparison in the case of stocks which were listed both on the BSE and on foreign stock exchanges as well. They came to the conclusion as a result of their research that on Budapest Stock Exchange the BLM, i.e. the size of the implicit cost is remarkably lower in a case of a particular stock than on other exchanges where the stock was simultaneously listed (Kutas and Végh, 2005).

In his research, Barra (2008) has examined the dynamics of the liquidity indicators based on *volume weighted transaction duration* and *capital weighted*

[16] Apart from these studies numerous Hungarian publications are available, which were concerned with stock exchange database analysis (e.g.: Fazakas and Juhász, 2009; Móricz, 2005), but I have only highlighted the ones that analysed the liquidity on BSE.

transaction duration according to the trades and quotes (TAQ) database of MOL within the framework of an ACD model (*autoregressive conditional duration).*[17] By duration he meant the expected time period during which a particular quantity or value of stocks can be bought/sold. In his thesis, Barra (2008) has presented the functioning of ACD models and the way we can predict duration-based liquidity indicators with the help of it, and how to create estimation for the future liquidity through it. One of the author's most important findings is that the Log-GGACD (1,1) model fits the data best; although it fit the examined data well, but there were periods regarding the examination of the out of sample data when the model did not work properly. The author has explained this with the fact that presumably there was a structural break in the database.

Margitai (2009) has also made a research relying on MOL TAQ database. His most important research questions were the following:

1. What are the underlying reasons behind the stylized facts which characterize the orderflow?
2. What is the relationship between liquidity and market efficiency?
3. Why is the price impact function concave?
4. What are the factors that influence the size of the spread?
5. What influences the formation of gaps between the price-levels in the order book?

One of the many answers he came up with is the one, that the sign of transactions is a long memory process (see Subchapter I/2.5 of my dissertation). Furthermore, the author has also shown that the better is the prediction concerning the direction and the size of an order, the lower price impact the order would have, which can be due to the compensatory role of liquidity strategy.

During his research, Margitai (2009) has also estimated an empirical price impact function based on the MOL TAQ database (see the Subchapter IV/1.4 of my dissertation). He has concluded that the more transaction he aggregated, the more concave shape the price impact function would have (which is similar to the experience in international researches).

Michaletzky (2010) has accomplished a time-series and cross-sectional analysis of different liquidity indicators on the TAQ database of the four biggest

[17] For more details about ACD models see Engle and Russell, 1998.

stocks traded on Budapest Stock Exchange (OTP, MOL, Magyar Telekom and Richter). Furthermore, he attempted to predict future liquidity with the help of the Hurst-exponent, which he did by analyzing the indicators: turnover and bid-ask spread predictions.

On one hand, one of Michaletzky's (2010) most important findings is that the intervals between transactions (*durations*) are predictable, however, in turbulent periods this effect is less significant. The author has also pointed out that in the case of each stock there was no big difference concerning the predictability of the duration, whilst the forecast of bid-ask spread was not significant in the case of none of the stocks. The author's other important achievement was that there was a strong positive relation between the relative spread and the turnover (measured in pieces), the extent of correlation was 0.82, which – according to his statement – indicates that liquidity improvement in one dimension is often accompanied with its deterioration concerning another dimension. Thirdly, his further interesting finding is that there is a strong positive relation (correlation is 0.82) between the percentile true range (*TR*) and relative spread, which indicates that the uncertainty appearing in the high price fluctuation increases the spread.

Finally, I intend to present Csávás and Erhart's (2005) research, which however is not based on the stocks of Budapest Stock Exchange, but on the data of Hungarian foreign exchange- and government bond markets. Regardless I consider this to be worthwhile to review, because they have also examined the relationship of liquidity and price fluctuation as Michaletzky (2010) did.

During the examination, the researchers have proceeded from the same observation that Michaletzky (2010) made, i.e. there is a strong positive relation between bid-ask spread and turnover. Csávás and Erhart (2005) have explained this phenomenon with volatility. According to their statement, as a consequence of increasing volatility market makers raise the spread in order to price their increased risk, while the augmented volatility entails turnover growth, especially in turbulent periods. According to their opinion, if the spread-growth is caused by the increasing volatility, it does not necessarily imply the decrease of liquidity. In order to make conclusions, we should know the reason of volatility increase (Grossman and Miller, 1988). Namely, the rise of volatility can be the consequence of the fact that the expectations concerning fundamentals change faster, or perhaps new pieces of information arrive at the market more quickly. In this case volatility is not harmful for liquidity, but it implies that the market fulfils its main function: the displaying of expectations in market prices (Csávás and Erhart, 2005, p. 24).

However, the authors have not found a model in the literature which could appropriately analyze the relationship between volatility and liquidity. For this reason they have applied the spread model which was also the basis of previous research (e.g.: Galati, 2000; Wei, 1994; Huang and Masulis, 1999, etc.). The model analyzed by them was the following linear regression, which they have completed with other factors in different phases of their research:[18]

$$\text{Spread} = \alpha + \beta_1 \cdot \text{volatility} + \beta_2 \cdot \text{turnover} + \beta_3 \cdot \text{concentration} + \varepsilon \qquad (9)$$

Taking this linear regression for basis, Csávás and Erhart (2005) analyzed the factors influencing the spread, during which their most important findings concerning volatility and spread were the followings:

- One of the strongest impacts on forint market bid-ask spread was exerted by volatility.
- The coefficient of the chosen volatility indicator[19] is positive. The 1 percentage point increase of the intraday fluctuation of volatility causes a 2 basispoints increase in bid ask-spread other conditions being equal.
- According to the results they could not clearly decide whether the spread-increase caused by volatility implies the deterioration of market liquidity. In their opinion it depends on the reason causing volatility increase.
- The decrease of volatility significantly lessens the spread, which is favorable for the investors because of the lower trading costs, and for the market makers because of the lower risk.
- They have divided volatility into two components: expected and unexpected components and thus they have also inserted it into the model. The authors have filtered the part from the volatility which had been expected for the given day

[18] I do not present these other factors in my dissertation, because during my research I will only apply equation (9), based on the method described in Subchapter II/2.4. For more details about the further models applied by these authors, see Csávás and Erhart's (2005) research.

[19] The authors have defined the volatility indicator in two different ways: in one case with the aid of GARCH model, in the other case they have observed the difference between the daily minimum and maximum price levels in percentage. I will give a more detailed description about these in Subchapter 2.4.

based on past information, and then they considered the rest as the unexpected component. From the expected and unexpected components of volatility only the unexpected one became significant, therefore the shocks affecting volatility are reflected in the spread. It may refer to the fact that only newly arriving pieces of information affect the spread change, whilst the impact of expected volatility is already included in the spread.

2.2. Database

The database of Budapest Liquidity Measure forms the basis of my research. The BLM values can be determined by the actual order book. During my research I have examined the data between 1st January, 2007 and 3rd June, 2011 from the BLM database which was created based upon the order book. In the examined period the database contains the BLM data for every second of each trading day from 9:02 AM until 4:30 PM when any change occurred in the order book. Furthermore, the database contains the BLM data of every security traded on BSE, on all the five order sizes (EUR 20 thousand, 40 thousand, 100 thousand, 200 thousand and 500 thousand). However, the database contains not only the BLM data, but also the three components of BLM on every transaction size: bid-ask spread, APM_bid and APM_ask. Furthermore, it contains some other data which provide information about the trade. Table 4 and 5 show a small part of the OTP BLM database on 12th September, 2007.

Table 4: The BLM database

Date	Time	LP (bps)	spread (bps)	APM_ bid1 (bps)	APM_ask1 (bps)	BLM1 (bps)	…	APM_bid5 (bps)	APM_ask5 (bps)	BLM5 (bps)
2007.09.12	10:00:01	4.36	8.72	0.00	5.28	14.00		35.47	14.71	58.90
2007.09.12	10:00:07	4.36	8.72	0.00	4.07	12.78		35.47	14.12	58.31
2007.09.12	10:00:15	4.36	8.72	0.00	0.13	8.85		35.47	13.88	58.07
2007.09.12	10:00:34	4.36	8.72	0.00	0.13	8.85		35.30	13.88	57.90
2007.09.12	10:00:36	4.90	9.81	0.00	0.13	9.94		34.73	13.88	58.42
2007.09.12	10:00:39	4.90	9.81	0.00	0.13	9.94		34.73	13.88	58.42
2007.09.12	10:00:49	4.90	9.81	0.00	0.13	9.94		28.94	13.88	52.63
…										

Source: my own edition based on the database of Budapest Stock Exchange

Table 5: Other data in the BLM database

Time	Mid price (HUF)	bid number	Bid price levels	Bid value (thHUF)	ask number	Ask price levels	Ask value (thHUF)	Last traded price (HUF)	Quantity	Turnover (thHUF)
10:00:01	9 176	517	173	1 186 121	728	249	2 703 746	9 180	238 280	2 186 499
10:00:07	9 176	517	173	1 186 121	729	250	2 717 531	9 180	238 280	2 186 499
10:00:15	9 176	517	173	1 186 121	730	250	2 719 367	9 180	238 280	2 186 499
10:00:34	9 176	518	173	1 186 578	730	250	2 719 367	9 180	238 280	2 186 499
10:00:36	9 175	518	173	1 186 571	730	250	2 719 367	9 180	238 280	2 186 499
10:00:39	9 175	518	173	1 186 571	731	250	2 719 967	9 180	238 280	2 186 499
10:00:49	9 175	519	173	1 204 871	731	250	2 719 967	9 180	238 280	2 186 499
…										

Source: my own edition based on the database of Budapest Stock Exchange

2.3. Research question

During a series of interviews about market liquidity,[20] market participants have told me that they also take liquidity into consideration as a significant risk factor for their investment decisions. According to them, market participants commonly categorize stocks into liquidity classes and they decide about their market entrance and strategy based on this. There are participants who are only willing to invest in liquid stocks, e.g. a significant part of technical analysts. However, there are those who are also willing to purchase illiquid stocks, for instance the passive fund managers. Furthermore, participants who are fundamental analysts are also willing to buy illiquid stocks. They act this way in the case when they assume that the fundamental value differs from the market value to such extent that it is worthwhile to buy/sell even if they face significant transaction costs caused by the lack of liquidity, because they will recover the loss by the rise/fall of market price. Concerning fundamental analysis, the interviewees' opinion was that the shorter the period in which somebody trades, the more significant role mathematics and statistics will have, whilst fundamentals are pushed to the background. They explained this with the fact that if e.g. somebody accomplishes a one-second or an even more frequent-period trading, she takes advantage of the inefficiency committed by those who trade for instance in one-day periods and do not constantly modify their portfolio as fresh news appear. However, those who trade on a daily bases profit from the mistakes

[20] The series of interviews was realized with the participation of Edina Berlinger, Ákos Gyarmati, Márton Michaletzky, Balázs Árpád Szűcs, Kata Váradi and Gábor Völgyes and its topic was market liquidity.

committed by those who trade only once a month, etc. The longer the period is in which we invest, the more fundamental analysis comes to the forefront and the role of mathematics and statistics will be pushed to the background. Therefore, time scale is very important when we take liquidity into consideration during a portfolio decision. As follows from the preceding information, the shorter the period in which market participants invest, the higher the value of a particular stock's liquidity will be for them.

In order to state whether a stock is considered to be liquid or illiquid, market participants use simple rules of thumb. The most commonly used indicators for assessing liquidity are the bid-ask spread and turnover (Szűcs and Váradi, 2012).

Based on the interviews, and on the Hungarian studies done in the past, I will compare the BLM to the two most commonly analyzed liquidity indicators, to the bid-ask spread and turnover. I examine to what extent these three liquidity indicators (i.e.: bid-ask spread, turnover and BLM) give similar results regarding liquidity and under what market circumstances can the use of bid-ask spread and turnover be misleading as far as liquidity is concerned.

In addition, I will also examine the relationship between volatility and liquidity, since according to the literature (Michaletzky, 2010; Csávás and Erhart, 2005) we can state that these two variables have a strong positive relation. I am going to examine the relationship between volatility and liquidity during a calm period and – based on this – how much predictive strength the growth of volatility has regarding the decrease in liquidity. After this, I observe what decrease in liquidity the growth of volatility caused on the market during the crisis period and I then make a comparison whether this value is higher or lower than it would have been estimated based on the calm period. If I come to the conclusion that the liquidity is lower than I estimated, then Csávás and Erhart's (2005) statement can be justified that only the volatility increasing effect of the new pieces of information is built into the bid-ask spread growth – and consequently into the liquidity decrease – as, the expected volatility is already reflected in the value of the bid-ask spread. Furthermore, based on the result I can also draw a conclusion whether the 2007/2008 crisis was actually also a liquidity crisis[21] or with the volatility increase only a „natural" decrease in liquidity was accompanied with.

The main questions I examine in the chapter are the followings:

- What average value does the BLM take on the five order sizes in the case of different stocks during the examined period?
- What kind of relationship does the BLM have with the two liquidity indicators which are most commonly applied by market participants, i.e. with the bid-ask spread and turnover?
- How strong is the relationship between liquidity and volatility of an asset?

I consider the examination of these questions as important above all for three reasons. On the one hand because by responding to these questions we can determine which one of the three examined liquidity indicators is worthwhile to use, and which one renders the most reliable result concerning liquidity. On the other hand, I consider this to be important because I intend to provide a basis for liquidity to be able to be traded as a product in the future – even with the help of an indicator as BLM –, and to be able to serve as an underlying asset for derivatives.[22] As a result the risk originated from liquidity could be hedged. However, to achieve this it is inevitable to know the relationship between volatility and liquidity. Thirdly, I find it important because when market participants execute a dynamic portfolio optimization on the market, then it is not sufficient to decide along the return-volatility dimension, they also have to include liquidity into the decision mechanism, since the market risk consists not only of the price-risk, i.e. the change of the mid price, but also of the liquidity risk. For this reason liquidity cannot be ignored during the optimization, and its relationship with return and volatility has to be borne in mind.

Based on the research questions in this phase of my research I am going to find answers for the following hypotheses:

[21] For more details on the crisis and the lack of liquidity that evolved during the crisis see Király (2008), Berlinger, Horváth and Vidovics-Dancs (2012).
[22] For the pattern by which volatility has started to be traded with see Berlinger et al. (1998).

H1: BLM, and the most commonly used liquidity indicators in practice (bid-ask spread, turnover) provide different ranking from the aspect of liquidity for individual stock:

H1/a: during a calm period, and

H1/b: during a crisis.

H1/c: in the case of a liquid and

H1/d: an illiquid stock.

H2: There is a positive relation between volatility (standard deviation, true range) and BLM.

2.4. Research methodology

Benefitting from previous Hungarian research I am going to examine BLM database based on the above presented Equation 9. The difference will be that I divide the linear regression into three parts and I will examine the impact of turnover and volatility separately. Furthermore, instead of concentration I am going to examine the bid-ask spread as an explanatory variable. In all three cases the dependent variable will be the BLM. All in all, the empirical analysis of BLM indicator can be divided into three main parts:

1. First, I present how the BLM database looks like, what are the average values between 1st January 2007 and 16th July 2010. In this part of the research I am going to put together a cross-sectional analysis.
2. Second, I determine the average BLM, bid-ask spread and turnover data for different periods – for a complete time series as well as before, during and after the crisis. After this I observe to what extent the three indicators provide a different ranking, which I am going to test with two rank correlation methods: Spearman's rank correlation method,[23] and Kendall's rank method.[24] Following

[23] Spearman's rank correlation: $\rho = 1 - \frac{6 \cdot \sum_{i=1}^{n} d_i^2}{n^3 - n}$, where $d_i = x_i - y_i$ is the difference of the rank number of the x and y variable by unit number i, and n is the number of the units of the statistical population. The value of the index can be between -1 and 1. If its value is -1 then the order is perfectly opposing, whilst if its value is 1, then the order is perfectly identical (Kerékgyártó and Mundruczó, 1995).

this, I determine the correlation of each indicator with one another, and with the help of a linear regression I observe the explanatory power of bid-ask spread and turnover concerning BLM. Finally, I also examine the connection between the change in the bid-ask spread/turnover and the change of BLM.

3. Third, I assess the relationship of liquidity – which will be quantified by BLM– with volatility. I am going to examine the connection with a linear regression. However, volatility can be measured in different ways, so I have defined it in my dissertation as follows:

    a. Standard deviation of the logreturn: $\sigma = \sqrt{T}\frac{1}{D}\sum_{d=1}^{D}(r_d - \bar{r})^2$, where r_d is the logreturn ($r_d = \ln\frac{P_d}{P_{d-1}}$), $\bar{r}$ is the average return during the given period, and *D* is the number of periods during a (*0,T*) time interval. If we estimate the standard deviation according to this, we assume that the time series on which we based the estimation is stationary, i.e. the distribution of the returns is equal to the long-term „average" distribution of the returns, which means that the expected value and the standard deviation are constant in time.

    b. Standard deviation estimated from GARCH model: If we assume that the time series of returns is not stationary, we can estimate the standard deviation of returns with the GARCH (*Generalized Autoregressive Conditional Heteroscedasticity*) model. GARCH models take the fact (which is commonly observed in practice) into consideration that the standard deviation of returns is persistent, i.e. if the standard deviation once increases, then its value remains high for a long period. This phenomenon causes the clustering of volatility (*heteroscedasticity*), which is the basis of GARCH models (Bollerslev, 1986).

[24] Kendall's method: $W = \frac{12 \cdot \sum_{j=1}^{m}\left(C_j - \overline{C}\right)^2}{m^3 \cdot \left(n^3 - n\right)}$ where $\left(C_j - \overline{C}\right)^2$ shows the sum of squares of each rank number sum's deviation from its mean, n is the number of the units in the statistical population, whilst m shows the number of ranking lists we compared. The value of the index can be between 0 and 1. If its value is 0, then the order is perfectly opposing, whilst if its value is 1, then the order is perfectly identical (Kerékgyártó and Mundruczó, 1995).

c. <u>Difference between the daily minimum and maximum price in percentage</u>: $\text{vol} = \frac{P_t^H - P_t^L}{P_t^L}$, where P_t^H is the daily maximum price, and P_t^L is the lowest one.

d. <u>True range:</u> $TR = \max(P_t^H; P_{t-1}^C) - \min(P_t^L; P_{t-1}^C)$, where P_t^H / P_t^L is the highest/lowest price experienced during the period, whilst P_{t-1}^C is the closing price at the end of the previous period (Wilder, 1978).

As I observe the relationship of BLM and volatility with the help of the linear regression, it is inevitable that volatility data should be available for every trading day. In the absence of data, the standard deviation of logreturn cannot be examined: the intraday prices should be known for this, but they are not at my disposal. Instead, I estimate the standard deviation for each day with the aid of the GARCH model. In this case I have the implicit assumption that the returns I observed are from the distribution which is assumed by the GARCH model during the estimation of the standard deviation, namely from Student's t-distribution in the current case.

I undertake the estimation of the standard deviation with the help of the following AR(1)-GARCH(1,1) model:

$$r_t = c + \phi r_{t-1} + \varepsilon_t \tag{10}$$

$$\varepsilon_t = \sigma_t \eta_t \tag{11}$$

$$\sigma_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + b_1 \sigma_{t-1}^2 , \tag{12}$$

in which Equation 10 is the equation of the expected value (conditional expected value), where r_t signifies the logreturn of the particular day, which depends on the logreturn of the preceding day, r_{t-1}. This is referred to as AR(1), i.e. an equation describing an autoregressive process in which the value of the return of a given day depends on the value of the return of one period preceding it. However, we can estimate the ε_t residuum value of this AR(1) process with a GARCH(1,1) process, where we receive the ε_t value as the product of σ_t conditional standard deviation and η_t (Equation 11), where η_t is a $\text{IID}(0,1)$[25] probability variable. However, for this we need to determine the conditional standard deviation, for

[25] IID(0,1) means the probability variables are independent, and identically distributed, where the expected value is 0 and the standard deviation is 1.

which we need the variance Equation 12 (conditional variance). Equation 12 gears the square of the conditional standard deviation (i.e. variance) to the variance of the previous period (σ_{t-1}^2), and the square of the residuum of the previous period (ε_{t-1}^2). As both the variance (σ_{t-1}^2) and the residuum (ε_{t-1}^2) are from the period directly preceding the current variance, therefore this process is referred to as GARCH(1,1) (Tulassay, 2009).

However, apart from the standard deviation values determined by GARCH model, I am also going to analyze another volatility index, the "true range" (TR). The reason why I use this index instead of the "difference of the daily minimum and maximum price in percentage" is because true range shows market volatility the best, as well as this is the index which is most commonly used by technical analysts to quantify volatility (Makara, 2004).

However, I am going to modify the TR formula previously presented in 3/d subpoint in order to be expressed in a percentile form, namely dividing the TR values by the average market price of a given day. Thus TR calculation will be the following, where P_t^M shows the average price of the particular day:

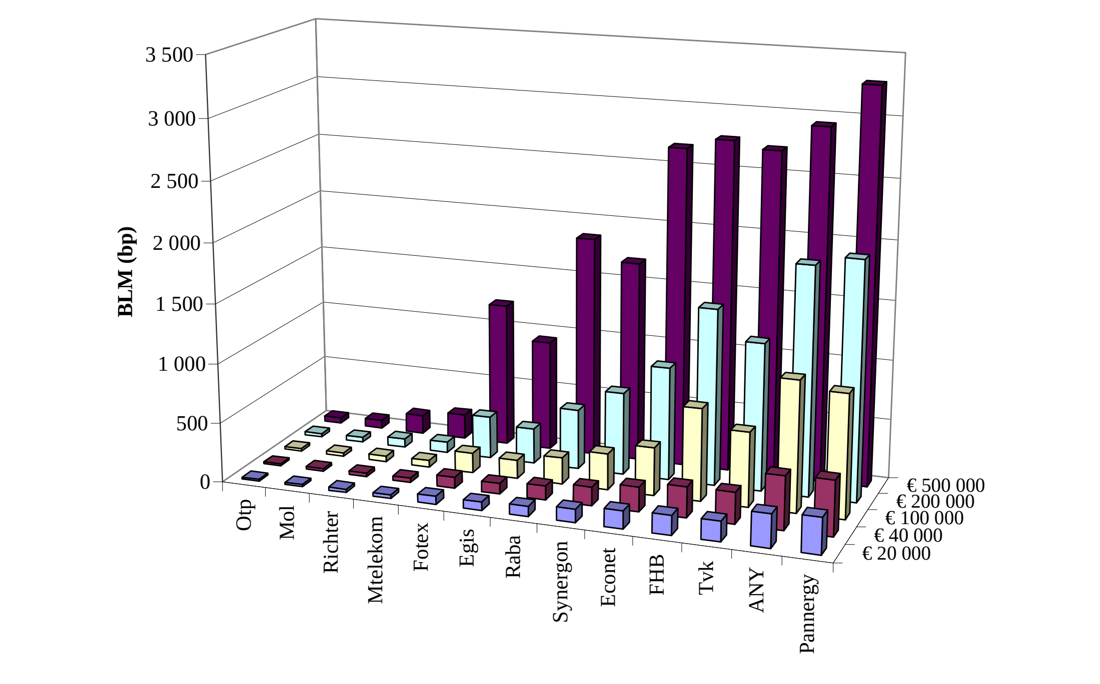
$$TR = \frac{\max\left(P_t^H; P_{t-1}^C\right) - \min\left(P_t^L; P_{t-1}^C\right)}{P_t^M} \tag{13}$$

2.5. Results

2.5.1. Average BLM values of the BUX shares

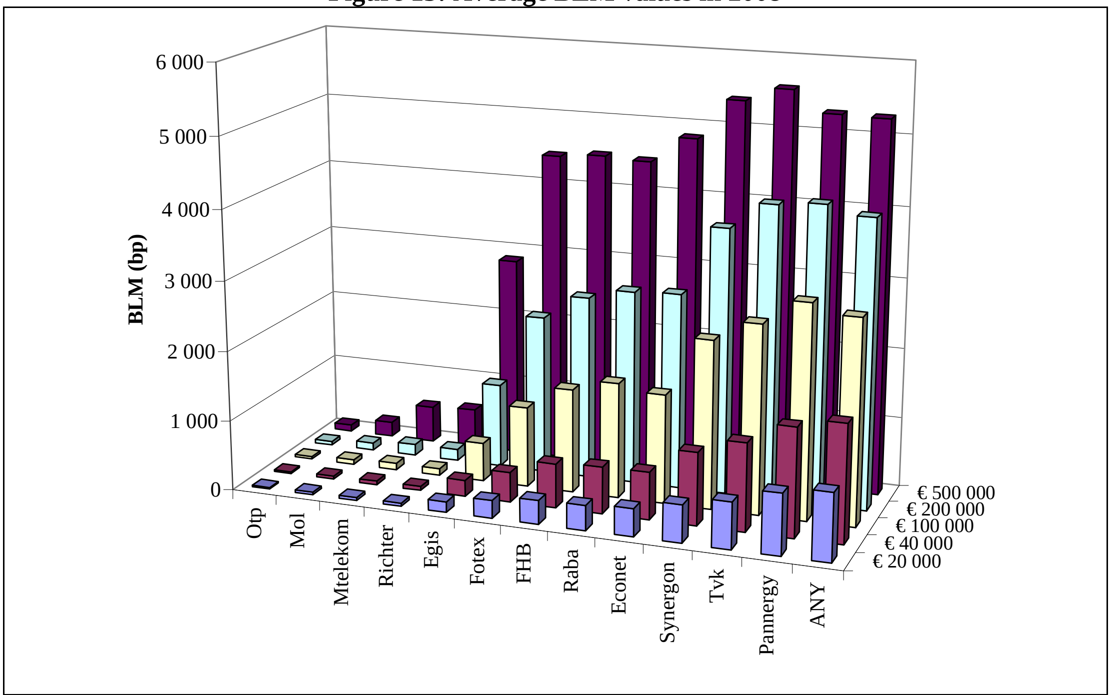
From the perspective of the investors it is important to know which instrument has the lowest value of liquidity measure, since the lower this figure the smaller the implicit cost the investors face when they buy/sell the stock. The following Figures (12; 13; 14; 15) show the average value of liquidity measures of stocks in BUX in years 2007-2010. In Figure 12 it is clearly visible that BLM values monotonously increase in the case of every stock, namely BLM1 shows the lowest, while BLM5 shows the highest value.

Figure 12: Average BLM values in 2007



Source: proprietary

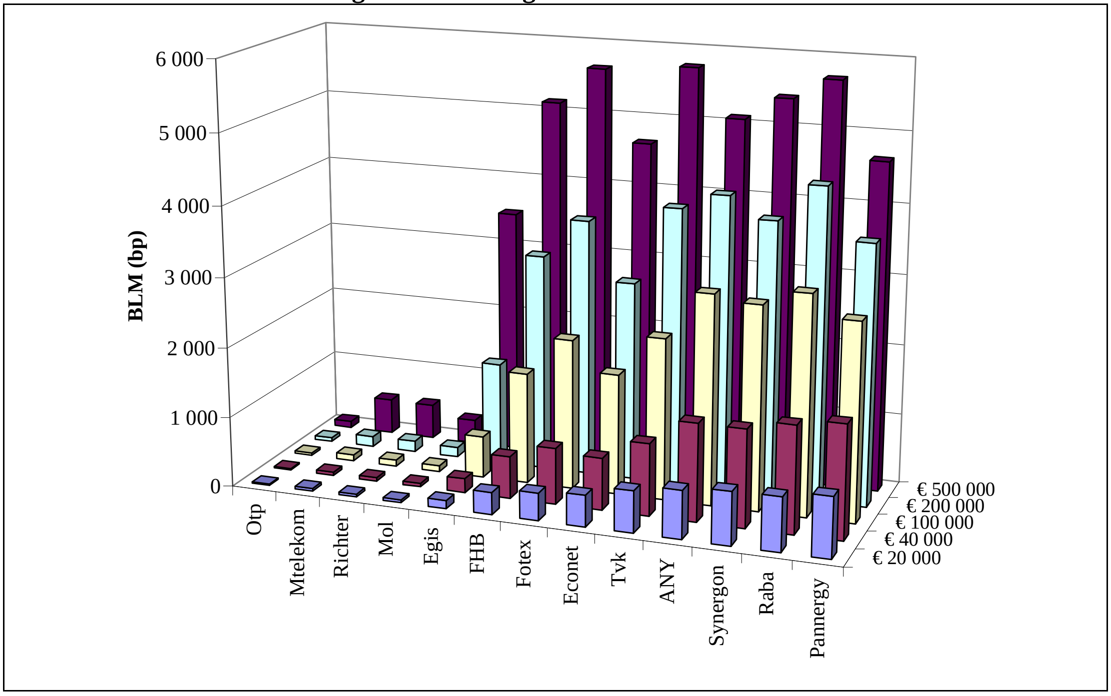
Figure 13: Average BLM values in 2008



Source: proprietary

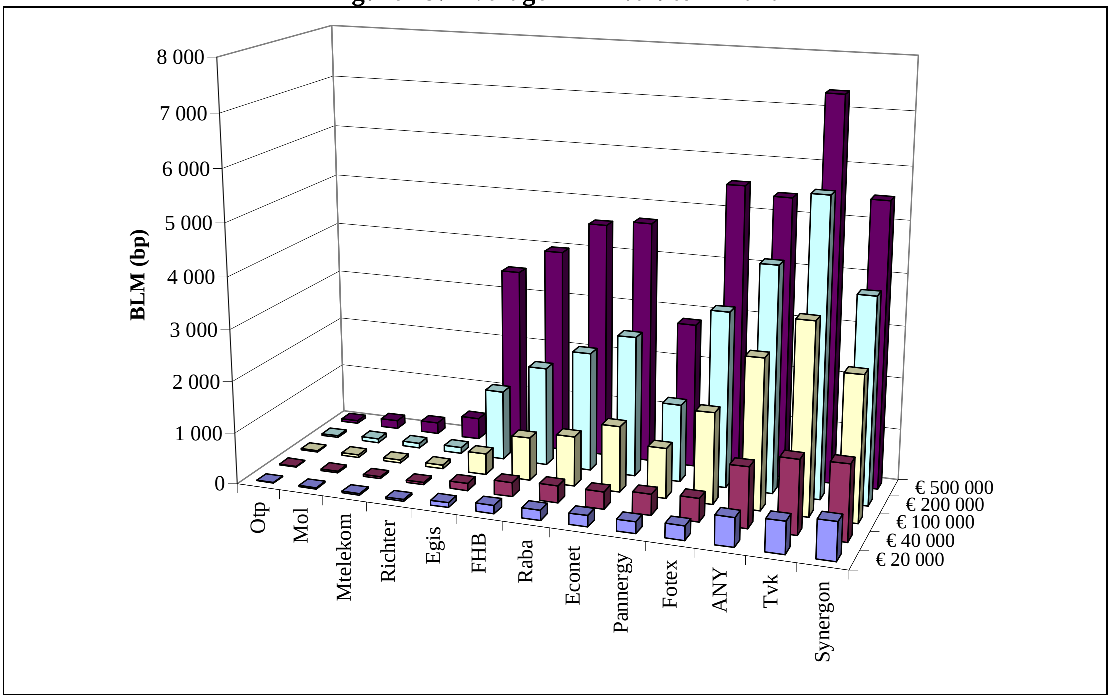
Furthermore, it is also conspicuous that the stock order formed on the basis of BLM1 value is not similar to the order of BLM3. This phenomenon is due to the fact that the order book of each stock may have different shapes. Whilst in the case of a stock (e.g. FHB) many orders are in the first few rows of the order book, it is possible that in the case of another stock (e.g. TVK) there are many in the higher levels of the book. Thus it can occur that FHB is more liquid on the first two order sizes. I found similar results concerning data in the other years.

Figure 14: Average BLM values in 2009



Source: proprietary

Figure 15: Average BLM values in 2010



Source: proprietary, published in Gyarmati et al. (2010a), p. 505.

In order to facilitate investment decisions for investors, it is worthwhile to place the BLM values of the examined stocks on a heat map, which includes the BLM values belonging to the different order sizes in a chart. The higher value the BLM takes, the darker coloring the particular cell gets, thus facilitating better transparency and quick decision making for investors concerning liquidity.

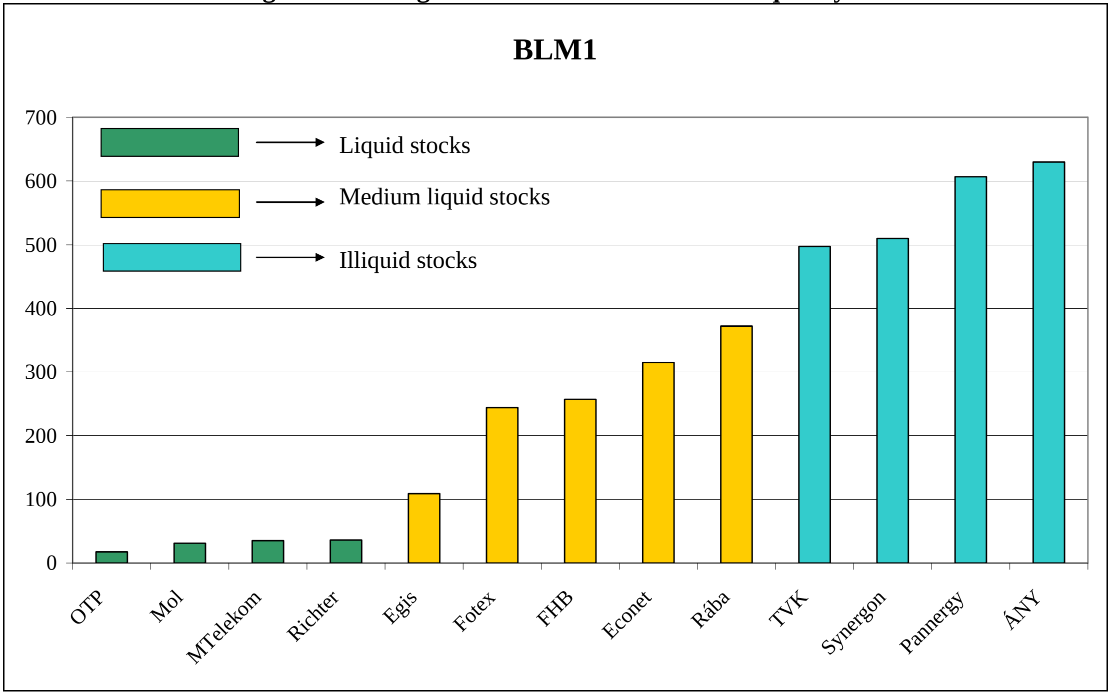
Table 6: Heat map

Heat map	**BLM1**	**BLM2**	**BLM3**	**BLM4**	**BLM5**
OTP	17	21	30	42	74
Mol	31	39	59	91	201
MTelekom	35	46	77	127	383
Richter	36	46	76	130	406
Egis	109	169	431	1,046	2,601
Fotex	244	444	1,250	2,302	4,058
FHB	257	464	1,214	2,327	4,116
Econet	315	512	1,237	2,279	4,157
Rába	372	705	1,563	2,535	4,109
TVK	497	937	2,151	3,521	5,107
Synergon	510	954	2,015	2,975	4,382
Pannergy	607	1,088	2,096	3,030	4,169
ÁNY	630	1,172	2,421	3,547	4,590

Source: own table, published in Gyarmati et al., (2010a), p. 507.

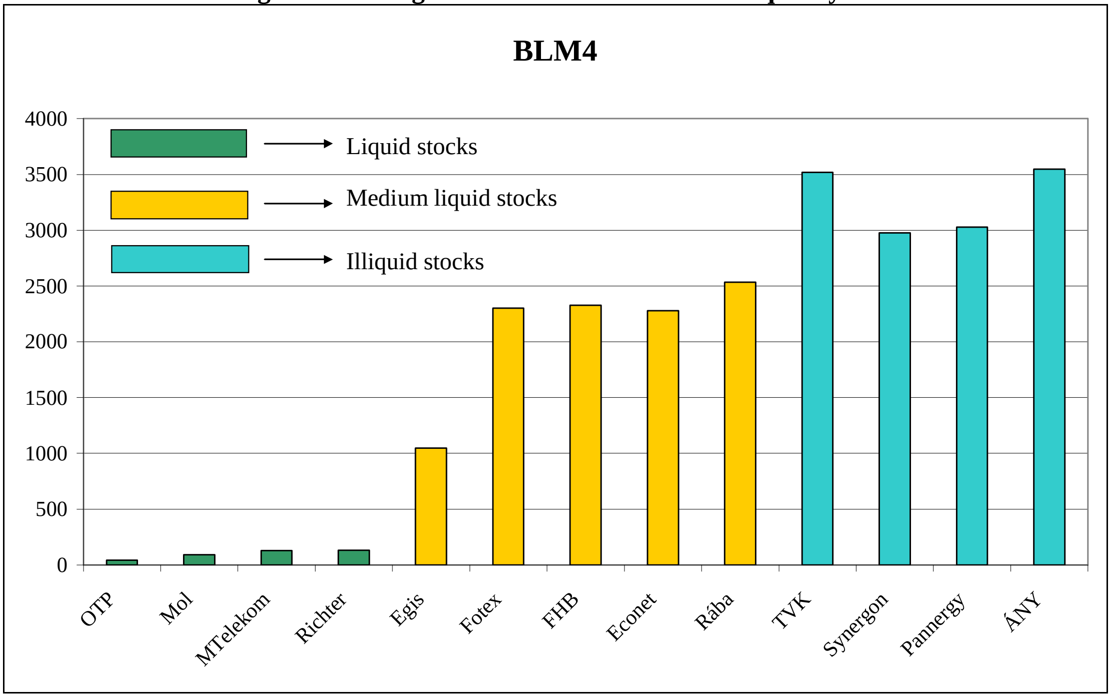
According to the heat map, I categorize the stocks into three groups from the aspect of liquidity: liquid, medium liquid and illiquid groups. The four bluechip stocks which received a white coloring on the heat map based on BLM1 level, i.e. OTP, MOL, MTelekom and Richter, are considered liquid stocks. For the determination of medium liquid stocks I did not consider BLM1 level, since the heat map did not show a significant color difference there. Therefore, in this case I took BLM4 values for basis, and thus it occurred that the following stocks are considered medium liquid stocks: Egis, Fotex, FHB, Econet, and Rába. I classified the other stocks into the illiquid category, i.e. the illiquid ones are the: TVK, Synergon, Pannergy and ÁNy. Figure 16 and 17 show the two columns of the heat map along which I divided the stocks into groups.

Figure 16: Categorization of stocks based on liquidity I.



Source: proprietary

Figure 17: Categorization of stocks based on liquidity II.



Source: proprietary

2.5.2. The relation between BLM and other liquidity indicators

The advantage of BLM compared to other liquidity measures is that it is able to measure liquidity along all static dimensions (tightness, depth, breadth), and thus it gives a more precise view about market liquidity situation. In this subchapter I examine to what extent BLM provides different results than the liquidity indicators most commonly used in practice: the bid-ask spread and turnover. Among static dimensions, bid-ask spread can measure liquidity in the dimension of tightness, while volume can measure depths and from dynamic dimensions it can also be applied to measuring immediacy.

Concerning the whole period (1st January 2007 – 16th July 2010), Table 7 shows the average values of each liquidity indicators, in which stocks are visible according to the ranking which was formed based on BML1.

Regarding the average of the whole period, it can be seen that different liquidity indicators provide different ranking concerning liquidity. The difference appears to be significant in the case of the turnover data, since in that case a difference in the ranking can be found in all the three liquidity groups, whilst based on the bid-ask spread a difference can only be found in the medium liquid and illiquid groups. In my opinion, the difference in ranking is a consequence of the fact that the indicators measure liquidity in different dimensions.

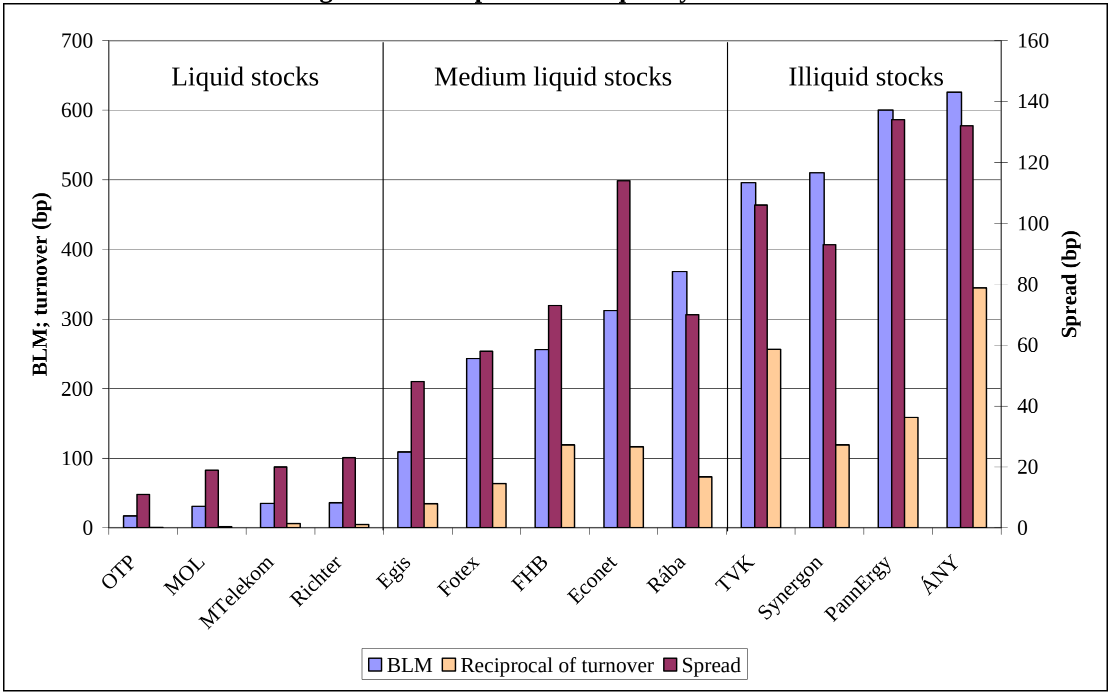
Table 7: Comparison of liquidity indicators based on the average values of the data of 01/01/2007-16/07/2010.

	Order based on BLM1 (bp)		Order based on bid-ask spread (bp)		Orders based on turnover (mHUF)	
OTP	17	1.	11	1.	14 090	1.
MOL	31	2.	19	2.	6 450	2.
MTelekom	35	3.	20	3.	***1 606***	***4.***
Richter	36	4.	23	4.	***2 140***	***3.***
Egis	109	5.	48	5.	288	5.
Fotex	243	6.	58	6.	157	6.
FHB	256	7.	***73***	***8.***	***84***	***9.***
Econet	312	8.	***114***	***11.***	***86***	***8.***
Rába	368	9.	***70***	***7.***	***137***	***7.***
TVK	496	10.	106	10.	***39***	***12.***
Synergon	510	11.	***93***	***9.***	***84***	***10.***
PannErgy	600	12.	***134***	***13.***	***63***	***11.***
ÁNY	626	13.	***132***	***12.***	29	13.

Source: proprietary

For the sake of a better comprehension, Figure 18 demonstrates the data of Table 7, where I ranked the stocks also according to BLM1. Instead of the turnover data itself, I displayed its reciprocal on the figure, because it is easier to demonstrate the turnover data in the same figure together with the BLM1 and the bid-ask spread.

Figure 18: Comparison of liquidity indicators

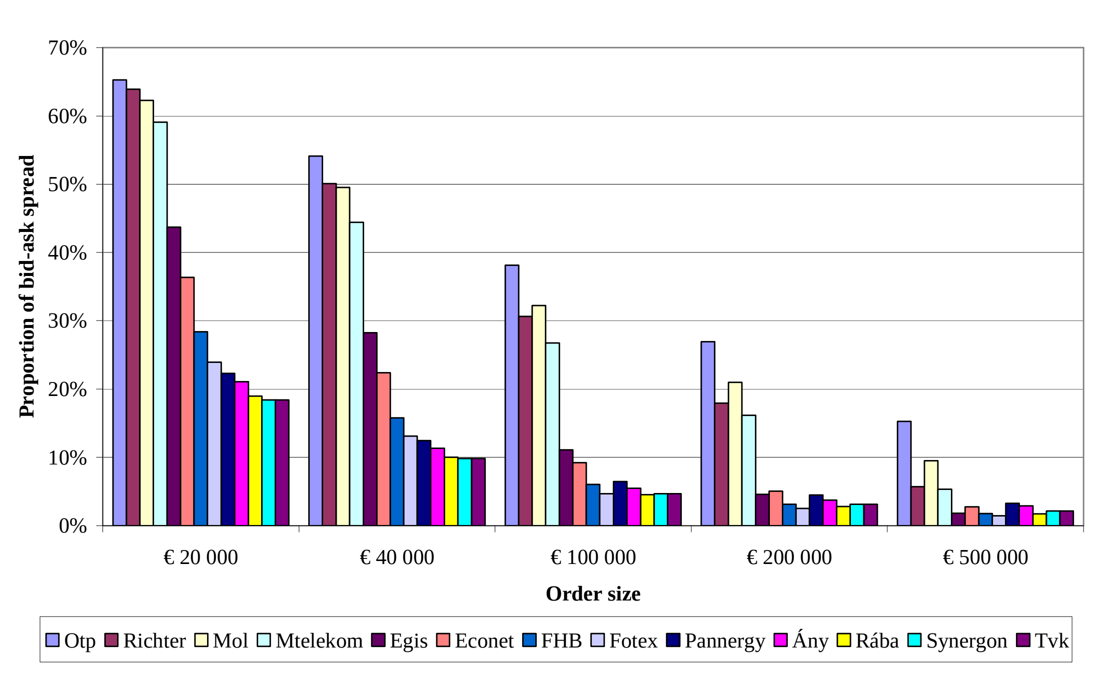


Source: proprietary

According to Table 7 and 18, it can be stated that in the group of liquid stocks the order based on bid-ask spread seemingly differs less compared to the order by BLM than in the case of the turnover. This is a consequence of the fact that the bid-ask spread is a component of BLM, thus it naturally influences the BLM value. It can also be observed on Table 7 that the less liquid a stock is, the less relative proportion bid-ask spread has within the BLM value, as the more significant the value of the adverse price movement will be within the BLM value.[26] For this reason the orders will differ in the more illiquid categories. Figure 19 shows the proportion bid-ask spread represents within each BLM value on different order sizes in the case of stocks in BUX, which thus shows in case of which stock can the value of the adverse price movement be considered as significant.

[26] $BLM = 2LP + APM_{ask} + APM_{bid}$ = bid-ask spread + adverse price movement on the bid side + adverse price movement on the ask side.

Figure 19: The average proportion of bid-ask spread within BLM values on different order sizes between 02/01/2007-16/07/2010.



Source: proprietary

The figure shows that the higher the size of the order we consider, the smaller the share proportion the spread represents within the BLM value, and the bigger the adverse price movement does. Furthermore, the more liquid the stock we consider, the higher the bid-ask spread share within the BLM value. For this reason in the case of liquid stocks the BLM and the bid-ask spread provide a nearly similar ranking for stock liquidity.

In the case of medium liquid and illiquid stocks it is interesting that whilst according to BLM1 and turnover the classification into the two big categories (medium liquid, illiquid) is the same – although the ranking differs within each category –, based on the spread the categorization is however dissimilar. For instance, according to BLM1 Econet belongs to the group of medium liquid stocks, while based on the spread and used the rules of thumb applied by investors, then we would slot the stock into the illiquid category.

With the help of rank-correlation I examined whether in the case of the bid-ask spread or in the case of the turnover, the BLM provides a similar result from the point of view of ranking. Carrying out the calculation according to Spearman’s rank correlation I received the result that the rank correlation value between BLM1 and the spread is 0.945, whilst this value between the BLM1 and the turnover is 0.956.

Therefore, even though according to the data it appears as though the ranking accords less in the case of BLM1 and turnover – as it differs in more locations –, based on the calculations, I concluded that the ranking differed more according to the BLM1 and the spread. This is the consequence of the fact that if there is difference in the ranking in the case of the spread and BLM1, then this difference is more significant there than in the case of BLM1 and turnover. This may cause a problem in case the different rankings have the consequence of slotting a stock into another liquidity category, which has happened for instance in the case of Econet.

I also determined to what extent the three indicators provide a similar order with another rank correlation method. This method is Kendall's rank correlation, with which I could examine the three indicators simultaneously. I received the result that the value of the index is 0.965, which shows the same as Spearman's rank correlation, i.e. that the order can be regarded nearly similar based on different indicators.

However, the whole examined period also contains the phase of the crisis started in 2007/2008. For this reason I considered it worthwhile to divide the time series into the following sections: before crisis (01/01/2007-16/10/2008), during the crisis (17/10/2008-03/04/2009) and after crisis (04/04/2009- 16/07/2010) phases,[27] and examine whether the same can be stated about the order formed according to the three indicators also for the three different periods than regarding the whole period. Table 8 contains the values of the rank correlation for each period.

Table 8: Rank correlation

Methods	Indexes	Whole period	Before crisis	During crisis	After crisis
Spearman's rank correlation	**BLM-spread**	0.945	0.956	0.907	0.896
	BLM-turnover	0.956	0.967	0.775	0.934
Kendall's rank method	**BLM-spread-turnover**	0.965	0.982	0.896	0.918

Source: proprietary

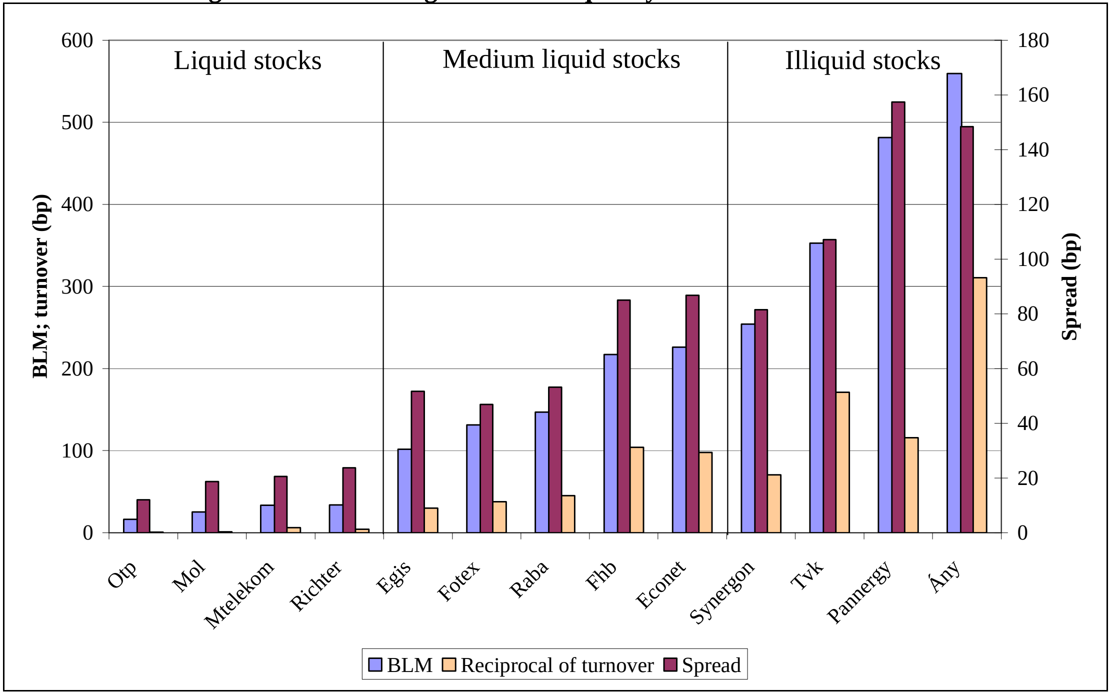
Based on Spearman's rank correlation, it can be stated that before and also after the crisis the connection was stronger in the ranking formed between the BLM1 and turnover than between BLM1 and the spread – although both could be considered

[27] I have accomplished the division into periods with the help of the bloxplot diagram and with the examination of structural breaks, about which I give a detailed description in Subchapter IV/2.4.4.

as strong. However, during the crisis this reversed and the strength of the relationship between BLM1 and turnover significantly reduced, while between BLM1 and the spread it did not decrease equally significantly. This entailed that between the rankings provided by spread and BLM1 the connection became stronger than between the rankings according to turnover and BLM1.

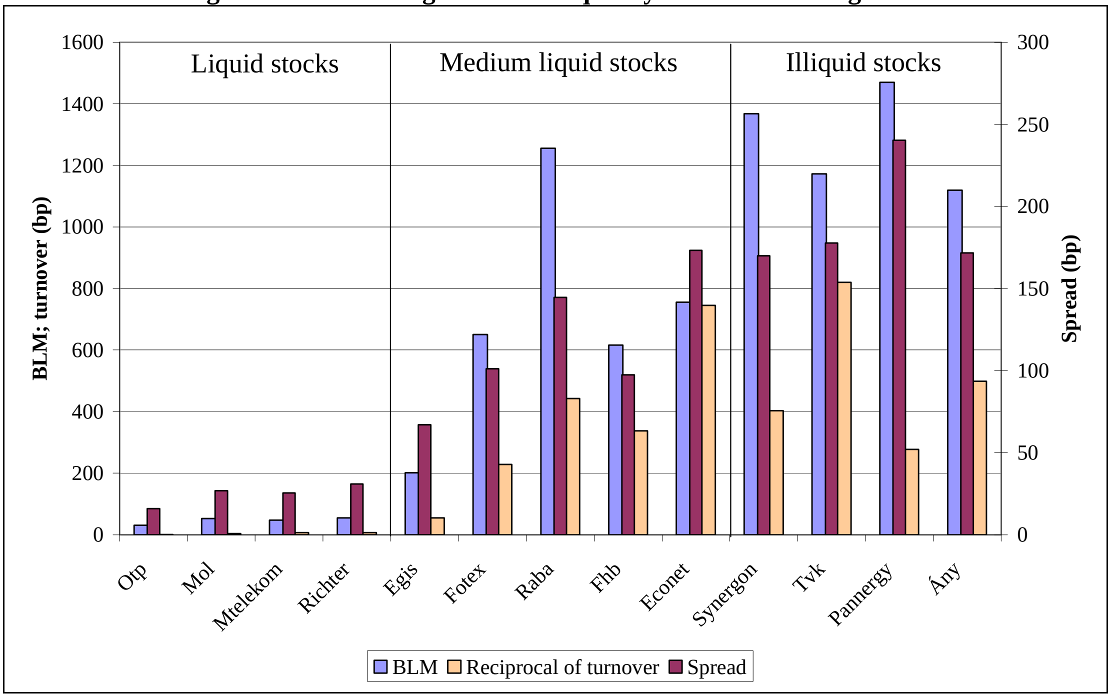
It can also be experienced in the case of the correlation index calculated with the help of Kendall's rank method that during the crisis the strength of the connection decreases, which again increased following the crisis. In order to comprehend this phenomenon it is worthwhile to observe the formed ranking in the three examined periods, which are shown in the following three figures (20; 21; 22). On these figures, I ranked the stocks according to the BLM1 value of the whole time series. I did this in order that it could be seen that the categorization of stocks can change in each period, and for this reason it can be important to often revise which liquidity category each stock belongs to. An instance for this during the crisis is Rába, which would have belonged to the group of illiquid stocks instead of the medium liquid ones. Furthermore, it is also visible on the figures (Figures 20-22) that based on different indicators we would have sorted the stocks into different liquidity groups, as we did for the whole examined period, e.g. in the case of Econet.

Figure 20: The average values of liquidity indicators before crisis



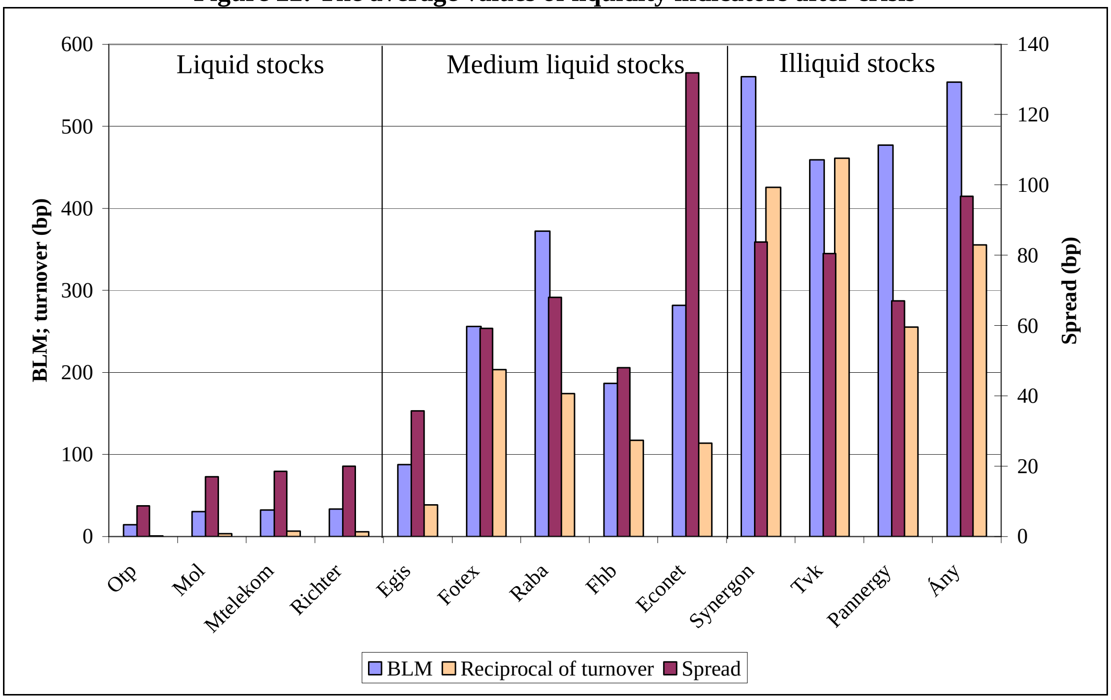
Source: proprietary

Figure 21: The average values of liquidity indicators during crisis



Source: proprietary

Figure 22: The average values of liquidity indicators after crisis



Source: proprietary

Table 9 summarizes the data of the above figures. It can be observed on the table how the economic crisis originated from the subprime crisis of 2007/2008 affected the values of liquidity indicators. It can be seen that as a consequence of the crisis the values of BLM and spread have significantly grown in 2008, as well which in some cases did not return to their pre-crisis level. The same appears in the turnover data, i.e. that the turnover slumped in the case of all stocks. However, while BLM1 and bid-ask spread did not return to their pre-crisis level only in some instances, for the turnover data it can be observed that except for OTP and MTelekom, the turnover of none of the stocks returned to its pre-crisis level. In the table the particular indicators of stocks whose liquidity did not return to their pre-crisis level after the crisis are highlighted.

Table 9: Average values of liquidity indicators

	BLM1 (bp)			Spread (bp)			Turnover (mHUF)		
	Before crisis	**During crisis**	**After crisis**	**Before crisis**	**During crisis**	**After crisis**	**Before crisis**	**During crisis**	**After crisis**
OTP	16	30	14	12	16	9	13,405	9,666	16,497
MOL	25	53	30	19	27	17	9,763	2,958	***3,038***
MTelekom	33	48	32	21	25	19	1,691	1,599	1,485
Richter	34	55	33	24	31	20	2,523	1,599	***1,801***
Egis	102	201	88	52	67	36	333	185	***261***
Fotex	131	651	***256***	47	101	***59***	263	44	***49***
Rába	147	1,255	***372***	53	145	***68***	221	23	***57***
FHB	217	617	186	85	97	48	96	30	***85***
Econet	226	755	282	87	173	***132***	102	13	***88***
Synergon	254	1,368	***560***	81	170	84	142	25	***24***
TVK	353	1,172	***459***	107	178	80	58	12	***22***
Pannergy	481	1,470	***477***	157	240	67	87	36	***39***
ÁNy	559	1,119	554	148	172	97	32	20	***28***

Source: proprietary

Therefore, on the whole we can state based on Figure 20-22 and Table 9 that during the examination of rank correlation the relationship can be considered as strong concerning the ranking formed by liquidity indicators. However, there are differences in the order which can be important during an investment decision. An instance for this is when we sort a stock into a different liquidity category because of the differing order. This phenomenon only occurs in the medium liquid and illiquid categories. In the case of liquid stocks i.e. the four bluechip stocks, the stocks can be considered as liquid according to all three liquidity indicators. It follows from this that if we regard the four bluechip stocks then the rules of thumb used by market actors – namely that they consider the bid-ask spread and the turnover as liquidity indicators – typically lead to a correct result in the sense that they sort the stocks into the liquid group. However, the same cannot be said about the sorting into the other two categories.

Another important conclusion is that during a crisis the rank correlation decreases, therefore it increases the inaccuracy of the categorization if we sort a stock into a liquidity group based on an inappropriate indicator.

Based on Table 7 and 9 a further interesting phenomenon can be observed, namely that the liquidity of each stock compared to each other highly differs according to different indicators. Having examined the four bluechip stocks, Table 10 shows this difference. For instance it can bee seen that based on BLM1 OTP is 1.82 times more liquid than MOL, whilst according to turnover data it is already 2.18 times more. What is even more significant that OTP is nearly 2 times more liquid than MTelekom from the aspect of BLM, while if we regard turnover, then OTP appears to be 9 times more liquid.

Table 10: Liquidity of stocks compared to each other

	BLM1	Spread	Turnover
OTP-MOL	182%	173%	218%
OTP-MTelekom	206%	182%	877%
OTP-Richter	212%	209%	658%
MOL-MTelekom	113%	105%	402%
MOL-Richter	116%	121%	301%
MTelekom-Richter	103%	115%	75%

Source: proprietary

This is essential because if traders decide what position they should take in each stock according to their respective liquidity then it is not the same according to which indicator they make such decision. Namely, based on BLM they would take two times bigger position in OTP than in MTelekom, while based on volume they would create a nine times bigger position.

Therefore it is important to check how strong the relationship is between the three liquidity indicators, since in spite of the fact that stocks are nearly similarly categorized in respect of liquidity, it does not necessarily mean that there is a strong relationship between each liquidity indicator.

2.5.2.1. Relationship between liquidity indicators

During the comparison of the three liquidity indicators, I considered it worthwhile to examine the correlation between the three indicators, i.e. to observe how strong the relationship is between them.

Table 11: Correlation of liquidity indicators between 02/01/2007 and 16/07/2010

Correlation between liquidity indicators		
	BLM1-Spread	BLM1-turnover
OTP	0.911	-0.092
Mol	0.884	-0.273
Richter	0.746	-0.241
MTelekom	0.919	-0.178
Egis	0.838	-0.328
Fotex	0.794	-0.313
Rába	0.736	-0.213
FHB	0.557	-0.099
Econet	0.738	-0.239
Synergon	0.648	-0.297
Pannergy	0.554	-0.095
TVK	0.694	-0.273
ÁNy	0.521	-0.105

Source: proprietary

According to the table it can be stated that there is a strong positive relationship between the BLM and the spread, but the less liquid the stock, the weaker is this relationship.

There is a weak negative relationship between the turnover and the BLM. Namely, when the turnover on the market is low/high, it does not predict well whether the liquidity would be also low/high according to the BLM or the spread. Therefore, the a conclusion can be drawn according to Table 11 that the BLM and the spread provide a similar result concerning liquidity based on daily data, but the turnover gives a significantly different result. As a consequence, in Subchapter 2.5.2.2 I am going to carry out a more detailed examination about the relationship between BLM and turnover data also in the case of intraday data. But before that, I present how the relationship changed between the BLM and the spread, and also between the BLM and the turnover before, during and after the crisis. With the help of linear regression, I examine the extent of the explanatory power of the spread and turnover concerning BLM in the three different periods. I made the examination in the case of a liquid (OTP), a medium liquid (Egis) and an illiquid (Pannergy) stock. As a result, I have concluded that compared to turnover, the spread had a higher explanatory power regarding BLM, which is shown by Table 12 which contains the R-squared values. In the table it can be seen that during the crisis the explanatory power decreased in the case of all three stocks, which did not return to its pre-crisis level in the case of liquid and medium liquid ones. Moreover, for OTP the turnover did not have a significant explanatory power concerning BLM after the crisis at all. Furthermore, it can be seen in the data – which we can also observe in Table 11 – that the less liquid a stock, the lower is the explanatory power of bid-ask spread. This phenomenon cannot be observed in the case of turnover, since there the explanatory power is higher before and after the crisis in the case of a medium liquid stock, while during the crisis it is higher for the liquid stocks – although this explanatory power is not considered as significant in any case.

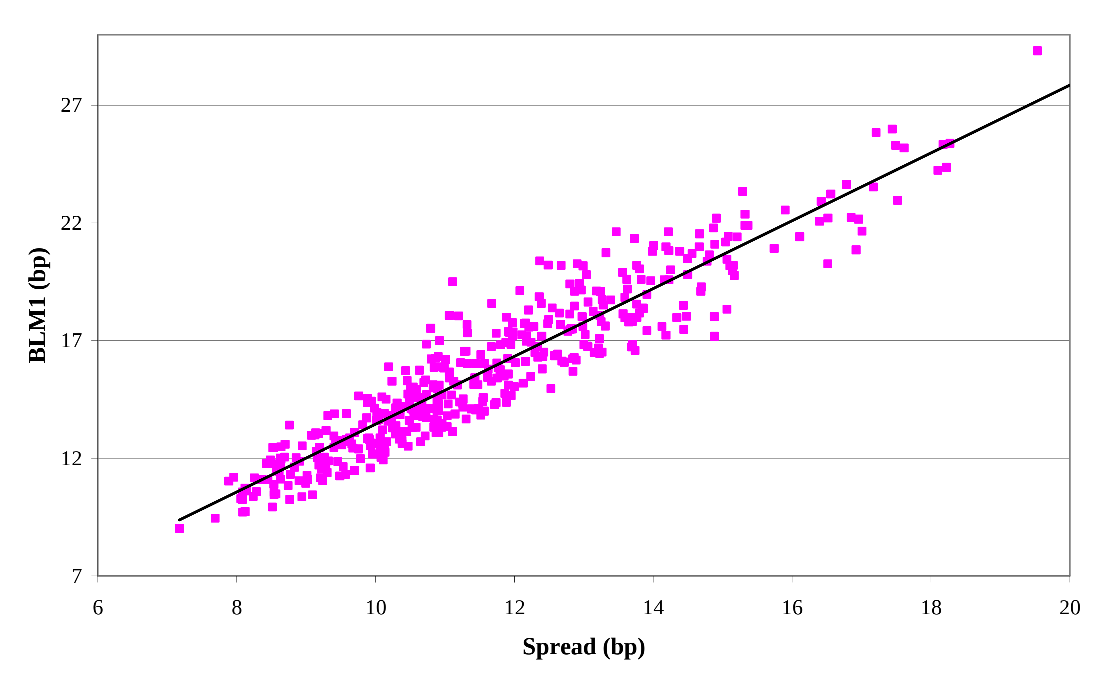
Table 12: Explanatory power of spread and turnover

	Spread-BLM			**Turnover-BLM**		
R-squared	**OTP**	**Egis**	**Pannergy**	**OTP**	**Egis**	**Pannergy**
Before crisis	0.924	0.766	0.421	0.019	0.126	0.007
During crisis	0.899	0.654	0.111	0.124	0.081	0.012
After crisis	0.875	0.674	0.641	**0.002**	0.159	0.020

Source: proprietary

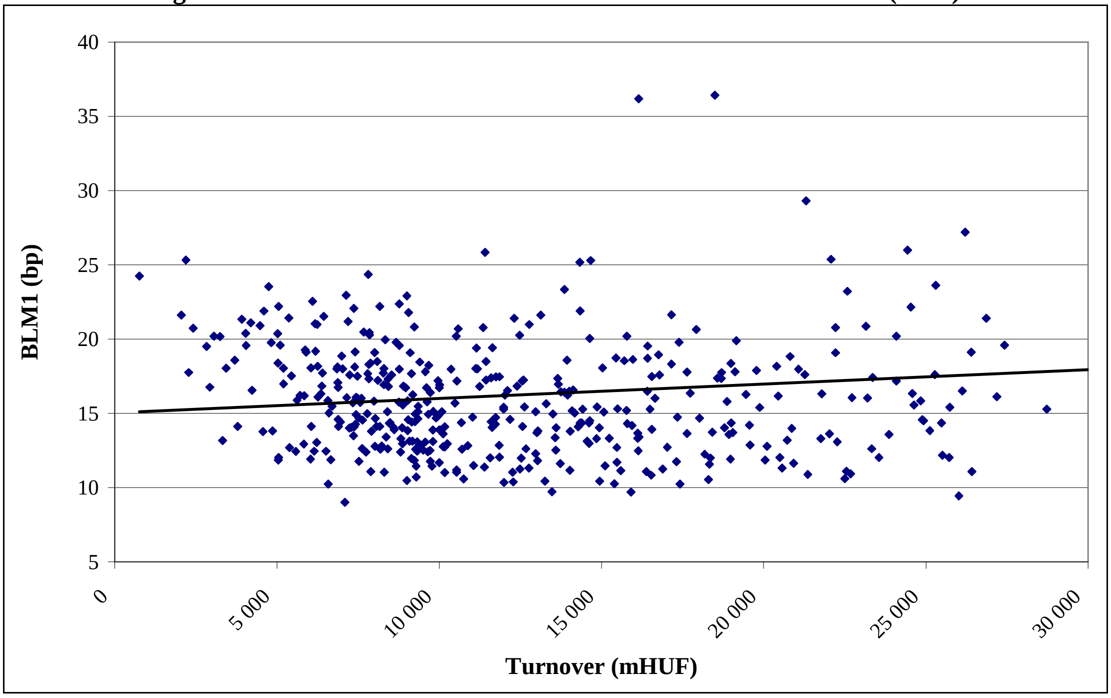
Figures 23-24 show the result of the linear regression for OTP in the case of bid-ask spread and turnover before the crisis. The result of the other two stocks appears to be similar, therefore I dispensed with its illustration.

Figure 23: Relation between spread and BLM before the crisis (OTP)



Source: proprietary

Figure 24: Relation between turnover and BLM before the crisis (OTP)



Source: proprietary

It is seen in the figures as well that the connection between the spread and the BLM is strong, while between the turnover and the BLM it is not. However, apart from the fact that the relationship between the indicators is not strong, it is worthwhile to observe what is characteristic for the change of these indicators, i.e.

when there is a high increase or decrease in the bid-ask spread or in the turnover, then what can be said about the BLM value. Thus I examined the extent to which the spread and turnover change explain the change of BLM.

I carried out the examination for all the three periods. Table 13 contains the results, where this time I have chosen another stock from the illiquid group: the Synergon instead of Pannergy. In Table 13 I bolded the values where the relationship was not significant.

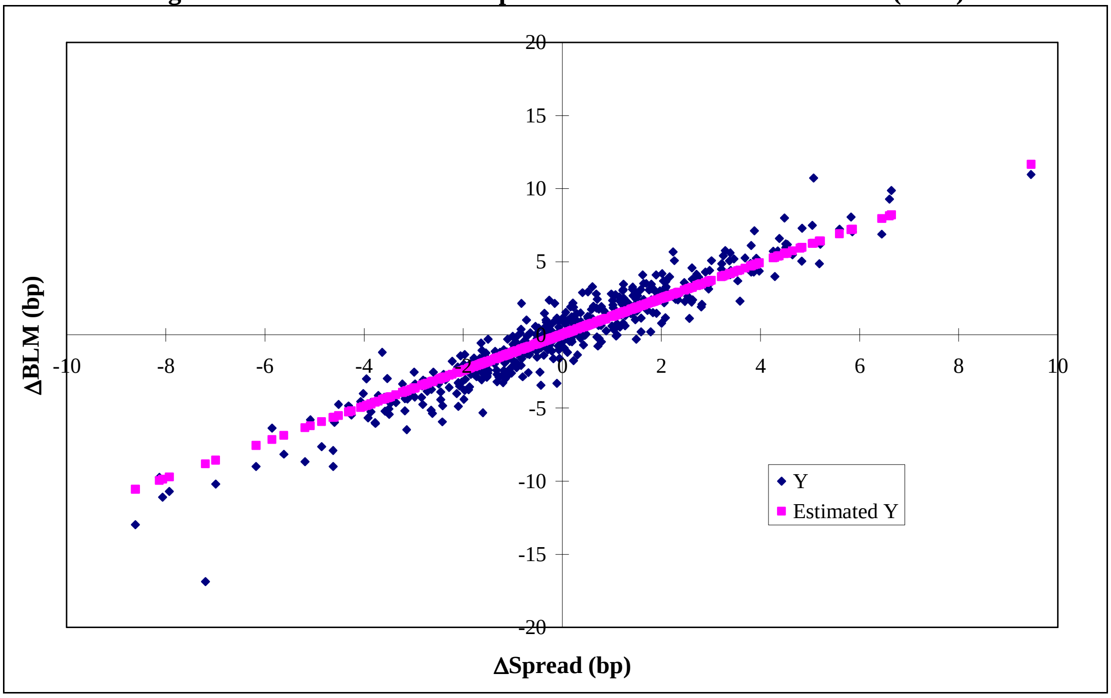
Table 13: Explanatory power of Δspread and Δturnover

	Δspread-ΔBLM			Δturnover - ΔBLM		
R-squared	**OTP**	**EGIS**	**Synergon**	**OTP**	**EGIS**	**Synergon**
Before crisis	0.925	0.401	**0.000**	**0.000**	0.024	**0.004**
During crisis	0.846	0.476	0.041	**0.005**	0.145	**0.003**
After crisis	0.876	0.402	0.021	**0.000**	0.036	**0.001**

Source: proprietary

From the results it can be seen that in the case of liquid and illiquid stocks the turnover change is not able to explain the change of BLM in any period. For medium liquid stocks it can, although the relation was not strong either.

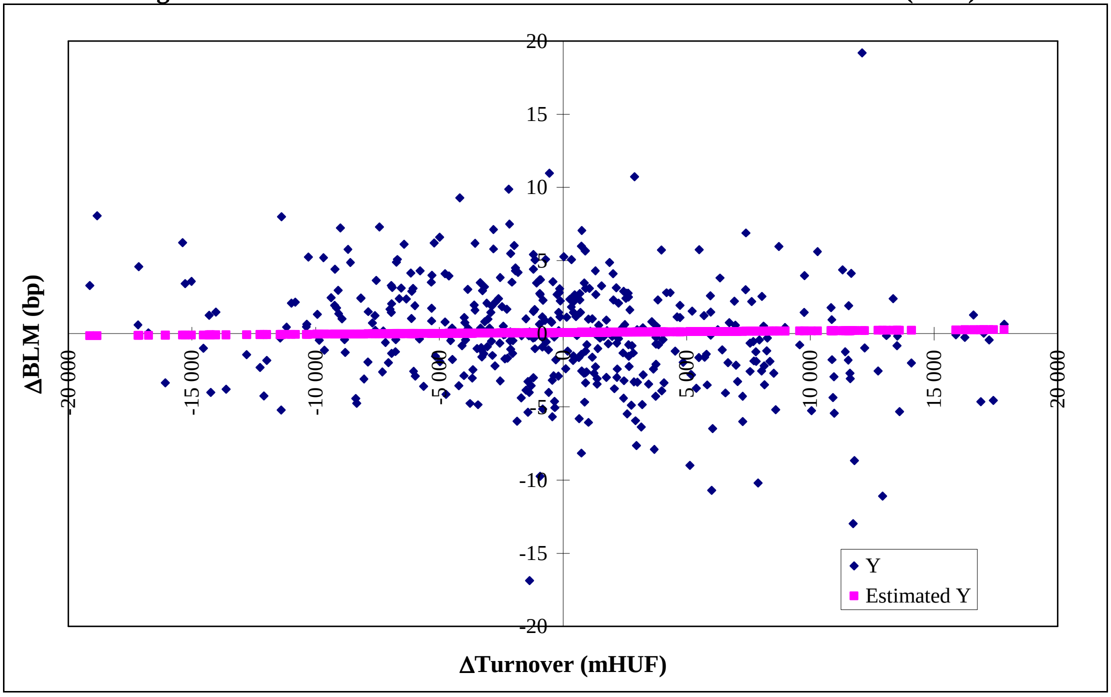
Figure 25: Relation between Δspread and ΔBLM before the crisis (OTP)



Source: proprietary

However, the explanatory power of spread change decreases with the liquidity of stocks, i.e. the more illiquid a stock is the less the spread change explains the BLM change. This is due to the above presented reason, namely that the more illiquid a stock, the lower the share bid-ask spread represents within the BLM value. I demonstrate the results as well, in the case of OTP for the pre-crisis period, which are shown by Figure 25 and 26.

Figure 26: Relation between ΔTurnover and ΔBLM before the crisis (OTP)



Source: proprietary

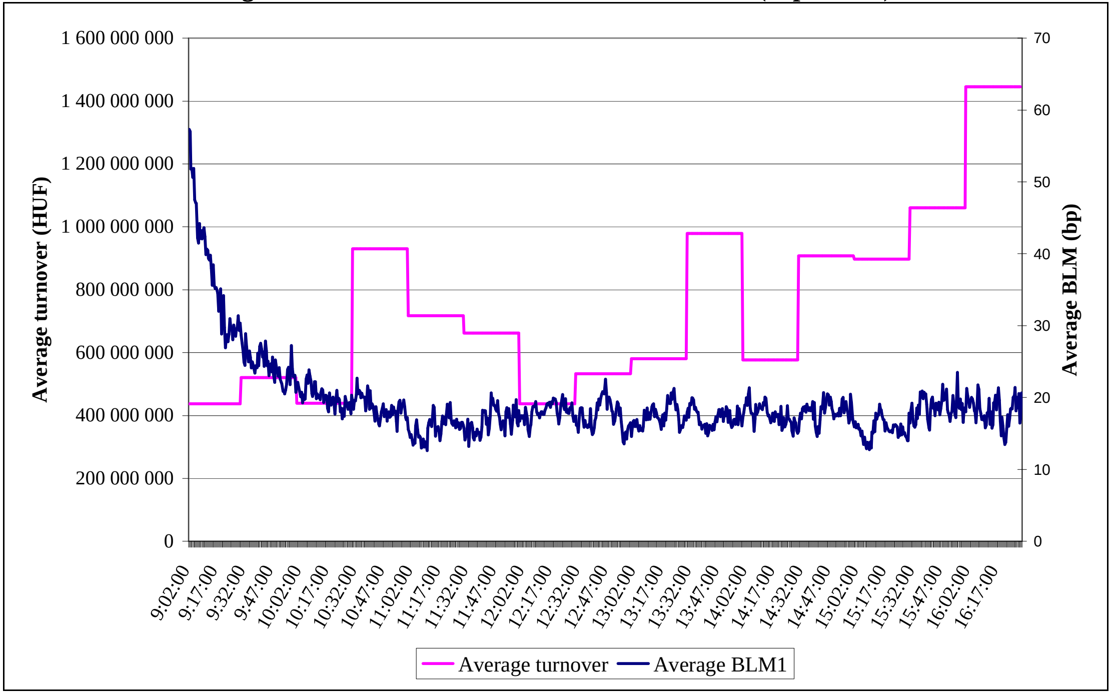
2.5.2.2. Relationship between liquidity and turnover based on intraday data

Regarding the relationship of BLM and turnover, we would have a prior expectation that the higher turnover a stock has the lower its BLM value, i.e. the better investment it appears to be from the point of view of liquidity. In the case of daily data we observed that the relationship between the two indicators is weak, however, it is also worthwhile to examine how this phenomenon arises intradaily, i.e. if the BLM value of a stock is low during the day, then whether it also has a high turnover at the same time.

I have carried out this examination on the four bluechip stocks, based on average intraday turnover and BLM data of September 2007, which are shown by

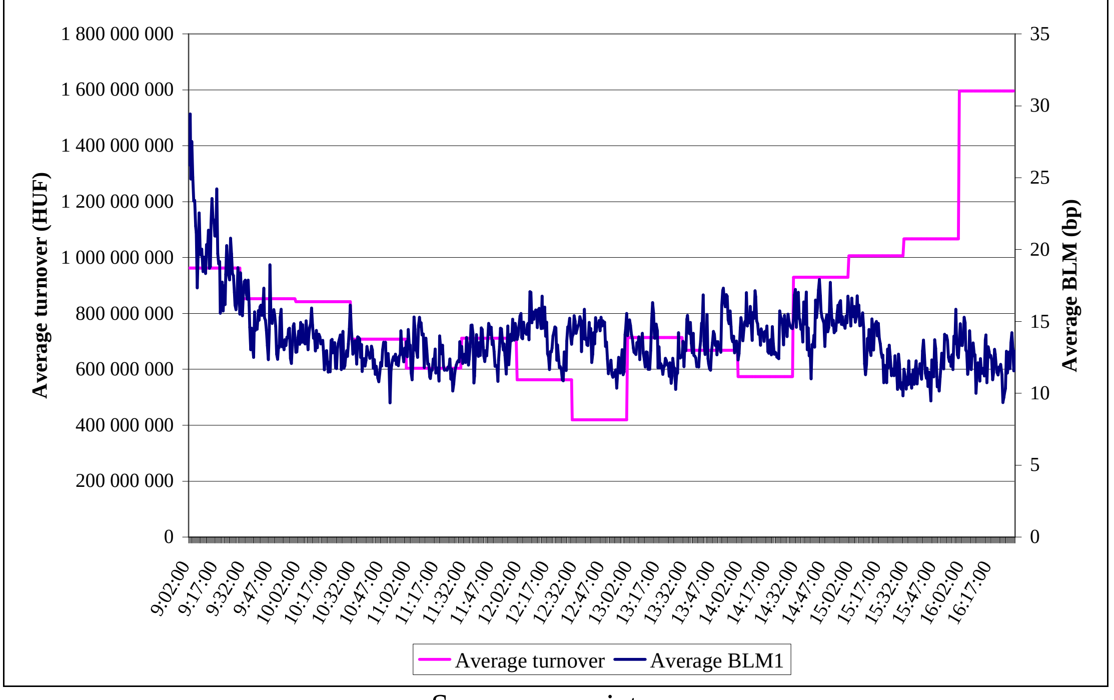
Figures 27-30. The average values were calculated in a way that I took the average of the BLM and the turnover data belonging to the same second of each day. The basis of the calculation were those days of September 2007 when there was trading on BSE.

Figure 27: BLM1 and turnover values of MOL (Sept. 2007)



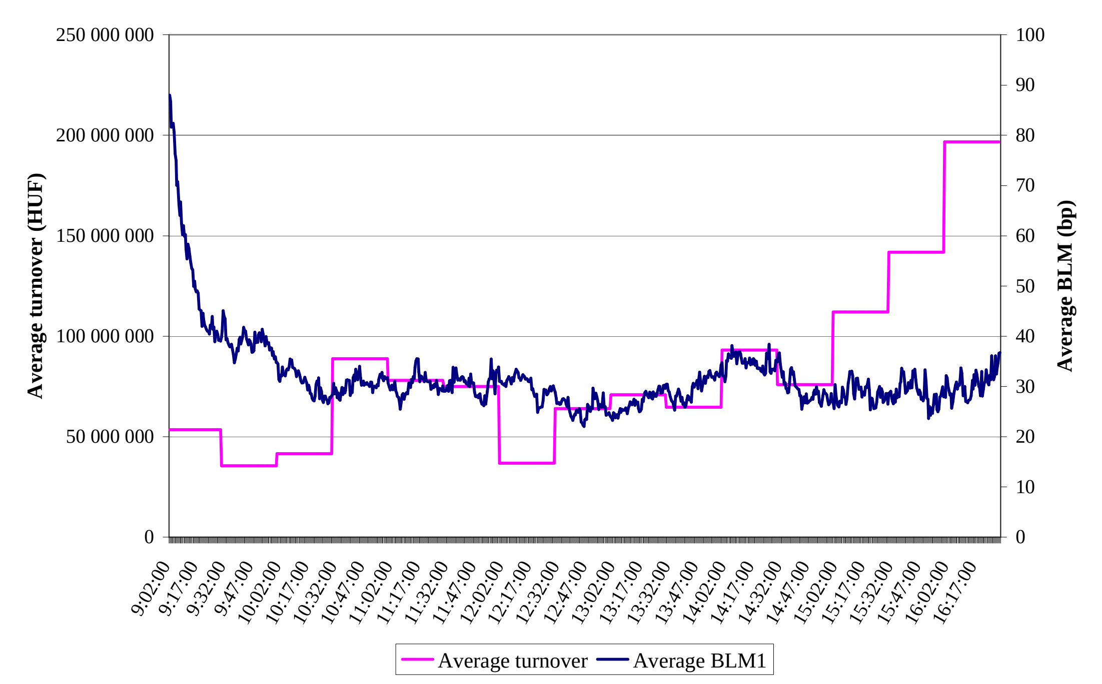
Source: proprietary

Figure 28: BLM1 and turnover values of OTP (Sept. 2007)



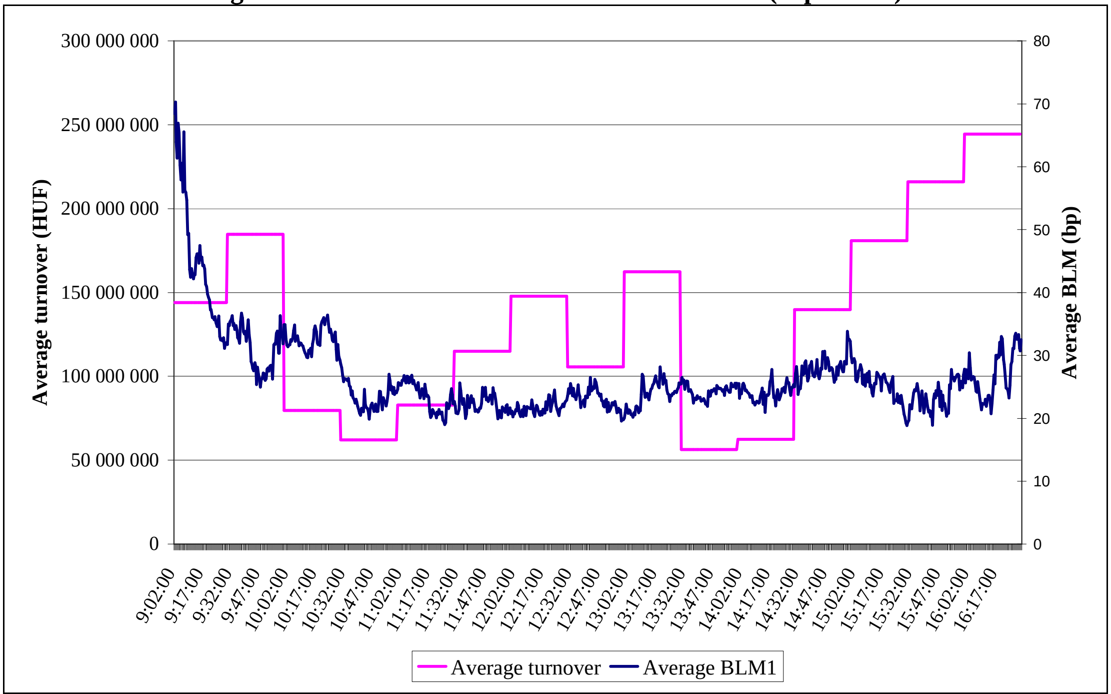
Source: proprietary

Figure 29: BLM1 and turnover values of MTelekom (Sept. 2007)



Source: proprietary

Figure 30: BLM1 and turnover values of Richter (Sept. 2007)



Source: proprietary

The intraday average values calculated from the data of September 2007 by no means support the hypothesis that the liquidity measured by BLM co-moves with the turnover. The tendency that the higher turnover goes together with low BLM values is not realized.

During the interviews with market participants, (Szűcs and Váradi, 2012) we experienced that as a rule of thumb they regard the formation of intraday turnover as though it formed according to a „U-shape", i.e. it is higher at the beginning and the end of the day than during the day. However, this „U-shape" can only be observed in the case of OTP, for the other stocks only the end-day increasing turnover is visible, which can be linked to the opening of the American stock exchange in all the four cases. The American stock exchange opens at 3:30 PM Hungarian time, which generates a significant turnover on BSE in the last trading hour. While this impact can be seen in the turnover data, it does not influence the value of BLM. While with the increase in turnover should be accompanied with the increase in liquidity, this is not reflected in the indicator.

Moreover, it can also be observed in the figures that the trading activity is low in the first hour after the opening, it intensifies at about 10 AM, thus the first hour of trading cannot be considered as typical for the daily average trade, therefore its BLM1 data do not provide reliable information about liquidity. Furthermore, it is also possible that the BLM1 values are higher in the first hour, because the investors build up the order book with their orders at that time. According to this, it can be stated that between turnover and BLM data there is no strong relationship even intradaily.

This finding is important for day traders (namely those who close the opened position by the end of the same trading day at the latest), because if they intend to decide at the beginning of the day based on turnover whether a stock is liquid or not, then it is not certain that they receive a correct result. Namely, based on Figures 27-30 there is also an instance that high turnover is accompanied with low liquidity (OTP, Richter), and also there is a further example that by low turnover the liquidity is also low (MOL, MTelekom). Therefore, the high turnover at the beginning of the day did not entail that the order book was built up faster, and thus the particular stock was more liquid.

2.5.3. Relationship between volatility and liquidity

In the classical portfolio theory of Markowitz (Markowitz, 1952) every investor optimizes in the standard deviation-return space in order to maximize utility. According to Markowitz, if we can assume that the distribution of returns is normal, then it is sufficient to know the expected value and the standard deviation, and based on them investors are able to carry out the optimization (Bélyácz, 2009, 2011). However, the model ignores an essential factor, namely that the product cannot be traded at the mid price. Therefore, it does not consider the transaction cost originated from the lack of liquidity. In case we take this additional transaction cost into consideration, then traders do not only have to solve a utility maximization problem in the standard deviation-return space, where the aim is to achieve the highest return with the lowest risk (Riecke et al., 1985), but they would also have to minimize the occurring costs simultaneously. For solving such a complex task, we need to know the relationship of liquidity compared to the standard deviation and the return. In the present chapter I do not provide a solution for the optimization task, I only present the connection between the three factors (liquidity, return, standard deviation).

The reason why I find the collective examination of the three factors important is because during the series of interviews we found that these are the three factors that market actors strive to predict. They create their strategy based on the forecast of return-standard deviation-liquidity, for instance how to accomplish the order splitting of big orders or where to put the stop limits exactly.

During the analysis I observed the correlation between the BLM and the standard deviation values estimated from the GARCH-model, and between the BLM and true range (TR) values in the case of three liquid, one medium liquid and one illiquid stock. Table 14 contains the results.

Table 14: Correlation between volatility and liquidity

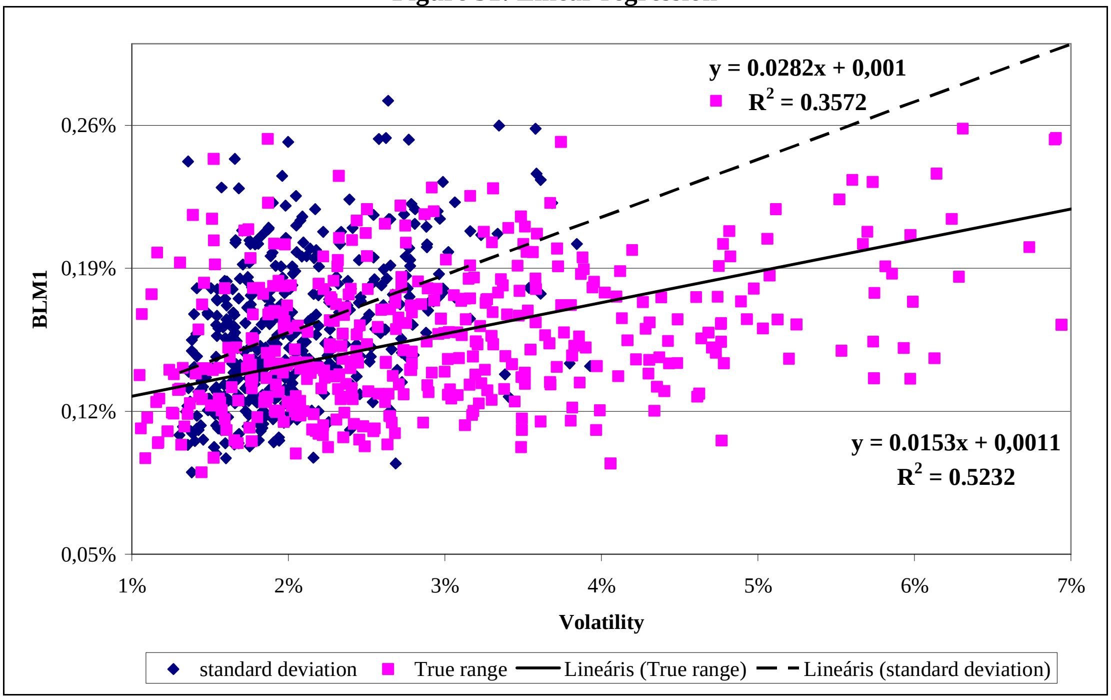
Correlation	BLM-TR			BLM-standard deviation		
	Before crisis	During crisis	After crisis	Before crisis	During crisis	After crisis
OTP	0.723	0.632	0.391	0.598	0.582	**0.606**
MOL	0.638	0.423	0.311	0.523	0.224	**0.414**
Richter	0.276	0.636	0.224	0.248	0.523	**0.403**
Egis	0.526	0.424	0.118	0.410	0.373	**0.162**
Pannergy	0.302	0.107	0.245	0.059	**0.179**	**0.246**

Source: proprietary

According to the data, the correlation between volatility and liquidity before and during the crisis is always bigger if we measure volatility with the true range and not with the standard deviation. During the crisis, it is only in the case of the illiquid stock that the correlation is higher between the standard deviation estimated from GARCH model and BLM than between the true range and the BLM. However, after the crisis the correlation in every case is higher between the standard deviation and the BLM. According to the data, it can also be observed that the more liquid a stock is, the higher the correlation tends to be between liquidity and volatility for every period.

In the case of OTP, apart from correlation I also examined the explanatory power of volatility concerning liquidity, and I observed what estimation could have been given for liquidity, based on the pre-crisis period assuming the knowledge of volatility. I accomplished the examination with linear regression, on one hand because this model is used in the literature, on the other hand because only the fitting of a very high (six) degree polynomial provided a better estimation than linear estimation, but also in that case only with a small percentage did the R-squared value improve. Thus applying the linear regression appeared to be justified.

Figure 31: Linear regression

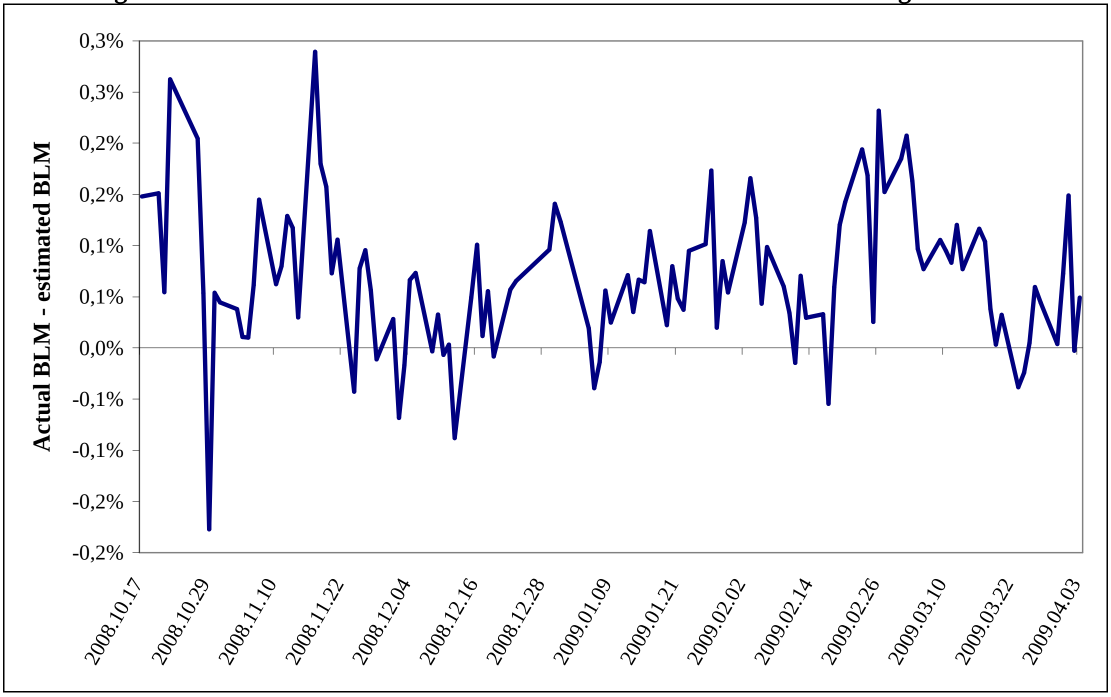


Source: proprietary

Referred to BLM, I examined the explanatory power of standard deviation and the explanatory power of true range separately. It can be stated based on Figure 31 that the explanatory power concerning the formation of liquidity is higher in the case of the true range, since there the R-squared value is 0.52, whilst in the other case it is only 0.36. For this reason, I am going to apply the linear regression estimated with the true range in order to assess the extent liquidity decrease could have been caused by a volatility rise such as the one occurred during the crisis.

Figure 32 shows the extent of the difference between the actual and the estimated liquidity. According to the figure it can be stated that almost every day (100 times out of 114 days) the estimated BLM was lower than the actual one, i.e. the shortage of liquidity was bigger than it could have been expected. Therefore, based on this the conclusion can be drawn that there was also a real liquidity crisis during the year 2008. Furthermore, it justifies Csávás and Erhart's (2005) statement that in the liquidity decrease the rise of the unexpected volatility is reflected.

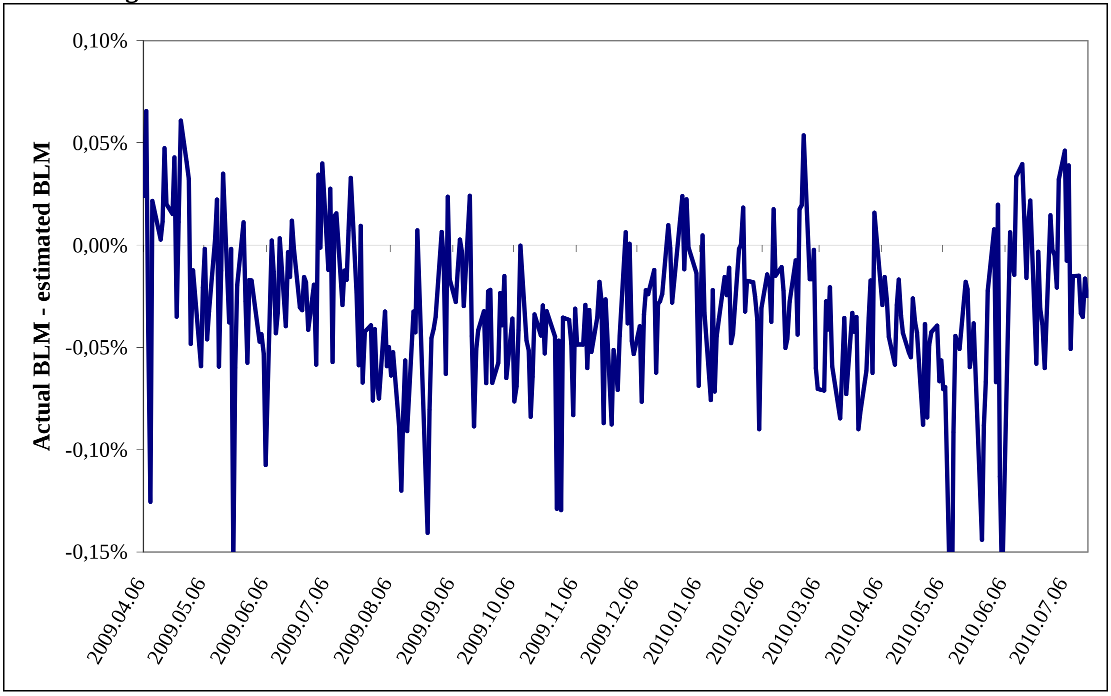
Figure 32: Difference between the actual and the estimated BLM during the crisis



Source: proprietary

I also examined for the post-crisis period what estimation we would give concerning liquidity. During this assessment, I experienced exactly the opposite as for the during-crisis period, i.e. we overestimated the shortage of liquidity almost every day based on the linear regression.

Figure 33: Difference between the actual and the estimated BLM after the crisis

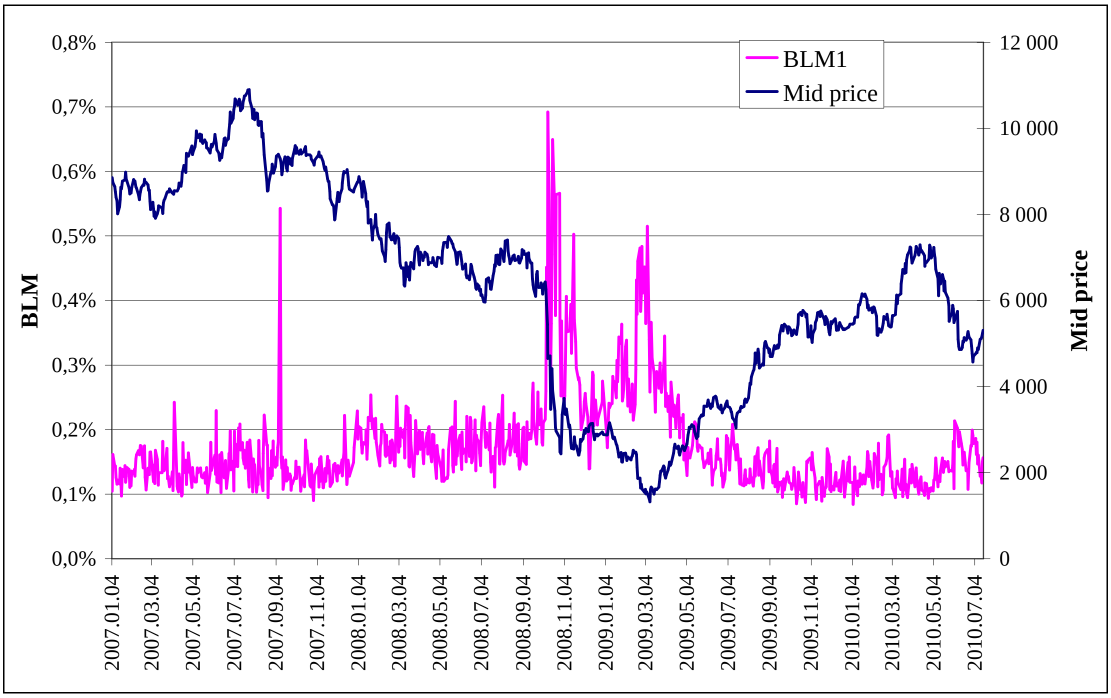


Source: proprietary

2.5.4. Time series of BLM

In my dissertation, I intend to use the BLM database for presenting how it could be utilized in risk management by supplementing a VaR-type model with it, and by estimating a virtual price impact function from it and then carrying out a statistical analysis on the estimated database. I present these in the following two (III and IV) chapters. However, for this I find it inevitable to describe how the BLM database evolves in time. Namely, I am going to carry out the examinations on the time series data of the price impact function based on this. The following figure shows the formation of daily BLM1 and daily price of OTP between 1 January, 2007 and 3 June, 2011:

Figure 34: Average daily BLM1 and price data of OTP during the period of 01/01/2007-03/06/2011



Source: proprietary

In the figure mean-reverting can be observed in the BLM1 time series, furthermore it is also to be seen that there is a correlation between the BLM1 of the preceding day and the BLM1 values of the current day , since it can be observed that typically low BLM1-value days are followed also by low BLM1-value days, and the same can be stated when the BLM1 takes a high value. It also appears in the figure that during the financial crisis of 2008 the value of the indicator significantly increased, which well reflects the shortage of liquidity on the market during this period.

According to this, in Subchapter IV.2 I am going to examine the database of the virtual price impact function estimated from the BLM time series from the angle whether a mean-reversion can be found in the time series, whether there is an autocorrelation in the time series data, and whether there is a structural break in the database as a consequence of the ongoing economic processes.

2.6. Conclusion

I have shown that BLM is a liquidity indicator which is able to measure the liquidity of the assets traded on the stock exchange along several dimensions, thus it provides a reliable view of the current liquidity situation of the market. The analysis also revealed, that the rankings based on the bid-ask spread, the turnover or the BLM are nearly the same, though the relation between the bid-ask spread and the BLM is stronger than between the turnover and the BLM. In sum, the BLM is an indicator which is easy to use, and can help investment decisions from the viewpoint of liquidity. Moreover it gives a more reliable picture of the assets'/market's liquidity, as opposed to a situation in which the investor would base her decision only on the turnover or the bid-ask spread. My main statements in this chapter and my answer to the first hypothesis are the following:

> **H1: BLM, and the most commonly used liquidity indicators in practice (bid-ask spread, turnover) provide different ranking from the aspect of liquidity for individual stock:**
>
> > **H1/a: during a calm period, and**
> > **H1/b: during a crisis.**
> > **H1/c: in the case of a liquid and**
> > **H1/d: an illiquid stock.**

S1: In the case of medium liquid and illiquid stocks, bid-ask spread does not give the same ranking as BLM, however the difference is not significant.

S2: In the case of liquid, medium liquid and illiquid stocks, turnover does not give the same ranking as BLM, however the difference is not significant.

S3: In a calm period i.e. before and after crisis, ranking differs less from the ranking provided by BLM based on turnover than from the one based on bid-ask spread.

S4: During a crisis, the ranking based on bid-ask spread differs less from the ranking provided by BLM than from the one based on turnover.

S5: During the crisis the rank-correlation has decreased between BLM and the spread and between BLM and the turnover.

S6: In the case of the medium liquid and illiquid stocks it would be worthwhile to take also the BLM into consideration as a liquidity indicator, because in their case the ranking in the wrong order is more significant. In respect of these stocks I have also shown during my analysis that there is a chance that a particular stock is sorted into a wrong liquidity category.

S7: In the case of liquid stocks, the values of BLM and the bid-ask spread returned to their pre-crisis level, while in the case of turnover it could only be observed in the case of OTP and MTelekom.

S8: In the case of medium liquid and illiquid stocks liquidity of some stocks did not return to the pre-crisis level according to the BLM and bid-ask spread, while it did not happen to any stocks according to turnover.

S9: Each stock’s liquidity related to one another can significantly differ in the case of different liquidity indicators.

S10: The correlation between bid-ask spread and BLM can be regarded as strongly positive, while the correlation of BLM and turnover shows a slightly negative relation.

S11: The less liquid a stock is, the lower the correlation between the liquidity indicators.

S12: The change of bid-ask spread has a strong explanatory power about BLM change in the case of a liquid stock, whilst in the case of medium liquid stocks this explanatory power is not significant. In the case of illiquid stocks, bid-ask spread change has very limited explanatory power, which cannot even be considered as significant before the crises.

S13: The turnover change cannot explain BLM change in the case of liquid and illiquid stocks, whilst it has also only a low explanatory power in the case of a medium liquid one.

S14: Turnover and liquidity do not co-move intradaily, for instance at the beginning of the day liquidity is low in every case regardless whether the turnover is big or small.

S15: BLM can be important for those market participants who invest in illiquid stocks or intraday.

According to my statements, I can conclude that I reject the first hypothesis, namely that the ranking of the different liquidity indicators differ notably. Though I reject the hypothesis, in the course of making investment decisions it is worth taking into account the differences among the ranking provided by the liquidity indicators.

Apart from the fact that the investors could base their trading strategy on the BLM from the view of liquidity, market participants can use this indicator for several other things, as well. For example, brokers would be able to optimize the order splitting of larger blocks of shares, or it could help traders to set the prices of the stop limits. Above these, BLM could be used for creating new derivative products, which would enable market participants to hedge liquidity risk. In relation to these possible applications, I have analyzed the relation between volatility and liquidity. I have based my second hypothesis on this analysis. In the following I am listing my statements, and my answer to the second hypothesis.

H2: There is a positive relation between volatility (standard deviation, true range) and BLM.

S1: On the Budapest Stock Exchange it has been justified, that there is a positive correlation between BLM and volatility, namely that the more volatile markets are, the transaction cost caused by the lack of liquidity is higher.

S2: The less liquid a stock is, the lower the correlation between liquidity and volatility tends to be.

S3: Before and during the crisis, the correlation between the true range and liquidity was stronger than the one between standard deviation and liquidity. However, after the crisis this has reversed.

S4: The crisis of 2008 can be regarded as a liquidity crisis based on the liquidity estimated from volatility, i.e. the estimated BLM value is lower than the actual BLM value.

S5: After the crisis, the estimated BLM value is typically higher than the actual value, i.e. liquidity is higher after the crises than it had been expected.

From my statements it follows, that I cannot reject the second hypothesis, namely that there is a positive relation between volatility and liquidity.

III. Liquidity adjusted Value-at-Risk

Liquidity risk becomes more and more important in risk management, i. e. during the crises of the past decades, market participants had to face that the lack of liquidity caused significant losses to them. This could be observed during the fall of two giant hedge funds, Long Term Capital Management (LTCM) in 1998, or even Amaranth Advisor in 2006. What contributed to their bankruptcy was that they took such big positions that were impossible to liquidate in a short period without a significant price impact, which resulted in significant losses for them (Jorion, 2007). Furthermore, also in the case of the subprime crisis between 2007 and 2008 it caused great losses that money-markets dried out and liquidity completely disappeared from the markets (Stange and Kaserer, 2009a).

Apart from market participants, regulatory authorities have also recognized that there is a need to take liquidity into consideration, as well when drafting new regulations.[28] Thus, the Basel II accord did not prove to be adequate for the regulation of financial institutions anymore, since it did not address the issue of liquidity management. During the crisis of 2007 and 2008, numerous reports and guide-lines were created referring to the handling of liquidity. As a consequence, the Basel Committee called upon the banks to use conservative methods when the assessing their assets from the point of their marketability. Besides, the Committee has also prescribed to the banks to integrate the costs, benefits and risks of liquidity into their pricing, performance assessment and into the process of accepting new products in the case of every significant business activity (Basel Committee, 2008). The Basel Committee has created the main scheme of Basel III regulatory standard in 2009, whose aim is to provide a regulatory framework concerning the capital requirement and liquidity of banks, thus expanding the Basel II regulation.

Referring to liquidity, the Basel III elaborated two indexes so that the banks could be more resistant during the periods when the lack of liquidity occurs on the market. One index incorporates the short-term liquidity of banks, whereas the aim of the other index is to regulate the refinancing of less liquid assets with appropriate long-term liability (Kovács, 2011). The first index is the so called LCR (liquidity

[28]About more detailed information on the relationship of the crisis and the regulations see: Antalóczy et al. (2009).

coverage ratio),[29] and the other is the so called NSFR (net stable funding ratio) index[30] (BIS, 2010).

However, I am not concerned with the liquidity of banks in my dissertation, thus I do not undertake the analysis of the indexes applied in the framework of the Basel III regulation, but my analysis is going to centre around the liquidity adjusted value-at-risk (LAVaR) models, since for market participants this index provides important information regarding risk.

A vast number of research were created during the past few years about how to include the concept of liquidity into risk management, and how to integrate liquidity into the conventional VaR models. For this, it is inevitable to determine a unified framework for the quantification of liquidity, which is a complicated task, namely liquidity in all asset classes is a concept which is highly difficult to quantify (Basel Committee, 2005). Chapter III/1 is concerned with the research which supplement the conventional VaR models with liquidity risk.

1. Literature of the LAVaR models

The Value-at-Risk (VaR) is a commonly used model in the risk management systems, since it is easy to use and to understand. The VaR measure shows us the maximum loss of the portfolio over a predefined time horizon (T) at a given significance level (α). It can be expressed either in forint or as a percentage of the portfolio value (Jorion, 2007). The significance level is usually 95% or 99%, while the time horizon can be anything, usually one day, one week, one month, one year, etc. There is a relation between the time horizon and the significance level, since the longer the time horizon, the lower significance level can be, because we require a lower security level in that case.

In order to be able to calculate the value of the VaR, we need to know the probability distribution of our position in the certain security/portfolio at time T. The

[29] $$\text{LCR} = \frac{\text{Stock of highly - liquid assets}}{\text{Total net cash outflows over the next 30 calendar days}} \geq 100\%$$

[30] $$\text{NSFR} = \frac{\text{Available amount of stable funding}}{\text{Required amount of stable funding}} \geq 100\%$$

(1-α)th percentile of this distribution will give us the value, from which our security/portfolio will be worth less with a probability of (1- α) at time *T* (Jorion, 2007).

$$P(V_t<K)= 1-\alpha \tag{14}$$

where the value of our position is *V* and the difference of its percentile (*K*) will give us the value-at-risk in forint. For example, if our portfolio's value is normally distributed at time *T*, which has an *m* mean value, and an *s* standard deviation, then *K* can be determined according to the next equation (Öcsi, 2007), where *N* shows the distribution function of the standard normal distribution:

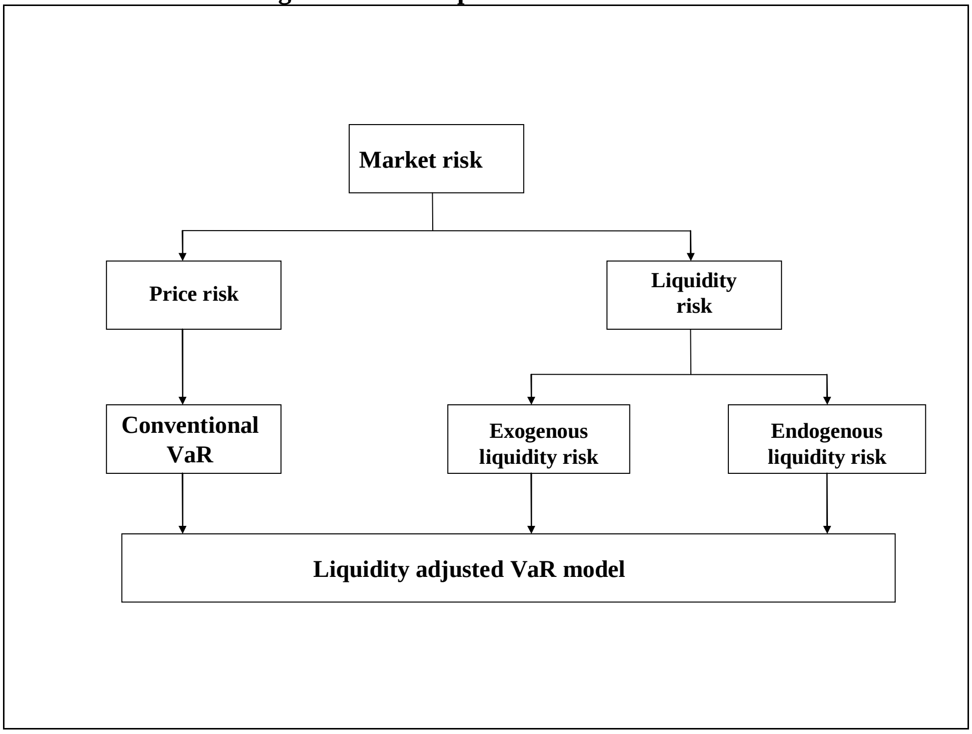
$$N\left(\frac{K-m}{s}\right)=1-\alpha \tag{15}$$

The conventional VaR calculation doesn't contain the total market risk, since it doesn't take into account the liquidity risk. The conventional VaR assumes that one can trade on the mid price within a fix time period. This is not true in case of real market situations. Because of this one needs to take into account, that it is not possible to trade on the mid price, and liquidity should be quantified. A variety of studies have showed that liquidity risk constitutes a significant share of total market risk therefore it is worth considering it. For instance, Bangia et al. (1999) state that in emerging markets models underestimate market risk by as much as 25-30% because of ignoring liquidity risk. Lawrence and Robinson (1997) reach a similar conclusion. According to their study neglecting liquidity risk may underestimate VaR by 30%. Stange and Kaserer (2009a) analyzed the data of the Deutsche Börse AG and found that conventional VaR measures underestimate risk by 25% even for liquid stocks. Finally, Dowd (2001) states that the costs of illiquidity may reach the extent of losses suffered from price fluctuations.

The results above all suggest that, when calculating VaR, above price risk we must take into account liquidity risk, therefore the conventional VaR model should be amended with the quantification of liquidity risk.

In sum, market risk can be split into two main parts: the price risk, namely that the mid price changes as a result of market processes; and liquidity risk, namely that market participants cannot trade on the mid price. Furthermore liquidity risk itself can be divided into two parts, to exogenous risk, and endogenous risk, which is shown at Figure 35.

Figure 35: Decomposition of market risk



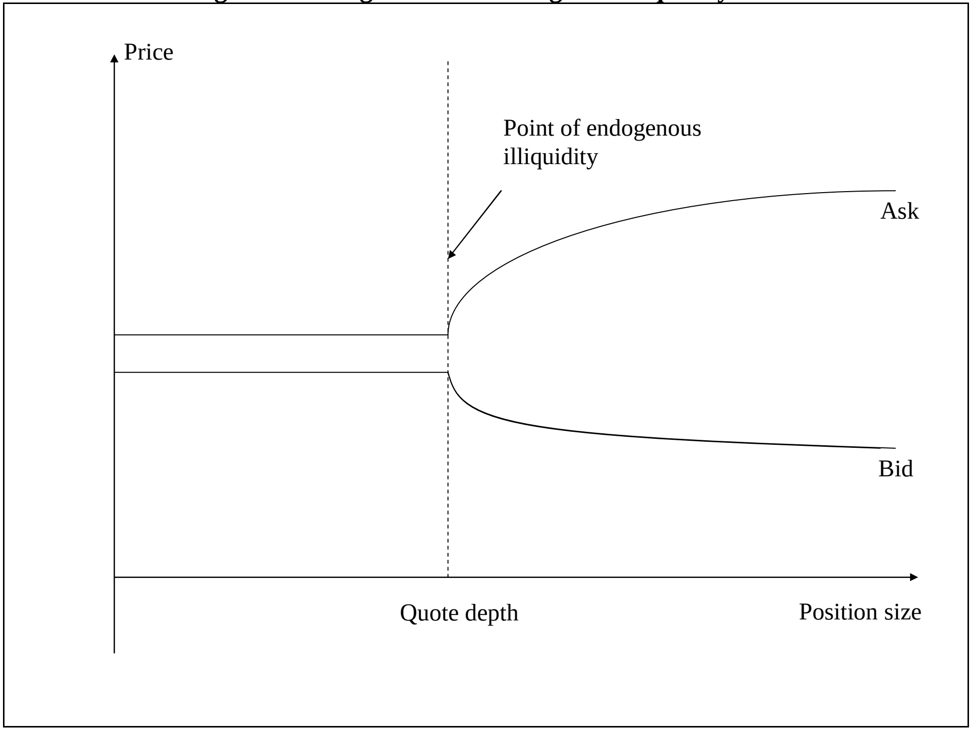
Source: Bangia et al. (1998), p. 3.

Exogenous liquidity risk stems from market processes, and is uniform for all market players. None of the individual market participants can influence exogenous liquidity, although their aggregate activity certainly can. This liquidity risk can be measured, for example, with the size of the bid-ask spread, the turnover, or the quantity of buy and sell orders available at the best levels. On liquid markets the bid-ask spread is quite stable, and small, while the quantity of the orders available on the best price level is usually high, and has a stable value as well. Besides these characteristics, it can be observed on liquid markets, that the turnover is high. In contrast, on illiquid markets – such as for example the emerging markets – the bid-ask spread is quite variable, and has a higher value, than in case of liquid markets. Also the quantity of the orders available on the best price level is more variable as well. Moreover it often happens that there are only a few orders on the market, and the turnover is lower than on liquid markets (Bangia et al, 1998, p. 4.).

To the contrary, endogenous liquidity can be different for each of the market participants. Its value depends on the size of the position a market player has on that given market. Usually the size of the position has an effect on the endogenous liquidity risk (Bangia et. al. 1998, p. 4).

The next figure describes the relationship between the size of the position, the endogenous and the exogenous liquidity:

Figure 36: Exogenous and endogenous liquidity risk



Source: Bangia et al. (1999), p. 5.

Before the appearance of the liquidity adjusted VaR models, market participants have taken liquidity risk into account in case of large illiquid positions, that they have calculated the VaR measure for a(n) – ad hoc – longer time period. The length of the time period was influenced by what market participants thought about the time which was needed to liquidate the whole position. In this case the variances and covariances were not calculated for the whole time period, but for the shorter time period, and then these values were multiplied by the square-root of time (Bangia et al., 1998). This approach didn't lead to the right result, and caused an over-estimation in the value of the variance and covariance (Diebold et al., 1998).

Numerous papers published in the last decade have adjusted conventional VaR calculation for liquidity risk. This group of models is named LAVaR (Liquidity Adjusted Value at Risk) models, and is usually divided into two large groups: i) models based on the data of the order book; and ii) models based on optimal execution. Liquidity adjusted VaR models can be further split into the subgroups as seen below:

- Order book based models:
    - Models considering exogenous liquidity risk,
    - Models considering endogenous liquidity risk,
    - Transaction or volume based models.
- Optimal execution based models:
    - Stochastic time horizon models,
    - Price impact function based models.

In the following I will introduce the models of the first group in detail, since my empirical research will be based on those LAVaR models. I have chosen a model that is based on the order book, because it has the advantage of not having to estimate numerous parameters as it is in the case of models based on optimal execution (Stange and Kaserer, 2009b).

1.1. Models considering exogenous liquidity risk

The first LAVaR model was created by Bangia et al. (1998), which become the reference point for all later models that estimate LAVaR based on the data of the order book. This model provided the market with a simple-, and easily applicable method, which enabled the market participants to incorporate liquidity risk into the VaR framework. The model they created is called BDSS in the literature, after the authors' names (Anil *Bangia*, Francis X. *Diebold*, Til *Schuermann*, John D. *Stroughair*).

The BDSS model quantifies only the exogenous liquidity risk, since it takes into account only the bid-ask spread. Hence, in this model the LAVaR value is the sum of the conventional VaR and the liquidity risk determined by the bid-ask spread. The LAVaR is calculated as follows:

$$\mathrm{LAVaR} = \mathrm{Pmid}_t\left[\left(1-e^{\mu-\alpha\sigma}\right)+\frac{1}{2}\left(\bar{S}+\alpha'\tilde{\sigma}\right)\right], \tag{16}$$

where Pmid_t is the mid price of the asset at time t, μ is the logreturn, α is a pre-defined percent of the logreturn's distribution, σ is the standard deviation of the logreturn, $\bar{S} = \frac{\mathrm{Pask} - \mathrm{Pbid}}{\mathrm{Pmid}}$, is the average relative spread, $\tilde{\sigma}$ is the relative spread's standard deviation, while α' is the pre-defined percent of the relative spread's distribution.

The practical advantage of the BDSS model is that it is easy to use, as bid-ask spread data are available for the market participants of various markets. However, there are also several disadvantages that inspired researchers to develop further models in this field. These shortcomings are as follows:

1. It assumes the distribution of spreads to be normal. The experiences in practice show that the distribution of the spreads is not normal, since it is fat-tailed and more skewed than the normal distribution as a consequence of the trends on the markets. In some cases researchers found that the distribution has several modes, which can happen because of regime switches (Bangia et al. 1998).
2. It ignores endogenous liquidity risk, hence it underestimates liquidity risk.
3. It assumes perfect correlation between liquidity risk and price risk. According to the model price is the lowest when spread is the largest. This way the model overestimates risk. Stange and Kaserer (2009a) give empirical evidence that this assumption is not correct. On a theoretical level, Francios-Heude and Wynandaeale (2001), Angelidis and Benos (2006) and Jorion (2007) criticize this assumption.

To address the first of the shortcomings of the BDSS model, it could make sense to use the empirical distribution of the bid-ask spread instead of the normal distribution. The problem with this however is, that we would need a long time series to estimate the distribution, and as a result, the time series could contain structural breaks, or have several modes, which should be taken into account when calculating VaR.

In the literature Ernst, Stange and Kaserer's (2008) model is well-known, which tries to solve the problem caused by the assumption of the normal distribution. This model is also based on the bid-ask spread, like the BDSS model, but in case of Ernst et al.'s model, the percentile of the distribution is estimated by the Cornish-Fisher estimation[31] instead the historical estimation. The basic of the estimation is also the normal distribution, but it takes into account the skewness and curtosis of the distribution. Ernst et al.'s (2008) model give a more precise result, than the BDSS model, but the other shortcomings of the BDSS are not being solved by this model either.

To handle the endogenous liquidity risk as well, the solution could be to use a LAVaR model that incorporates the whole order book, like for example the model of Francois-Heude and Wynendaele (2001) or Giot and Gramming (2005). I will introduce these models in more detail in Subchapter 1.2.

The third critique of the BDSS model, namely that there is a perfect correlation between exogenous liquidity risk and price risk, could be solved by estimating the real correlation from real market data.

Models similar to Bangia et al.'s (1998) model can be found in the Hungarian literature as well. Radnai and Vonnák (2009) have examined during the analysis of Basel III regulation, the possibility to specify capital requirements for those assets which can be found in a bank's trading book. This capital requirement would serve to cover the possible losses caused by illiquidity. The authors have suggested using the bid-asking spread, since it is a good indicator of liquidity. According to their opinion the capital requirement should be a linear function of the bid-ask spread, or it could be determined with internal model based on the spread's historical distribution (Radnai and Vonnák, 2009, p. 252).

1.2. Models considering endogenous liquidity risk

The most important feature of the models considering endogenous liquidity risk is that they not only quantify liquidity risk with the bid-ask spread but also with a quantity weighted spread measure (Stange and Kaserer, 2009b). In other words, they

[31] More detailes about the Cornish-Fisher estimation see: Jorion, 2007, p. 273.

take into account that a transaction is not necessarily executed at the best price level. Accordingly, the spread values are weighted with the quantities at the given price levels. These models can incorporate a liquidity measure, e.g. the BLM or the XLM, as these give the price of liquidity for a predefined trade size.

These models give a better result, since they quantify exogenous and endogenous risk as well, and they are a more general approach than the BDSS model is.

The first model that dealt with endogenous liquidity risk was elaborated by Francois-Heude and Wynendaele's (2001). Their model is based on the BDSS model, with the difference that they used the first five levels of the order book, not only the best level. As a result the authors could measure the price impact of different transaction sizes, in case when the transaction is fulfilled on the first five price levels. Their model uses intradaily data. In Francois-Heude and Wynendaele's (2001) model the following equation gives the liquidity adjusted VaR measure:

$$\text{LAVaR} = \text{Pmid}_t \left[\left(1 - \left(1 - \frac{\overline{\text{Sp}}(Q)}{2} \right) * \left(e^{-\alpha\sigma} \right) \right) + \frac{1}{2} * \left(\text{Sp}_t(Q) - \overline{\text{Sp}}(Q) \right) \right], \tag{17}$$

where Pmid_t is the mid price at time t, $\overline{\text{Sp}}(Q)$ is the average spread at Q volume, $\text{Sp}_t(Q)$ is the value of the spread at time t, at Q volume, α is a given percent of the distribution of mid price, and σ is the standard deviation of th return.

The next important research in this field was carried out by Giot and Gramming (2005). They based their model also on intraday data, but they analyzed stock portfolios as well. The authors examined price impact of an investor buying and selling a certain amount of stock. This price impact, namely, that what the price will be for those who give market order, will depend on the actual order book. The authors have called this measure CRT (cost of round trip), which was first introduced by Irvine et al. (2000). This measure was calculated as a weighted average spread. Giot and Gramming (2005) defined the LAVaR as follows:

$$\text{LAVaR} = 1 - \exp\left(\mu_{\text{rnet}(q)} + \alpha\sigma_{\text{rnet}(q)} \right), \tag{18}$$

where rnet(q) is the net return, $\mu_{rnet(q)}$ is the expected net return, α is a given percentile of the net return[32], while $\sigma_{rnet(q)}$ is the standard deviation of the net return. The net return was defined as follows:

$$r_{net(t,q)} = r_t * \left[1 - \left(\frac{\sum_i \frac{a_{i,t} n_{i,t}}{n} - \sum_i \frac{b_{i,t} n_{i,t}}{n}}{P_{midt}} \right) / 2 \right], \tag{19}$$

where r_t is the return of the mid price, $a_{i,t}/b_{i,t}$ is the ask/bid price on level i, $n_{i,t}$ is the ask/bid volume on level i, and $Pmid_t$ is the mid price at time t.

The model has two important deficiencies. The first one is that it does not use the empirical distributions, but t-distribution. The second one is that it does not take into account the fact, that liquidity can be different on the bid and ask side of the order book. Stange and Kaserer (2009a) give a solution for the first problem, by using an empirical distribution. These authors have determined the LAVaR for the daily XLM database. The researchers have pointed out, that it is not proper to simply ad liquidity risk to the conventional VaR measure, since it causes the over-estimation of the total risk, because we ignore the correlation between liquidity- and price risk. A shortcoming of the model is that it assumes the symmetry of the book, that is, the transaction costs arising from illiquidity are equal on the sell and the buy side.

Qi and Ng (2009) offer a solution for the second deficiency, by estimating liquidity for both sides of the order book. Namely they calculate liquidity risk for the bid and ask side, quantifying it through the VWAP (volume weighted average price). They named their model LAIVaR-nak (liquidity adjusted intraday VaR), since they calculate the VaR intradaily. In their study they pointed out that it is worth assuming an asymmetric order book since price movements are not symmetric: drops are always more significant and drastic than price increases.

In their model Bt(v) and At(v) means the weighted average price for a given volume (v) for the bid and ask side for a given short period.

[32] The distribution of the return was assumed to be a student-t distribution.

$$B_t(v) = \frac{\sum_j B_{i,t} v_{i,t}^{BID}}{\sum_j v_{i,t}^{BID}}, \qquad A_t(v) = \frac{\sum_j A_{i,t} v_{i,t}^{ASK}}{\sum_j v_{i,t}^{ASK}} \tag{20}$$

where v is a given volume, which will be bought/sold at time t, and at least the first j levels of the order is needed to fulfill the transaction the way, that $v \leq \sum_{\min(n)} v_{i,t}$. Namely that the volume of the transaction is smaller than the sum of the volumes in the first i levels of the order book. The indicators in Equation 20 show the immediate transaction cost of the bid and ask side (Qi and Ng, 2009).

Finally, Erwan's (2001) model is worth mentioning as well, in which the author also develops the BDSS model by using weighted average spread. The interesting thing about the article, that the author shows, that the endogenous liquidity risk is about the half of the market risk in case of illiquid stock, so it shouldn't be ignored.

1.3. Transaction or volume based models

I am going to present two models that are not based on the order book, but are using past transaction data to estimate liquidity risk. The major advantage of these models is that they can be used on markets where there are no order books. The models are proposed in Berkowitz (2000a, b) and Jarrow and Potter (2001).

The basic assumption of the LAVaR model of Berkowitz (2000a, b) is that liquidity can be estimated by a linear regression, where the regressor is liquidity, while the regressand is the transaction price. The regression is the following:

$$P_{TA,t+1} = P_{mid,t} + C + \theta N_t + x_{t+q} + \varepsilon_t, \tag{21}$$

where $P_{TA,t+1}$ is the price after the transaction, θ is the coefficient of the regression, which measures liquidity, N_t is the number of sold stocks, C is the constant, ε_t is the residuum, and x_{t+1} is the effect of the risk factor's change to the mid price.

This model assumes that the risk factor and the liquidity are independent, which means, that the correlation between return and liquidity is 0. The advantage of the model, that it is not necessary to know the order book to be able to calculate the LAVaR, so it can be used on markets, where there isn't any order book. But its disadvantage is that if we want the estimation of the regression to provide a reliable result, we need to have sufficient quantity of available data. Berkowitz (2000a, b) applies his model to intraday data in order to have enough data.

Jarrow and Potter (2005) display a model based also on a regression. The difference between their model and the Berkowitz model is that they only perform the estimation based on the data from a period of one particular market turmoil. A further difference is that Jarrow and Potter (2005) do not consider other risk factors in their model therefore they do not have to presume zero correlation between liquidity and return. Moreover, the authors use relative changes, hence they consider the data of the previous period, as well. In their case the regression looks like the following:

$$\log\left(\frac{P_{TA,t+1}}{P_{TA,t}}\right) = \left(\mu_{rt} - \frac{1}{2}\sigma_{rt}^2\right) + \theta\left(N_{t+1} - N_t\right) + \varepsilon_t, \tag{22}$$

where μ_{rt} is the expected value of the mid price's return, while σ_{rt}^2 is the variance of the mid price's return.

There is another significant group of order book based models, the models based on volume weighted price impact. I would highlight the work of Cosandey (2001). The essential feature of the model, that the author estimates price impact from volume data. The price is a function of the number of shares traded (N), while the investors can only trade with a predetermined quantity (Q). So the price is calculated like: P=Q/N. If we assume that the number of traded shares (N) are constant, then the mid price will be: P_{midt} (ΔN) = Q/(N+ΔN) = P_{midt}*(N/(N+ΔN)). Namely, the price impact will be the linear function of the traded volume. The calculation of the LAVaR in his model will be the following:

$$\text{LAVaR}(\Delta N) = \text{perc}\left(r_{t+1}\frac{N}{N+\Delta N}\right), \tag{23}$$

where „perc" means the percentile from simulated distributions. In Cosandey's (2001) model, the change of the mid price and the change of the volume were modeled together.

1.4. Stochastic time horizon models

There is another group of LAVaR models next to the models based on order book, the models based on optimal execution. Two types of models belong to this group, the models based on stochastic time horizon and the models based on price impact functions. The essence of these models is to provide an optimal execution strategy for market participants, where the strategy is built on the optimal balance between the transaction cost of trading and the cost of delay caused by not executing the transactions immediately. In case the market participant waits with the execution she has the chance to face a better market liquidity, so her transaction will cause a smaller price impact.

From the models based on stochastic time horizon I would like to present two models, one of them were developed by Lawrence and Robinson (1997), and the other one by Haberle and Persson (2000). The model of Lawrence and Robinson (1997) is based on the assumption, that the shorter the time horizon used in the calculation of VaR, the more VaR underestimates the possible losses. So their model contains the cost of illiquidity and the cost of delay, though the authors do not give the exact calculation of the cost of delay. The researchers give an optimal time-horizon for the execution of the transaction, where this optimal time-horizon depends on the volume of the transaction and the market liquidity. The other shortcoming of the model in addition to the lack of defining the calculation of costs, that it does not take into account the time variations of liquidity (Francois-Heude and Wynendaele, 2001).

Haberle and Persson (2000) assume that a certain proportion of the daily turnover can be liquidated without notable price impact, but this proportion can be different for every asset. The value of that certain turnover is called price-neutral value. The authors do not provide any clue how to estimate the exact proportion. They state that the proportion should be defined empirically.

1.5. Price impact function based models

The other group of the models based on optimal execution, are the models based on price impact function. Models in this group are made by Jarrow and Subrahmanyam (1997, 2001), Berkowitz (2000), Hisata and Yamai (2000), Almgren and Chriss (2000), Almgren (2003), Dubil (2003), Glosten, Jagannathan and Runkle (1997), Bertismas and Lo (1998), and Engle and Ferstenberg (2007).

The essence of these models, that they assume the time-horizon to be fixed under which market participants can liquidate their positions. The authors quantify on the one hand how the market price has changed during this time-horizon, on the other hand they ask the question: what would be the optimal trading strategy on this same time-horizon which would minimize the transaction cost of trading caused by illiquidity.

The most important shortcoming of the models based on optimal execution, that they can hardly be used in practice. This has several reasons. For example on the one hand in practice it is usually not always possible to wait with the transaction and not to execute an order immediately or within short period of time. During crisis it can be especially risky for the market participants to wait with a transaction. On the other hand the parameters of the optimization should be stable in order to realize the return by postponing the transaction otherwise it is possible to achieve a worse return compared with immediate execution. Thirdly the optimization depends on the estimation of several parameters, which are difficult to handle in practice (Stange and Kaserer, 2009b).

1.6. Testing practical applicability of the models

All the models presented so far have their own respective advantages and disadvantages. A particularly important question is that which model works best on real market data, which provides the best and the most reliable result. Ernst et al. (2009) have prepared a study comparing the estimates of the models based on order book data. I have singled out the following models:

- Models based on endogenous liquidity risk:
    - Stange and Kaserer (2009)

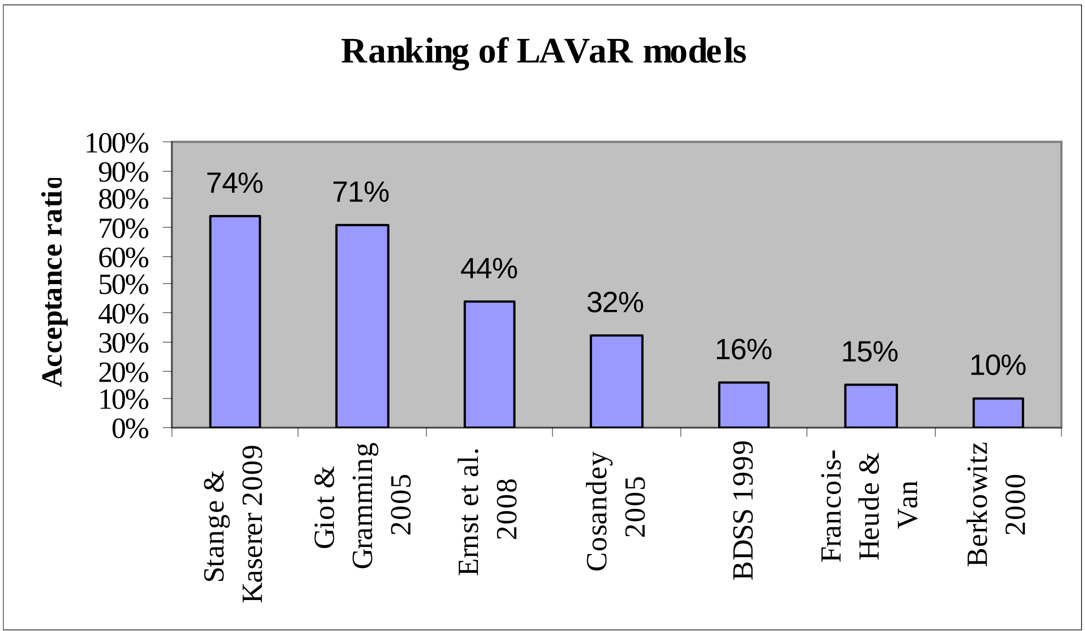
- Giot and Gramming (2005)
- Francois-Heude and Van Wynendaele (2001)

- Models based on exogenous liquidity risk:
  - Ernst et al. (2008)
  - BDSS (1999)
- Models based on transaction
  - Berkowitz (2000)
- Models base on volume
  - Cosandey (2005)

The various models were tested by Ernst et al. (2009) on real market daily data from the July 2002 –December 2007 time period. The authors studied the returns and the risks predicted by the various models. During their test they assumed that the positions must be liquidated immediately at the prevailing order book.

In the test the authors compared the experienced returns with the risk forecasted by the different models. LAVaR has been estimated with a 99% confidence interval, which means that the realized returns may have exceeded the value estimated by the model in 1% of total occurrences (Ernst et al., 2009). Figure 37 shows the acceptance ratios of the models, namely the percentage in which the models were able to properly forecast the stocks' risk.

Figure 37: Ranking of LAVaR models



Source: Ernst et al. 2009, p. 13.

The results of Ernst et al. (2009) demonstrate that those models produced the best results in terms of predicting which have taken endogenous liquidity risk into account. These models significantly overperformed the others.[33]

Summing it up it can be stated that amongst the order book based LAVaR models, the ones utilizing a liquidity measure, such as the XLM, give the best forecasts. Accordingly, I will build a LAVaR model in the empirical part of my dissertation which incorporates endogenous liquidity risk as well. The basis of my model will be the work of Giot and Gramming (2005) and Stange and Kaserer (2009a). The difference will be that I will build it on a Hungarian database for single stocks and for different stock portfolios.

[33] For more results see Ernst et al. (2009).

2. Empirical research: building an own LAVaR model

Besides the statistical analysis of BLM I will build a theoretical model in my dissertation. With the aid of this theoretical model the possible loss caused by the lack of liquidity is easily quantifiable. With building a LAVaR model I would like to show a possible application of the BLM, which is one of its most promising applications. Accordingly in this chapter I will introduce a liquidity adjusted VaR model for single securities and for stock portfolios. My hypotheses will be the following:

> **H3: Market risk can be underestimated at least by 5% even for liquid stocks at the order size of EUR 20,000 on the Budapest Stock Exchange, if we do not take the liquidity risk into consideration.[34]**

> **H4: In case of stock portfolios not only price risk but liquidity risk can be diversified.**

2.1. Research method

The work of Giot and Gramming (2005), Stange and Kaserer (2009a) were the starting point of my own model, who made their models based on XLM type measures.

The technical tools I have used to estimate LAVaR were the same for single stocks and for stock portfolios. The difference in the analysis is that for portfolios it is not sufficient to know the BLM values at five different order sizes (20, 40, 100, 200, and 500 thousand Euros), since in this case not the value but the quantity of the stocks is fixed. Accordingly, we must have BLM figures for all “q”-s. This can be carried out in two simple ways: 1) to use linear interpolation based on the available BLM data for each day, or 2) to use a linear regression.

In my modeling I have taken the second approach, since I will use the same approach during the estimation of the price impact function in the fourth chapter of my dissertation. Obviously, this is a serious simplification, but based on the available

[34] I have chosen 20,000 EUR, because this was the smallest available transaction size, on which the BSE calculates the BLM.

data, it is appropriate for a first approximation. Linear regression is a practical, easy to use method, and qualitative consequences can surely be drawn from the analysis. Furthermore daily BLM data can be well approximated by a straight line.

In my dissertation I have calculated the conventional and also the liquidity adjusted VaR, in order to be able to compare them. In order to account for the clustering volatility of returns and net returns, I have fitted an AR(1)-GARCH(1,1) model, where I have used t-distribution. The sample used to estimate the model was the first 2.5 years (1st January, 2007-15th July 2009), while the last year (16th July 2009-16th July 2010) was used as a control period. I calculated the daily 95% and 99% VaR using forecasts from the GARCH model. I used a rolling window of 2.5 years to continuously re-estimate the GARCH model, i.e. I have estimated a GARCH model for the first 2.5 years and have made a forecast for the next year, and then I have repeated the procedure while rolling the sample period with one day.

The test of the correction of the risk forecasts was done in the following way: the predicted VaR values for both net and normal returns were compared to the corresponding values of the control period and empirical exceedance frequencies were calculated. Then the significance of deviation from the theoretical frequencies was determined statistically using the Likelihood Ratio Test of Kupiec (1995). The test is the following. Let N_u denote the number of days when the net returns exceeded the forecasted LAVaR values, and N the number of days in the sample. Then the empirical exceedance frequency is N_u/N, and let α denote the theoretical frequency. The test statistics using this notation is the following:

$$LR = -2\ln\left((1-\alpha)^{N-N_u} \cdot \alpha^{N_u}\right) + 2\ln\left(\left(1-\frac{N}{N_u}\right)^{N-N_u} \cdot \left(\frac{N}{N_u}\right)^{N_u}\right) \qquad (24)$$

Under the null-hypothesis of H_0: $\alpha = N_u/N$ the test statistic is chi-squared distributed with one degree of freedom. I used the test uniformly on confidence level of 95%, thus H_0 was accepted if LR $\leq$ 3.84. This test will reject the model if the empirical exceedance frequency is significantly below the theoretical value (model overestimates risk) or significantly above (model underestimates risk).

2.2. Value-at-Risk calculation

The starting point of a LAVaR model is a conventional value at risk (VaR) calculation frequently used in everyday risk management. VaR calculation can be carried out according to the following formulae, for the returns (Equation 25) and the prices (Equation 26):

$$\mathrm{VaR}_{return}^{\alpha,\Delta t} = r_t^{\alpha,\Delta t} = \mu_{t+\Delta t} + \sigma_{t+\Delta t} q_{1-\alpha}\,, \tag{25}$$

where returns are considered on continuous time horizon, thus $r_t^{\Delta t} = \ln\left(\frac{P_{mid}^{t+\Delta t}}{P_{mid}^{t}}\right)$, $\mu_{t+\Delta t}$ is the expected value of the return in Δt time, $\sigma_{t+\Delta t}$ is the standard deviation of the estimation, and $q_{1-\alpha}$ is the $1-\alpha$-th quantile of a chosen distribution.

$$\mathrm{VaR}^{\alpha,\Delta t} = \frac{P_{mid}^{t} - P_{mid}^{t} * \exp\left(r_t^{\alpha,\Delta t}\right)}{P_{mid}^{t}} = 1 - \exp\left(r_t^{\alpha,\Delta t}\right), \tag{26}$$

where P_{mid}^{t} is the mid price at time t, while $P_{mid}^{t+\Delta t} = P_{mid}^{t} * \exp\left(r_t^{\Delta t}\right)$. If , for example, $\mathrm{VaR}^{95\%,1day}$ = 5%, then with 95% probability the loss in one day due to the change in mid price will not be larger than 5% (Jorion, 2007).

2.3. Liquidity adjusted returns

The basic idea of a LAVaR model is to incorporate the liquidity measure into the returns, and to determine the VaR value based on these new returns, as follows:

$$\mathrm{LAVaR}^{\alpha,\Delta t}(q) = 1 - \exp\left(r_{actual,t}^{\alpha,\Delta t}(q)\right), \tag{27}$$

where $r_{actual,t}^{\alpha,\Delta t}(q)$ is the net return including the BLM figure, thus allowing for the implicit costs of trading at a given „*q*" sized trade (Stange and Kaserer, 2009a). The net returns in Equation 27 is not given by Stange and Kaserer (2009a) in detail during the calculation of LAVaR, they just introduce the results. According to this in Subchapter 2.3.1 I show broadly how I determined net return for a single stock, which is my own result. Beyond Stange and Kaserer's (2009a) work I also determine in Subchapters 2.3.2 and 2.3.3 the net return of two different stock portfolios, to an equal volume stock (EVS) portfolio and for equal value stock portfolios as well, which is also my own result.

2.3.1. Determining the return for a single stock

For a single stock the return taking only the price risk into account at a given "*v*" trade size is as follows:

$$r_{hypothethicc} = \ln\left(\frac{P_{mid,t} \cdot v}{P_{mid,t-1} \cdot v}\right) = \ln\left(\frac{q_t}{q_{t-1}}\right), \tag{28}$$

where $r_{hypothethic}$ denotes the return one would realize if trading with the asset were possible at the mid price. Accordingly, $P_{mid,t} \cdot v$ and q_t stand for the value we were to get for selling „*v*" quantity of stock, if they were traded at the mid price. $P_{mid,t-1} \cdot v$ and q_{t-1} denotes the same but a period earlier.

One must take the implicit cost of trading into consideration to calculate the net or *actual return*. For this, first based on Equation 29 the weighted average price should be determined:

$$b_t(v) = \frac{\sum b_{k,t} \cdot v_{k,t}}{v}, \tag{29}$$

where $b_t(v)$ is the weighted average price on the bid side of the book for a given „*v*" quantity, $b_{k,t}$ is the price in the order book at level k at time t, while $v_{k,t}$ is the quantity available at level *k* of the order book at time *t* and „*v*" is the total quantity to be traded.

The total proceeding from selling a stock at time t is $b_t(v) \cdot v$. This can be expressed as follows:

$$b_t(v)\cdot v = q_t^{net} = q_t \cdot \left(1 - \frac{BLM(q_t)}{2}\right), \tag{30}$$

where q_t^{net} stands for the value we get when selling the stocks, and q_t is the value we would get if we were able to trade at the mid price. This latter must be adjusted for the transaction cost stemming from illiquidity, which is represented by the BLM. During the adjustment I take only half of the BLM, since the BLM represents the implicit transaction costs of turning around a position at the same time. By doing this I implicitly assume that the bid and the ask sides are symmetric. This assumption can be released, although presuming symmetry for daily level data is not a substantive restriction.

Based on the above, the actual return is to be determined by the following formula:

$$r_{actual} = \ln\left(\frac{b_t(v)\cdot v}{P_{mid,t-1}\cdot v}\right) = \ln\left(\frac{q_t^{net}}{q_{t-1}}\right) = \ln\left(\frac{q_t^{net}}{q_t}\cdot\frac{q_t}{q_{t-1}}\right) =$$

$$= \ln\left(\frac{q_t \cdot \left(1 - \frac{BLM(q_t)}{2}\right)}{q_t}\cdot\frac{P_{mid,t}}{P_{mid,t-1}}\right) = \ln\left(1 - \frac{BLM(q_t)}{2}\right) + r_{hypothethic} \tag{31}$$

The actual return has been split into two components; the first showing the effect of illiquidity: $\ln\left(1 - \frac{BLM(q_t)}{2}\right)$; the second is the return one was to realize if trading at mid price were possible: $r_{hypothethic}$.

2.3.2. Determining returns of an equal volume stock portfolio

The return of a portfolio consisting of "n" number of stocks is calculated similarly to the return of a single stock. This is showed by Equation 32:

$$r_{hypothethic} = \ln\left(\frac{\sum P^i_{mid,t} \cdot v_i}{\sum P^i_{mid,t-1} \cdot v_i}\right) = \ln\left(\frac{\sum q^i_t}{\sum q^i_{t-1}}\right) \tag{32}$$

During my analysis I calculate an EVS *(Equal Volume Stock)* portfolio's returns. This is a portfolio comprising of the same amount of each stock, that is v_i = v.

For calculating the actual returns I need the value of the portfolio at different times:

- The value of the portfolio at time t if there perfect liquidity: $\sum_{i=1}^{N} q^i_t = q_t = \sum v_i \cdot P^i_{mid,t} = v \cdot \sum P^i_{mid,t}$;
- The proceedings from selling the portfolio at time t, considering transaction costs arising from illiquidity: $q^{net}_t = \sum b_t(v_i) \cdot v_i = \sum q^i_t \cdot \left(1 - \frac{BLM(q^i_t)}{2}\right)$;
- The value of the portfolio at the previous period's mid price: $q_{t-1} = \sum P^i_{mid,t-1} \cdot v_i = v \cdot \sum P^i_{mid,t-1}$.

Determining the three values above is necessary, as during the calculation of the portfolio's return I will once again split the return into two components: the first coming from illiquidity, the second arising from the change in mid price. In order to determine the return from illiquidity I need the value of the portfolio with and without transaction costs. For the return from the change in mid price I need the value of the portfolio at time t and in the previous period, supposed that there is no loss from illiquidity. I arrive at the following actual return:

$$r_{actual} = \ln\left(\frac{\sum b_t(v_i)\cdot v_i}{\sum P^i_{mid,t-1}\cdot v_i}\right) = \ln\left(\frac{q_t^{net}}{q_{t-1}}\right) = \ln\left(\frac{q_t^{net}}{q_t}\right) + \ln\left(\frac{q_t}{q_{t-1}}\right) =$$

$$= \ln\left(\frac{\sum\left(q_t^i\cdot\left(1-\frac{BLM(q_t^i)}{2}\right)\right)}{q_t}\right) + \ln\left(\frac{v\cdot\sum P^i_{mid,t}}{v\cdot\sum P^i_{mid,t-1}}\right) = \ln\left(\frac{v\cdot\sum\left(P^i_{mid,t}\cdot\left(1-\frac{BLM(q_t^i)}{2}\right)\right)}{v\cdot\sum P^i_{mid,t}}\right) + r_{hypothethic} =$$

$$= \ln\left(\frac{\sum\left(P^i_{mid,t}\cdot\left(1-\frac{BLM(q_t^i)}{2}\right)\right)}{\sum P^i_{mid,t}}\right) + r_{hypothethic} \tag{33}$$

In the above I took advantage of the fact that I used EVS type portfolios, and therefore, I could simplify with „v”. Naturally, the above formula can be used for non EVS type portfolios, but then „v_i”-s will not let us to simplify the equation. In the next subchapter I show how the return calculation changes if I want to determine the LAVaR value for value weighted portfolios.

2.3.3. Determining returns of an equal value stock portfolio

In case of the equal value stock portfolio I assume that the value of the portfolio is constant for the whole period, for example EUR 20,000. I will sign the fix value proportion of each stock with w_i. According to this I will define the value of the portfolio ($q_{portfolio}$) as follows:

$$q^t{}_{portfolio} = \sum_{i=1}^{n} w_i\cdot q_i^t = \sum_{i=1}^{n} w_i\cdot P^t_{mid,i}\cdot v_i^t \tag{34}$$

I will be able to define the number of stock I need to have in my portfolio in order to have the required value of each one of the stocks. To determine v_i will be determined as follows:

$$q_i = w_i \cdot q \quad \rightarrow \quad v_i^t = \frac{w_i \cdot q}{P_{mid}^t} \tag{35}$$

The calculation of net return will be the same as it was in case of the equal volume portfolio, the difference will be, that the number of the stocks will change from time to time as the mid price changes, in order to keep the same value of each stock. This is the reason why I have used the assumption, that the value of the portfolio is the same every time. The return will be the following:

$$r_{actual} = \ln\left(\frac{\sum b_{+1t}(v_i)\cdot v_i}{\sum P_{mid,i}^t \cdot v_i}\right) = \ln\left(\frac{q_{t+1}^{net}}{q_t}\right) = \ln\left(\frac{q_{t+1}^{net}}{q_{t+1}}\right) + \ln\left(\frac{q_{t+1}}{q_t}\right) =$$

$$= \ln\left(\frac{\sum\left(q_i^{t+1}\cdot\left(1-\frac{BLM\left(q_i^{t+1}\right)}{2}\right)\right)}{q^{t+1}}\right) + \ln\left(\frac{\sum v_i^t \cdot P_{mid,i}^{t+1}}{\sum v_i^t \cdot P_{mid,i}^t}\right) =$$

$$= \ln\left(\frac{\sum\left(v_i^t \cdot P_{mid,i}^{t+1}\cdot\left(1-\frac{BLM\left(q_i^{t+1}\right)}{2}\right)\right)}{\sum v_i^t \cdot P_{mid,i}^{t+1}}\right) + \ln\left(\frac{\sum v_i^t \cdot P_{mid,i}^{t+1}}{\sum v_i^t \cdot P_{mid,i}^t}\right) \tag{36}$$

After determining returns, the v_i data should be updated, in order to keep the w_i weights of the portfolio for the t+1-th period. This can be done according to Equation 35.

Besides the LAVaR values I will also determine in Subchapter 2.4 the relative liquidity impact, while in Subchapter 2.5, I will show the diversification effect of liquidity risk.

2.4. Relative liquidity impact

The LAVaR measure represents the total market risk that includes both price risk and liquidity risk. Identifying the share of liquidity risk within the LAVaR is easily performed using Equation 37:

$$\lambda(q) = \frac{LAVaR^{\alpha,\Delta t}\left(\sum q_i\right) - VaR^{\alpha,\Delta t}\left(\sum q_i\right)}{VaR^{\alpha,\Delta t}\left(\sum q_i\right)} \tag{37}$$

In the literature $\lambda(q)$ is named relative liquidity impact or relative liquidity measure (Giot – Gramming, 2005). This measure shows the maximum loss due to illiquidity at a given confidence level for a predefined time period. During my calculations I will determine this indicator as well for both single stocks and for stock portfolios. In case of the stock portfolios I will also determine the value of liquidity risk diversification, which to the best of my knowledge no one before me has done with this method. I show the calculation in Subchapter 2.5.

2.5. Diversification

In case of portfolios it is an important question whether the liquidity risk can be diversified. I have determined the Equation 38 – no one has used this before me as per the literature –, which shows whether liquidity risk can be decreased in case of portfolios. Equation 38 will help us to address this issue.

$$\gamma(q) = \frac{\left(\sum LAVaR^{\alpha,\Delta t}(q_i) - LAVaR^{\alpha,\Delta t}\left(\sum q_i\right)\right) - \left(\sum VaR^{\alpha,\Delta t}(q_i) - VaR^{\alpha,\Delta t}\left(\sum q_i\right)\right)}{\left(\sum VaR^{\alpha,\Delta t}(q_i) - VaR^{\alpha,\Delta t}\left(\sum q_i\right)\right)} \tag{38}$$

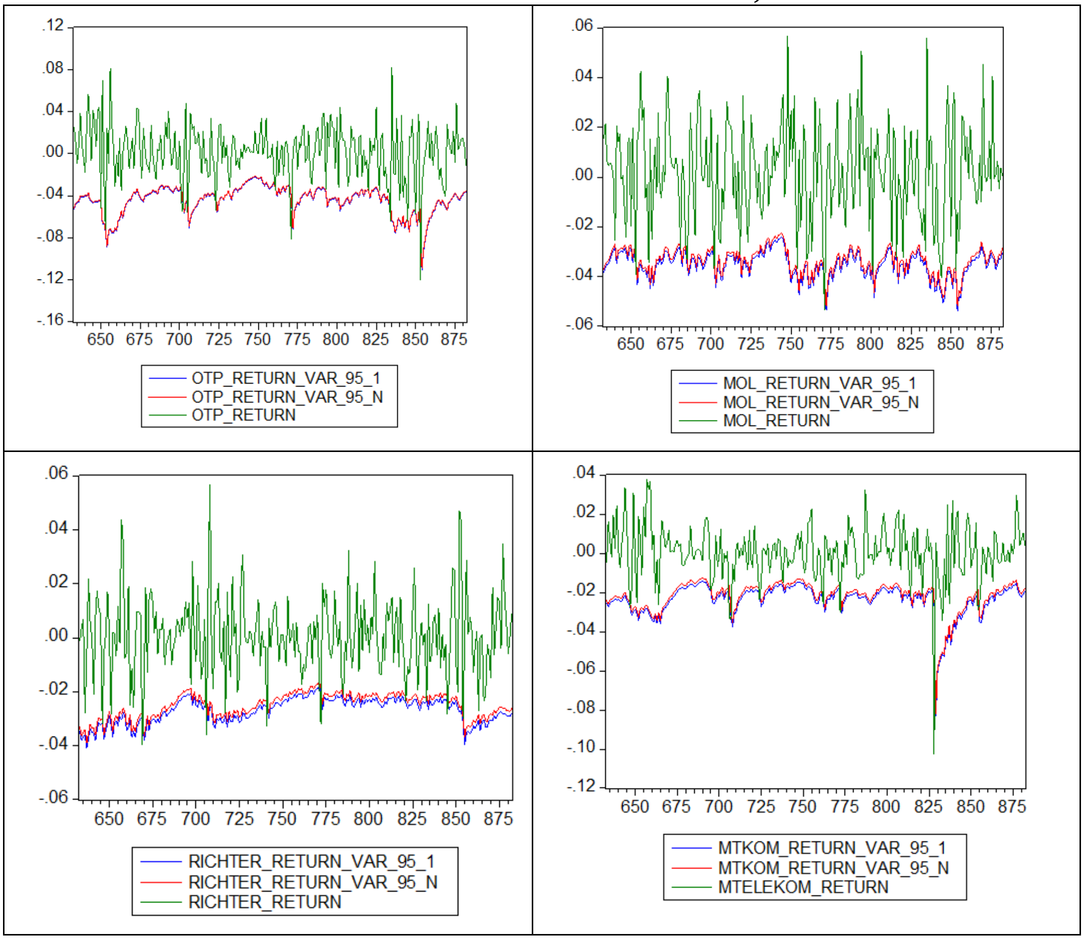
The $\gamma(q)$ shows the additional effect of diversification, as a percentage of the price diversification impact of portfolios, if we consider illiquidity. Namely, the $\left(\sum LAVaR^{\alpha,\Delta t}(q_i) - LAVaR^{\alpha,\Delta t}\left(\sum q_i\right)\right)$ formula gives us the difference between LAVaR values (at a given confidence level and for a predefined time period) for single stocks added together and for portfolios. The $\left(\sum \mathrm{VaR}^{\alpha,\Delta t}(q_i) - \mathrm{VaR}^{\alpha,\Delta t}\left(\sum q_i\right)\right)$ formula uses the same logic for conventional the VaR measure. As a result, Equation 38 demonstrates the diversification effect as a percentage of the price diversification impact.

2.6. Results

2.6.1. Single stocks

I will show the LAVaR calculation for the four bluechip stocks of the Budapest Stock Exchange, to the OTP, Mol, Richter and MTelekom.

Figure 38: The conventional and liquidity adjusted VaR forecasts, compared to the actual returns on the transaction size of EUR 20,000

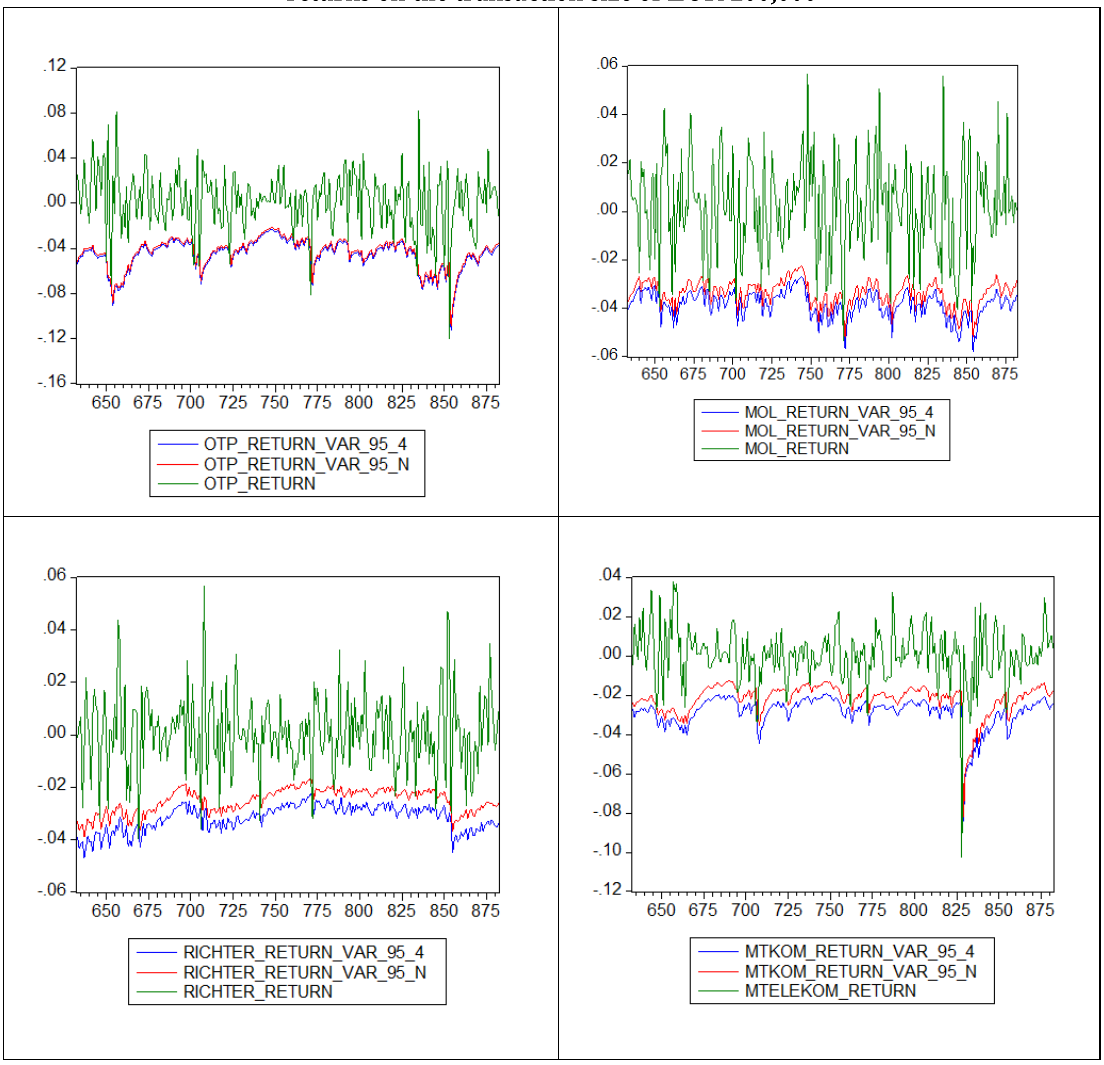


Source: proprietary

In Figures 38 and 39, I plotted the VaR forecasts and normal returns for the final year and the different stocks. I plotted both the LAVaR and the conventional VaR values in order to be able to make comparison and to see the difference between the two.

On Figure 38 I show the 95%, one day VaR estimations for order size of EUR 20,000, while Figure 39 shows the same for EUR 200,000 (1 denotes the 20,000, while 4 denotes 200,000). On both figures the numbers on the horizontal axis show the time of the forecast (e.g. 650 means the forecast for the 650th trading day from 01.01.2007.), while the numbers on the vertical axis are the percentage values.

Figure 39: The conventional and liquidity adjusted VaR forecasts, compared to the actual returns on the transaction size of EUR 200,000



Source: proprietary

As it can be seen in Figure 38, in the case of OTP there is no significant difference in the conventional and LAVaR values, which exactly indicates that OTP is a very liquid stock, its liquidity risk is low. In case of Mol and MTelekom the situation is quite the same, though there is a little difference between the two VaR values. The difference is the largest in case of the Richter. In this case even for the smallest order size, there is clearly a visible difference between the two forecasts, and this increases drastically if we move to the larger order sizes.

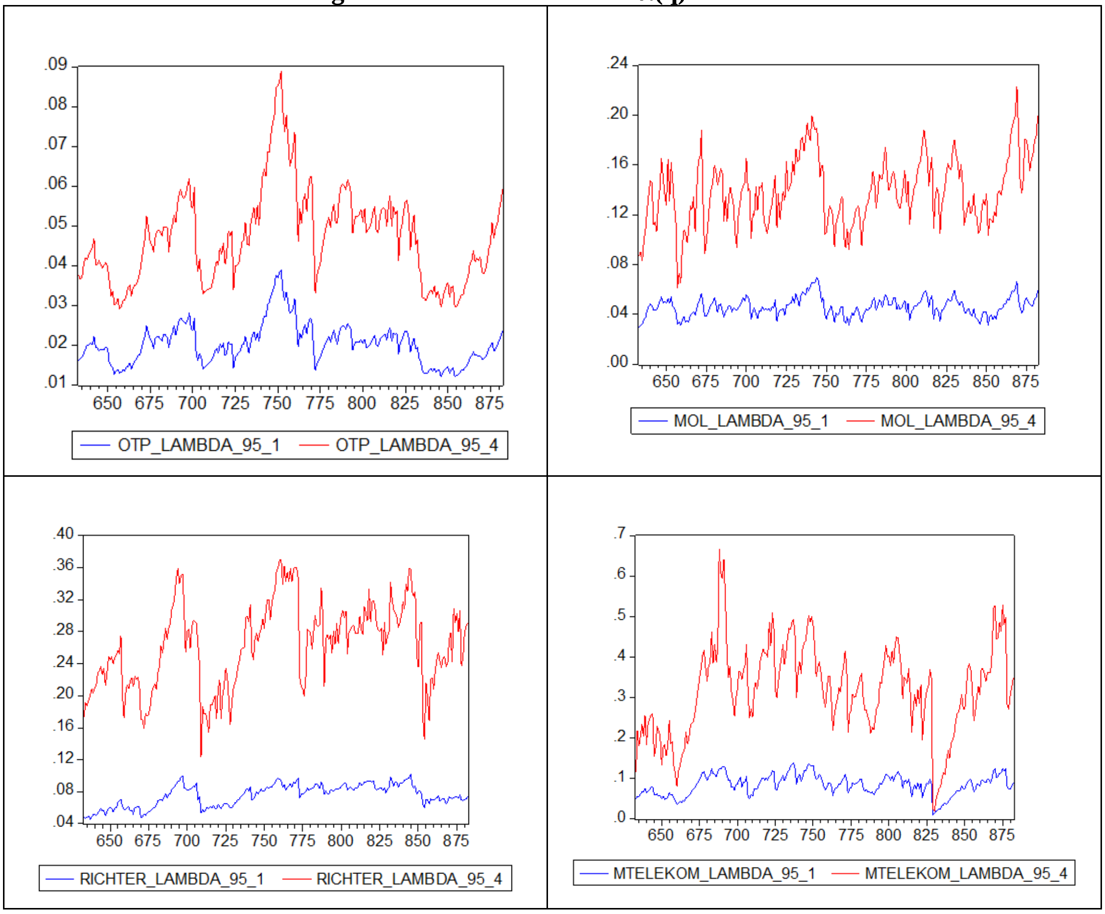
The difference between the four bluechip stocks is more visible if we analyze the VaR measures on higher order sizes, for example on EUR 200,000, as it is shown in Figure 39. The difference between OTP and the three other stocks increase notably. This means, that Mol, Richter and MTelekom are less liquid, than OTP, so they have a higher liquidity risk.

During the test of exceedances for OTP and MTelekom both the conventional and LAVaR forecasts work properly, the empirical values do not differ significantly from the theoretical 5%. This means that in the case of OTP and MTelekom by taking liquidity into account we do not lessen the accuracy of forecasts. For Richter the situation is similar, only 99% LAVaRs for 100 and 200 order size are inaccurate, this is probably due to the mentioned calculation problem of BLM. In the case of MOL both the 99% traditional VaR and LAVaR values are inaccurate, we get too strict forecasts – instead of the expected 1% exceedance there are in fact no exceedances at all. This is probably due to the used sample as it contains the entire period of the 2008 crisis.

To illustrate the difference between the conventional and LAVaR better, I looked at the time series of the $\lambda(q)$ relative liquidity measure, defined in Subchapter 2.4, for the different stocks. In the Figure 40, the $\lambda(20)$ and $\lambda(200)$ measures are plotted simultaneously. These figures show the percentage difference between the forecasts of the previous figures (the horizontal axis shows the time of the forecast, the vertical axis shows the value of the measure).

Based on the relative liquidity measure, it can be stated, that with the increase of the transaction size, the liquidity risk is increasing as well. This is quite clear, since the liquidation of a bigger positions have greater cost as well.

Figure 40: Time series of the λ(q) indicator



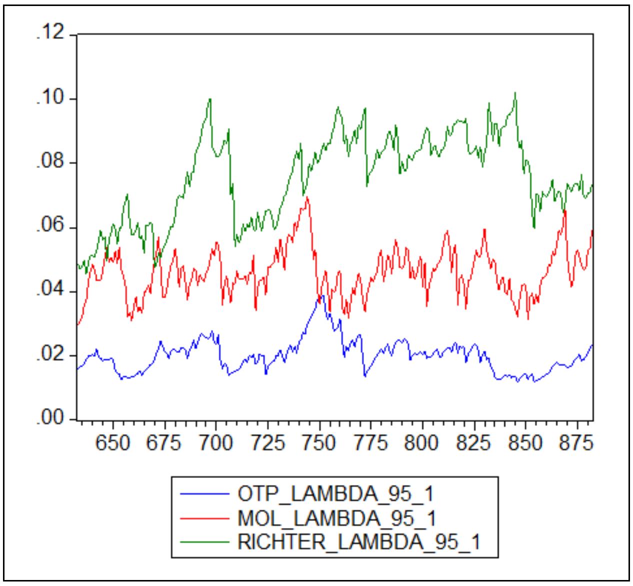
Source: proprietary

In case of the OTP if we examine the values of $\lambda(q)$ we see that for the smallest order size liquidity risk is always above 1%, but can be up to 4%, while for the order size of EUR 200 thousand liquidity risk is always above 3% and can go up to as high as 9%. This is the added risk we ignore if we concentrate only on mid price risk. While these values may not be very large, we should bear in mind that OTP is (one of) the most liquid stock(s) at BSE.

In case of the Richter, liquidity risk is significantly greater, even for the smallest order size it is always above 4%, but often reaches 8%, while for the order size of EUR 200 thousand it is mainly above 20%. This backs up numerically our previous conclusion from Figure 38 and Figure 39 that Richter is much less liquid than OTP and it has significant liquidity risk. In case of Mol and MTelekom I can conclude the same.

In Figure 41 the relative liquidity measures of the major Hungarian stocks are compared for the smallest order size.

Figure 41: Time series of λ(q) in case of OTP, MOL and Richter



Source: proprietary

The liquidity ranking of OTP-MOL-Richter is clearly visible, as expected.[35] The significant difference among them, however, shows that only OTP is a really liquid stock at BSE.

It is worth looking at the average values of the above relative liquidity measures for the different stocks and order sizes. Table 15 summarizes these values. The average values clearly show the liquidity ranking of the stocks. OTP proves to be the most liquid again (has the smallest liquidity risk by far).

Table 15: λ(q) values for different stocks for different order sizes

95%	OTP	MOL	Richter	MTelekom	99%	OTP	MOL	Richter	MTelekom
20	2.03%	4.61%	7.54%	8.46%	20	1.25%	3.07%	4.65%	4.78%
40	2.41%	5.76%	9.57%	11.29%	40	1.47%	3.90%	6.40%	6.38%
100	3.36%	8.91%	15.71%	18.78%	100	2.03%	6.25%	11.33%	10.71%
200	4.72%	13.86%	26.29%	31.98%	200	2.83%	10.03%	18.00%	18.49%
500	8.40%	29.75%	91.74%	133.52%	500	5.04%	22.24%	60.73%	97.43%

Source: proprietary, Gyarmati et al. (2010b), p. 531.

In the table above the methodological error of the BLM appears; I get unrealistic values for Richter and MTelekom (even above 100%!) for the largest order size. This phenomenon is the consequence of the order book not being deep enough, i.e. total limit orders in the book do not reach EUR 500 thousand on average, thus

[35] I didn't show MTelekom's relative liquidity measure, since it is quite similar to Richter's.

orders of this size could not be executed in reality (this means that there should be 'infinite' or 'n.a.' in the table).

As a conclusion I can say that the above results show that liquidity risk is not irrelevant, it is highly advised to take it into account when calculating VaR measures.

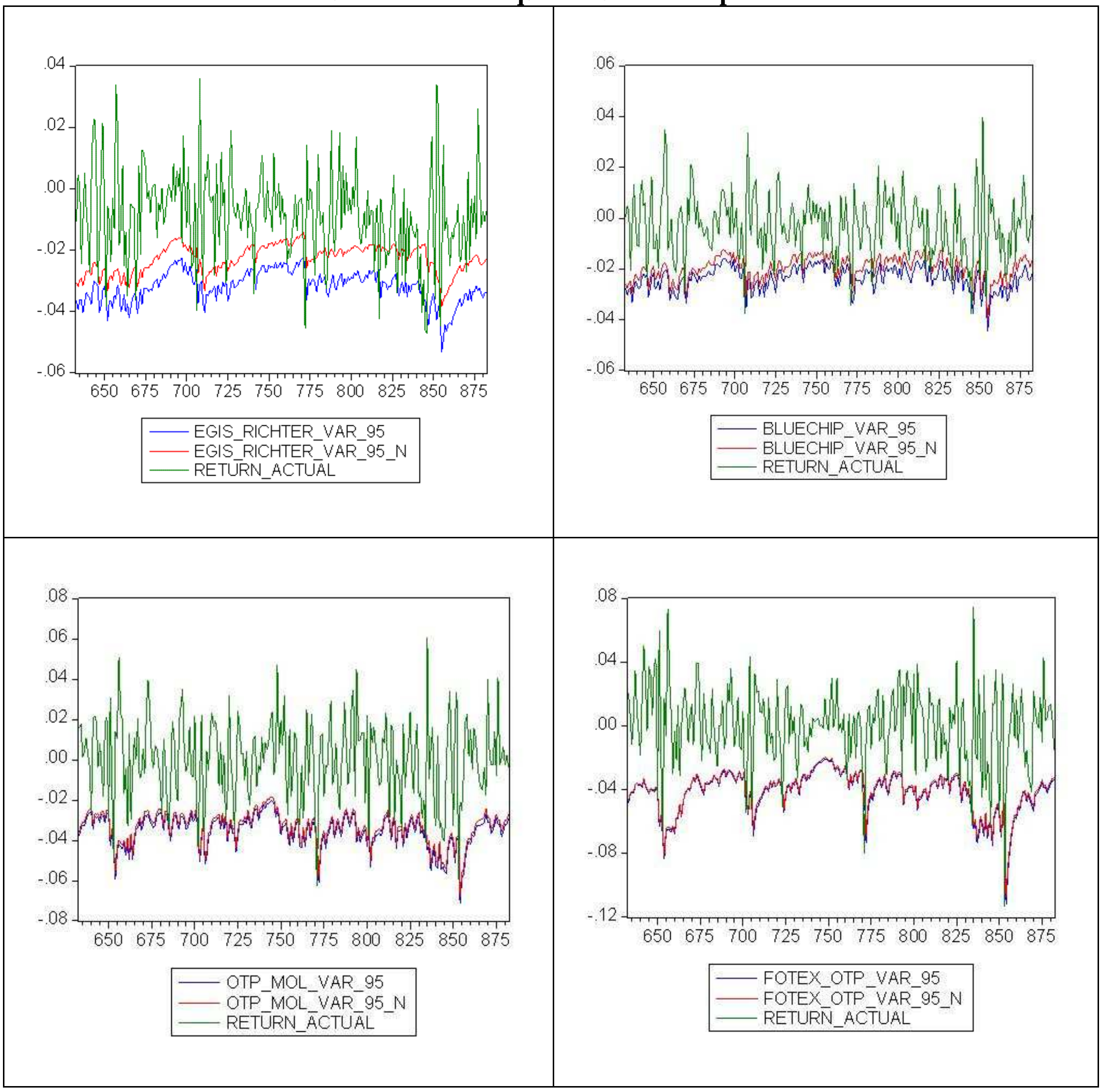
2.6.2. Portfolios

I have calculated LAVaR and conventional VaR for four different portfolios. The four portfolios are the following:

1. 1,000 – 1,000 Egis and Richter. The aim of this portfolio is to assess the behavior of LAVaR for portfolios of the same industry. Moreover, I wanted see whether liquidity risk can or cannot be diversified within an industry.
2. 1,000 – 1,000 OTP and MOL. In case of the two most liquid stock on the BSE my objective was the same: is there room for diversifying illiquidity? I wanted to see if the liquidity of the two stocks behave the same way, or not, and liquidity risk can indeed be mitigated by constructing portfolios.
3. 1,000 – 1,000 each from the blue chips of the BSE: OTP, MOL, Richter and MTelekom. I had the same objective as for the OTP-MOL portfolio, but I broadened my analysis.
4. 1,000 – 1,000 OTP and Fotex. The aim was to show that liquidity risk is significantly mitigated if I couple an illiquid stock with a liquid one into a portfolio. For OTP and Fotex I have also examined a larger portfolio's LAVaR, consisting of 100,000 stocks each.

The different VaR and LAVaR values were calculated at a 95% significance level and for a one day time period. The results are shown in Figure 42, where the horizontal axis is the same as it was in the case of single stocks, namely the number of days passed since 1st January, 2007. The vertical axis shows the VaR („*egis_richter_var_95_N*”), the LAVaR („*egis_richter_var_95*”) and the actual returns („*return_actual*”) as a function of time.

Figure 42: The conventional and liquidity adjusted VaR forecasts, compared to the actual returns in case of equal volume stock portfolios



Source: proprietary

Since each of the different portfolios included 1,000-1,000 stocks, they have different values, hence the different figures cannot be compared to each other directly. As we can see in Figure 42 the liquidity adjusted VaR values are higher than conventional VaR values for each of the portfolios.

The additional risk of illiquidity quantified by $\lambda(q)$ is shown in Table 16. These figures, however, cannot be compared to each other directly due to the different portfolio values, as mentioned earlier.

Table 16: $\lambda(q)$ values

Portfolio	v_i	$\lambda(q)$
Egis – Richter	1,000	40.6%
Bluechipek	1,000	21.3%
OTP – MOL	1,000	7.7%
OTP – Fotex	1,000	3.2%

Source: proprietary

The $\lambda(q)$ value shows that for a portfolio of for example 1,000 Egis and 1,000 Richter shares, the additional risk is 40.6%, which we ignore if we calculate only price risk.

In case of stock portfolios besides the relative liquidity impact I have quantified the diversification effect, so I have determined the $\gamma(q)$ figures, listed in Table 17.

Table 17: $\gamma(q)$ values

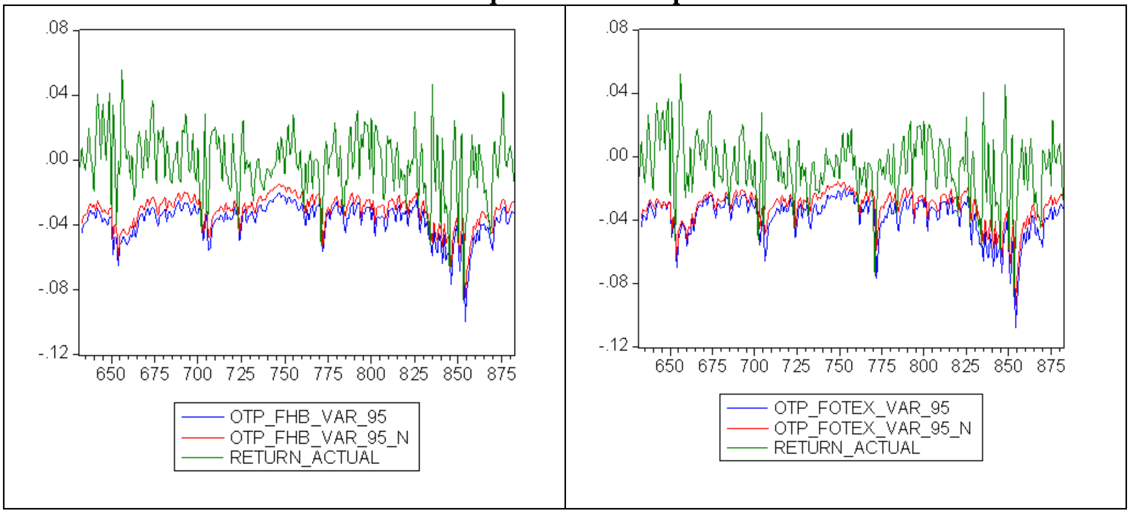
Portfolio	v_i	$\gamma(q)$
Egis – Richter	1,000	58.24%
Bluechipek	1,000	4.80%
OTP – MOL	1,000	2.52%
OTP – Fotex	100,000	336.07%

Source: proprietary

Based on Table 17 I state that significant diversification is possible by forming portfolios. In case of companies operating in the same industry, such as Egis and Richter, the diversification impact for a portfolio of 1,000-1,000 shares, respectively, is 58.24% larger if we account for illiquidity risk. For bluechips and the OTP-MOL portfolios the diversification impact of considering illiquidity is not as remarkable. This is due to the fact that they have similar and also relatively the best liquidity. These results, however, cannot directly be compared to each other due to the portfolios being volume weighted. The last row of the table shows a portfolio of much larger number of elements, a portfolio of 100,000-100,000 OTP and Fotex, respectively. In this case, as was expected, the diversification impact of accounting for illiquidity is huge: 336.07%.

I have determined the LAVaR values for value weighted portfolios as well. I have chosen two portfolios, in which the weight of the stocks was 50-50%. The value of the total portfolio was fixed during the whole time. The results are shown in Figure 43:

Table 43: The conventional and liquidity adjusted VaR forecasts, compared to the actual returns in case of equal value stock portfolios



Source: proprietary

The advantage of the value weighted portfolio, that the different portfolios are comparable, since their value is the same.

In case of the OTP-FHB portfolio, the value of λ(q), namely the risk we do not take into account if we are calculating only a conventional VaR is 21.14%, while in case of OTP-FOTEX it is: λ(q) = 20.46%-kal.

2.7. Conclusion

In this chapter I have shown how to make liquidity adjusted value at risk model. I have deducted how to define net return (sum of the returns caused by mid price change and illiquidity), which is the basis of the LAVaR calculation. I have pointed out that taking liquidity risk into consideration causes a significant increase in risk even in the case of the most liquid stocks. This means that liquidity risk shouldn't be ignored either in case of single stocks, or in the case of stock portfolios.

BLM and the method built upon it provide a simple and quick way to display liquidity in the capital requirement. Paying attention to the deficiencies and calculation problems of the index, the findings should be handled with precaution, but the presented model can appropriately reflect the essential empirical observations (e.g. OTP is the most liquid stock), thus in every case I recommend its integration into risk management systems. Based on the results in sum I cannot reject the H3 hypothesis.

H3: Market risk can be underestimated at least by 5% even for liquid stocks at the order size of EUR 20,000 on the Budapest Stock Exchange, if we do not take the liquidity risk into consideration.

In case of portfolios liquidity risk can be diversified. It worth to have more stocks in the portfolio, since not only the price risk, but liquidity risk can be reduced by diversification. Based on this I cannot reject H4 hypothesis.

H4: In case of stock portfolios not only price risk but liquidity risk can be diversified.

IV. Virtual price impact function

1. Literature of the price impact functions

One of the most important concepts of market liquidity is the price impact (or market impact), and the price impact function (or market impact function). Despite the fact that the analysis and modeling of the price impact function is getting discussed wider nowadays, in the literature of market liquidity only a few pieces of research have analyzed the value of transactions' price impact, i.e. the additional costs which cannot be paid as an explicit cost – e.g. brokerage fees, different exchange fees, etc – of trading. In subchapter 1.1 I introduce these results.

1.1. Value of price impact based on empirical research

Before introducing the literature of the price impact functions, it is worth analyzing the price impact of transactions on the market. One prominent study of the field is prepared by Torre and Ferrari (1999). The authors estimated the total transaction costs of trading with the stocks of the S&P 500 index. The authors have estimated the transaction cost to be 25 cents by assuming buying and selling of 10,000 pieces of stocks with a median mid price of 400 dollars. Torre and Ferrari (1999) estimated that the composition of this 25 cent is built up as follows: execution costs 5 cents, while the remaining 20 cents can be accounted for as price impact. From these 20 cents, 7 cents cover the half of the bid-ask spread, while the adverse price movement is responsible for 13 cents. It is remarkable, that the adverse price movement equals the half of the total transaction cost.

According to the data of ITG Global Trading Cost Review, in the last five years the average transaction cost of American corporations with high capitalization was 23 basispoints (bps). From this amount 9 bps were the fees, while 12 bps were the straightforward consequence of the price impact (Ferraris, 2008).

The above examples show that the largest part of the transaction costs is caused by the price impact. The examples explicitly highlight that the price impact is indeed important and that market participant should be aware of this fact. Had they take the price impact into account during trading, they could save substantial amounts of money.

1.2. Virtual and empirical price impact functions

Market participants can get information about the price impact from the price impact functions. These price impact functions show the expected relative price-shift caused by a particular order. Knowing the price impact is essential for the market participants, since they can predict the price impact concerning their orders in the future, i.e. the expected additional cost caused by price shift; or they can build a dynamic portfolio optimization by creating a trading algorithm based on the function.

There are two different price impact functions, the virtual and the empirical price impact functions. The *virtual price impact function (vPIF)* shows that if we want to fulfill the transaction immediately, what would be the difference between the last price level in the order book, on which our order has been realized, and the actual mid price in the time the order was given. In this case it is called marginal price impact, which can be valuated according to Equation 39.

$$\mathrm{vPIF}(q) = \frac{\text{Price level of the last order}}{\text{Mid price in the second the order was given}} - 1 = \frac{P^{t}_{\text{last}}}{P^{t-1}_{\text{midprice}}} - 1 \qquad (39)$$

The function shows the marginal price impact of an immediate execution (Bouchaud et al., 2008; Bouchaud, 2010a; Gabaix et al., 2003).

Besides marginal price impact one can define a virtual price impact function, which gives the average price impact of the order. The calculation of the average price can be carried out with the aid of the order book. In this case we calculate the ratio of the average price to the actual mid price on the market.

$$\mathrm{vPIF}(q) = \frac{\text{Average price of the order}}{\text{Mid price in the second the order was given}} - 1 = \frac{P^{t}_{\text{average}}}{P^{t-1}_{\text{midprice}}} - 1 \qquad (40)$$

The average price impact is a crucial information for the market participants, since it gives them the implicit cost of trading, namely the transaction cost which they have to pay because of illiquidity.

A third kind of virtual price impact can be calculated as well, namely by the quantifying how the mid price has changed during a transaction. To be able to calculate it, we have to define the mid price before and after the transaction. The mid price can be determined based on the order book, namely it will be the half of the sum of the best bid and best ask price. The price impact will be the following in this case:

$$\mathrm{vPIF}(q) = \frac{\text{Mid price after the transaction}}{\text{Mid price in the second the order is given}} - 1 = \frac{P^{t}_{\text{midprice}}}{P^{t-1}_{\text{midprice}}} - 1 \qquad (41)$$

The vPIF won't give us the actual values of the price impact, it gives us only the answer to the question what would be the marginal-, the average-, or the real price impact if we like to carry out a transaction immediately. The name 'virtual price impact function' stems from this fact. If a market player assumes on the basis of the virtual price impact function, that the planned transaction would change the market price notably, than most probably he does not add the transaction to the order book in one amount. Instead, he splits the order into lots and submits the order when he considers the price impact to be smaller. Accordingly, the virtual price impact, shown by the function only occurs, if the market player indeed submits the market order and it is executed immediately.

Virtual price impact function can easily be estimated from the actual order book, since it can show a stock's liquidity at different order sizes. To make it easier to understand the definition of the virtual price impact function, I show the calculation of the different price impacts in simple numerical example. To be able to calculate the price impacts, it is necessary to know the actual order book. In this case a fictional order book will represent it, as it can be seen in Table 18:

Table 18: A fictional stock's order book

Order book			
Bidsize	**Bidprice**	**Askprice**	**Asksize**
300	9,980	9,990	200
622	9,970	10,000	300
400	9,960	10,010	220
721	9,950	10,020	200
1,200	9,940	10,030	100

Source: proprietary

Let's assume that a given investor would like to buy the fictional stock for HUF 7,000,000. The order of the investor is executed on the first three price levels of the order book. On the first level he can buy 200 stocks for the price of HUF 9,900. On the second level he can buy another 300 stocks for HUF 10,000 each. After this, he has HUF 2,002,000 left to buy stocks on the third price level, for HUF 10,010 each. This means, that he can buy another 200 stocks. I have summarized the elements of the executed buy order in Table 19:

Table 19: Execution of a buy order

Buy order	Number	Price	Value (HUF)
Executed on the first level	200	9,900	1,998,000
Executed on the second level	300	10,000	3,000,000
Executed on the third level	200	10,010	2,002,000
Sum	**700**		**7,000,000**

Source: proprietary

At the time, when the order was given, the mid price was HUF 9,985, since this is the arithmetic average of the best bid and ask price. In this numerical example the marginal price impact can be calculated according to Equation 42, which is based on Equation 39. To be able to calculate the marginal price impact one has to know the last price level, where the order was executed. So the marginal price impact will be the ratio of this price and the mid price in the second the order was given.

$$\mathrm{vPIF}(q) = \frac{P^{t}_{\mathrm{last}}}{P^{t-1}_{\mathrm{midprice}}} - 1 = \frac{10010}{9985} - 1 = 0.25\% \qquad (42)$$

This result means, that if an investor wants to buy stocks for HUF 7,000,000, then according to the actual order book, the relative price change of his order will cause 0.25% marginal price impact.

Based on the order book, it is easy to calculate the average price impact as well, since it can be seen that how many stocks can be executed on each price level. The average price impact can be calculated as it is shown in Equation 43, which is based on Equation 40. The volume weighted average price can be found in the numerator, while in the denominator we can see the mid price at the time the order was given.

$$\mathrm{vPIF(q)} = \frac{P^{t}_{\mathrm{average}}}{P^{t-1}_{\mathrm{midprice}}} - 1 = \frac{\frac{200 \cdot 9990 + 300 \cdot 10000 + 200 \cdot 10010}{700}}{9985} - 1 = 0.15\% \qquad (43)$$

This result means, that if an investor wants to buy stocks for HUF 7,000,000, then according to the actual order book, the relative price change of his order will cause 0.15% average price impact.

In order to be able to define the price impact of the mid price change, we need to define the mid prices. After the executed HUF 7,000,000 buy order – if we assume that no other orders were executed in the meanwhile –, the mid price became HUF 9,995, which is the average of the best bid, namely the HUF 9,980 and the best ask, the HUF 10,010 order in the order book. The prompt mid price when the order was given was HUF 9,985, so the real price impact will be as it is shown in Equation 44.

$$\mathrm{vPIF(q)} = \frac{P^{t}_{\mathrm{midprice}}}{P^{t-1}_{\mathrm{midprice}}} - 1 = \frac{9995}{9985} - 1 = 0.10\% \qquad (44)$$

The investor’s HUF 7,000,000 buy order has increased the mid price with 0.10%, so this was the price impact of the buy order.

In contrast, the *empirical price impact function* (ePIF) shows the actual price impact that can be measured from real transaction data. Namely the previous three numerical examples show the price impact in case the orders are given immediately. But this price impact will not necessarily occur in the market, it will depend on the investor's decision, whether to submit the order immediately, or rather wait and submit it only later. The empirical price impact function cannot be estimated from the order book, only from real trading data. From real trade data only the change of the mid price can be seen, so only the following price impact can be defined:

$$ePIF(q) = \frac{\text{Mid price after the transaction}}{\text{Mid price in the second the order is given}} - 1 = \frac{P^{t}_{midprice}}{P^{t-1}_{midprice}} - 1 \qquad (45)$$

From the trading data one cannot quantify the average price, since only the mid price is included in the trading data, so it cannot be seen, that what were the prices on which the certain parts of the order had been executed. The only thing one can see is the mid price before and after the order was submitted.

Empirical price impact function can be estimated from past trade, which means that the order book cannot be used, rather the trades and quotes (TAQ) database. TAQ database contains the information of the mid prices. In the previous numerical example Equation 41 shows the empirical price impact in the case the order is in reality submitted. One of the main differences between the empirical and virtual price impacts is that the empirical price impact function is never being estimated only from one single order on each order size, but from the average of single or aggregated transactions through a longer period. In case of single transactions, the professionals estimate how an order of a certain value/volume changes the mid price on average during a longer period, like e.g. a year. In the case of aggregated transactions, the estimation is a little bit more complicated. The aggregation can be done by time (e.g. order in a five minute interval) or by order numbers (e.g. 20 consecutive orders). After this aggregation they calculate the average mid price change on different order sizes for a longer period (e.g. a year). Analyst can calculate the price impact in case of the virtual and empirical PIF as well in the function of the volume (number) or in the function of the total value (EUR, HUF, etc.).

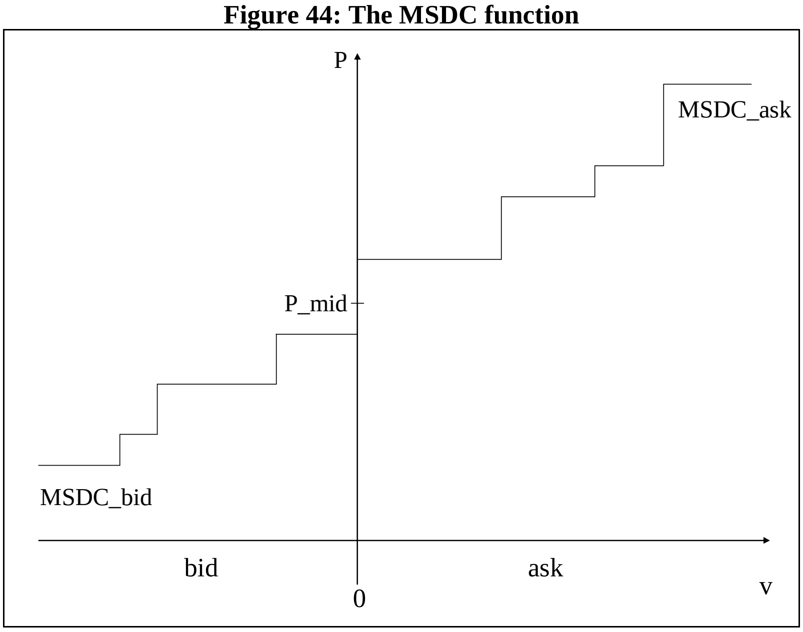
In sum the most important difference between the virtual and empirical price impact function is, that the virtual price impact function can be estimated from the actual order book, and one can estimate the immediate marginal/average price impact, or the impact for the mid price change. Therefore on the one hand the virtual price impact can be calculated for every second. On the other hand, the empirical price impact can be estimated for executed orders, and only for the mid price change, since in the TAQ database which contains trading data one can see only the information regarding the change in mid price. Moreover the ePIF cannot be estimated for every second, since it shows the average price impact of a longer period, so it cannot be used for time-series analysis.

In my empirical research I will estimate a virtual price impact function, so I think it is important to show the estimation of the function in more details in a full chapter. Related to this, I will introduce a new concept, namely the marginal supply-demand curve, which shows the actual state of the order book.

1.3. Marginal supply-demand curve

Marginal Supply-Demand Curve (MSDC) is an important concept during the estimation of the virtual price impact function, since the MSDC will represent the order book during the estimation. The MSDC shows the order book's actual status, that is, the price levels and the volume of orders on each price level. According to this the MSDC shows the price on which a transaction's last order was fulfilled, where the value of the transaction is „*v*" (which can be measure in volume or value) (Acerbi, 2010). The MSDC is shown in Figure 44:

Figure 44: The MSDC function



Source: proprietary

Having the MSDC function at my disposal, the total transaction cost of a buy order (mid price plus implicit costs) can be determined as follows:[36]

$$\text{Total Cost}(v) = \int_0^v \text{MSDC}(x)dx \tag{46}$$

A transaction's total cost can be determined by the MSDC(v) function with Equation 46 only in the case, when MSDC(v) interprets the order book at a given moment. Note that MSDC(v) could be defined as the average data of a longer time period's order book. In my dissertation I will define the MSDC(v) based on a certain second's order book and not on an average order book for a period T.

Supply Demand Curve (SDC) is a closely related concept to the MSDC. The SDC differs from the MSDC. The SDC shows the average price of a transaction. In contrast, the MSDC represents the transaction's marginal price. Thus, the SDC does not show the new mid price after the transaction. Instead, it captures the average price a market player has to pay for a transaction. The relation between the SDC and the MSDC can be defined as follows:

$$\text{SDC}(v) = \frac{1}{v}\int_0^v \text{MSDC}(x)dx$$

$$\text{MSDC}(v) = v * \frac{d\text{SDC}(v)}{dv} \tag{47}$$

There is an important difference between the two functions: the MSDC is never a continuous function, while the SDC is always continuous.

With the aid of the MSDC and SDC the marginal (Equation 48) and the average (Equation 49) virtual price impact can be defined as follows:

$$\text{vPIF}(v) = \frac{\text{MSDC}(v)}{P_{mid}} - 1 \tag{48}$$

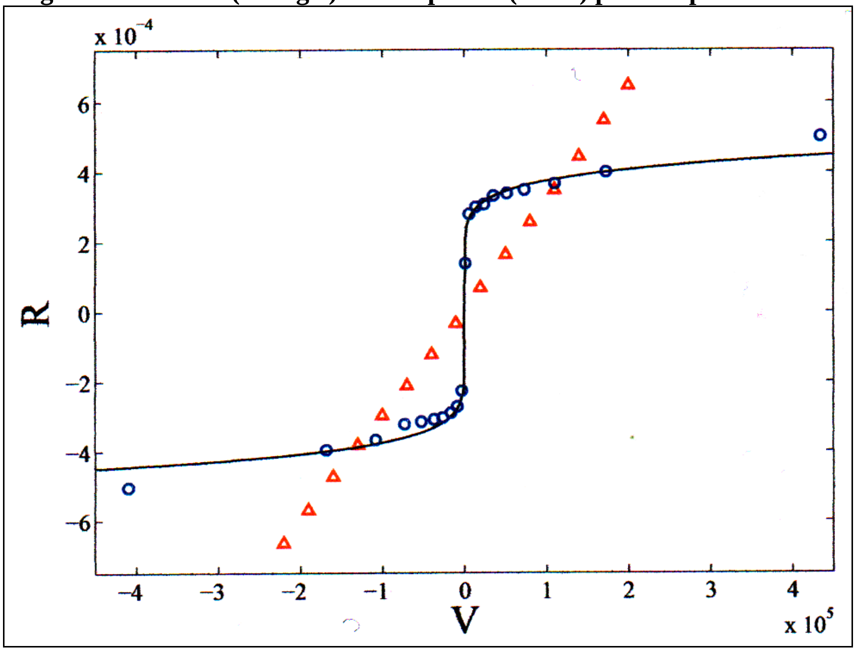
[36] It can be written like Equation 46, because during the estiamtion of the vPIF I will estimate the MSDC(v) from the total implicit cost. The same will be true for the deduction of Subchapter 2.3.

$$vPIF(v) = \frac{SDC(v)}{P_{mid}} - 1 \tag{49}$$

1.4. The shape of the price impact function: empirical facts

As a consequence of the different estimation of the virtual and empirical price impact functions, we can get very different shapes for the functions. Figure 45 shows the relation of the virtual and empirical price impact functions, which were estimated from real market data. On *x* axis the size of the transaction can be seen, while on *y* axis the relative change of the mid price.

Figure 45: Virtual (triangle) and empirical (circle) price impact function



Source: Bouchaud et al., 2008, p. 38.

On the figure it can be seen that the vPIF can be estimated almost with a straight line, while the ePIF's shape can be estimated with a concave curve in case of the ask side of the curve. According to the empirical facts, researchers have identified different shapes for the PIF-s, and reasons for these shapes. Usually researchers analyze the shape of the PIF on the ask side of the curve. The different shapes can be the result of several reasons. Mainly the price impact of transactions depends on the order size and on the time horizon of the analysis. In Tables 20-23 I have summarized the most important findings on limit order markets. The first three tables summarize the findings for empirical price impact function, while the fourth table contains the researches carried out on virtual price impact functions.

In the initial studies, the researchers plot the price impact functions without defining its functional form. The results of these studies are summarized in Table 20. Most of the researchers identify the price impact functions with positive slope and with a concave form. However, the studies differ in relation to the change of the function's slope.

Table 20: Initial studies on the shape of the price impact function

Authors	Examined stock exchange	Period	Shape of the PIF	Remarks, specialties
Hasbrouck (1991)	Data of NYSE, AMEX and regional exchanges	62 days from 1989	Positive slope, concave function.	The price impact is delayed. The PIF haven't been formalized.
Hausman, Lo & MacKinlay (1992)	10 randomly chosen American stock	1988	Positive slope, concave function with decreasing growth.	The PIF haven't been formalized.
Biais, Hillion & Spatt (1994)	Stock of Paris Bourse CAC 40 index	29/Oct-26/Nov 1991.	Almost a straight line, slightly concave function, which has a greater slope on the best price levels, than on other levels.	The PIF haven't been formalized.
Niemeyer & Sandas (1995)	30 stocks of the Stockholm Stock Exchange's OMX index	03/Dec /1991-02/Mar /1992.	Nonlinear function, which is not so slope at the best price levels.	The PIF haven't been formalized.
Kempf & Korn (1999)	DAX futures, aggregated in every 5 minutes	17/Sept /1993-15/Sept /1994.	Nonlinear function, the concave function flattens on the sides: the large orders have relatively smaller price impact than the small orders.	The authors just analyzed the relation between the order size and the price impact on the best price level.
Evans & Lyons (2002)	DM/USD & Yen/USD, daily aggregation	1/May-31/Aug /1996.	Strong positive relation: the net order flow explains a notable portion of the exchange rates' volatility.	The authors define quantity as the difference of the buyer or seller initiated signed orders.

Source: proprietary

Table 21 shows the most important results of those studies that examine the price impact function on the level of single transactions. All the authors make efforts to define the functional form of the empirical price impact function. The majority of the studies identify a strongly concave function which differ in respect of the parameters. However, on different markets the price impact function can be formalized differently.[37]

Table 21: Price impact of single trades

Author	Examined stock exchange	Period	Shape of the PIF	Remarks, specialties
Lillo, Farmer & Mantegna (2003)	1000 stock from New York Stock Exchange, which have the highest capitaliza-tion	1995-1998	Concave function, which cannot be described with a power law function. The slope of the function changes in the function of order size.	With appropriate average calculation and with the rescaling of the axis, 1000 stocks' price impact can be put to the same curve. The higher capitalized corporations stocks' price impact is smaller by the same transaction size.
Bouchaud & Potters (2002)	Stocks of Paris Stock Exchange and LSE	1/June-15/July 2002.	Logarithmic relation.	The small transactions' price impact is relatively larger, than the large transactions'. The price impact of trading is quasi-permanent, which means that the market players look at the trading as new information.
Farmer & Lillo (2004)	Three stocks of London Stock Exchange	May 2000-Dec. 2002.	The price impact function can be estimated with a power-law function, where the exponent is 0.26.	The authors highlight the difference between the price impact on the NYSE and on the LSE.
Lim & Coggins (2005)	Australian Stock Exchange 300 stock with the highest capitaliza-tion	2001-2004	Relation can be estimated with a power law function.	Trading the same amount, the price impact is smaller in case of corporations that have higher capitalization.

[37] The power law function is concave/convex if the exponent is smaller/greater than 1. If the exponent equals 1, than the power law function is a straight line.

Hopman (2007)	Stock of Paris Bourse CAC40 index; price impact for a 30 minute interval.	04/Jan/ 1995-22/Oct/ 1999.	The function can be estimated as a power-law concave function, where the exponent is between 0.37 and 0.47.	The exponent's value in case of market orders is 0.37, an order which is between the bid-ask spread is 0.38, while in case of a limit order it is 0.47.
Zhou (2011)	23 stock from the Shenzen Stock Exchange	2003	The executed orders' price impact function is a power-law function, where the exponent is 0.65 on the bid side, and 0.69 on the ask side. The partly executed orders' price impact is constant in case of small values.	With normalizing the returns and the quantities, independently from the capitalization, the price impact functions can be brought together to one curve.
Cont, Kukanov & Stoikov (2011)	TAQ database (NYSE, AMEX, NASDAQ) 50 randomly chosen stocks	April 2010.	The price impact in the function of the imbalance of the bid-ask side is linear.	The slope of the linear price impact function is inversely proportional to the market depth.

Source: proprietary

Table 22 summarizes the results of the studies which estimated the price impact function with aggregated transactions. The second column of the table the aggregation level is shown as well. It can be seen, that researchers have arrived at different results, and formalized the ePIF differently. Bouchaud et al. (2008) state, that these differences can be the consequence of the differences in markets, assets, time, and aggregation level. On shorter time horizon the price impact is nonlinear (on high aggregation level), but the price impact becomes linear on longer time horizon, and also slope of the curve decreases on higher level of aggregation.

Table 22: Price impact of aggregated trades

Authors	Stock exchange & aggregation	Period	Shape of the PIF	Remarks, specialties
Gabaix et al. (2003, 2006)	1000 biggest stocks of the TAQ database, the aggregation is based on 15 minutes intervals	1994-1995	Growing, concave price impact, which can be described with a square-root function.	The authors state the large price movement is the consequence of the large transactions. In contrast Farmer and Lillo (2004) say that the large changes in price are the consequence of the lack of liquidity. See Farmer and Lillo (2004) of this discussion.
Plerou et al. (2002)	116 most traded stocks of New York Stock Exchange, aggregated for 5 to 195 minute intervals	1994-1995	Defining two different price impact functions, on one hand in the function of the imbalance of order numbers (φ), and on the other hand in the function of the volume imbalance (Ω). In both cases the function is a concave, tangent function, which flattens with in the case of higher imbalance.	φ: is the difference between the orders given by the sellers and buyers; Ω: is the difference of the number of the seller and buyer initiated orders. If Ω is close to 0, the price impact $\langle G\rangle_{\Omega} \sim \Omega^{1/\sigma}$ can be written with a power-law function, where the exponent increases with the decrease of σ, by increasing Δt.
Almgren et al. (2005)	30 thousand transaction of Citigroup US, aggregated for 30 minutes interval	Dec. 2001-June 2003.	Defining two different price impact functions. The permanent price impact is linear. The temporary price impact is a concave power-law function with an exponent of 0.6.	Only the linear permanent price impact guarantees the market to be arbitrage free, and to the price impact to be independent from time.
Hopman (2007)	Stocks of Paris Bourse CAC40 index, 7 aggregation level: 10 min, 30 min, 1 day (without night), 1 day (with night), 1 week, 1 month, 3 months	04/Jan/1995 - 22/Oct/1999.	Estimating it with linear regression. The daily aggregation provided the best result, with R^2=43.5%. The slope of the line decreases with the aggregation level.	The author defines the order flow on different time intervals with a square-root function: $\mathrm{SQRT} = \sum_{i \in bid} v_i^{0,5} - \sum_{i \in ask} v_i^{0,5}$

Margitai István (2009)	Budapest Stock Exchange: MOL, aggregation: 5 and 20 transactions	251 tran-saction days from 8/Mar/ 2007	Estimated with square-root function. With the increase of aggregation level, the exponent of the function is increasing, and the function flattens.	The result he got is consistent with the empirical literature.
Bouchaud, Farmer & Lillo (2008)	Stocks of NYSE and LSE, aggregation of transactions: N=1, 8, 64, 512	2000-2002	With increasing the aggregation level, the price impact function flattens. Increasing linearity: with the increase of N, around the balance of order values, the price impact function becomes linear. Decreasing slope: The slope of the linear regression decreases with increasing N.	The relation is true for the aggregation of the transactions (N) and for the signed imbalance of the value (Q) as well.

Source: proprietary

In addition to the empirical researches summarized in Table 22, Bouchaud's (2010a) research is worth mentioning in which the author summarizes the most important characteristics of the price impact function. The author concludes based on the result of past research, that the price impact function is nonlinear, concave and can be estimated with a power law distribution which has an exponent smaller than 1. This exponent is increasing with the increase of the aggregation level: on single transaction level the exponent is between 0.1 and 0.3, and if the aggregation is based on aggregating around 1,000 transactions, then the exponent will be close to 1.

In the literature it is an accepted view, that the number of transactions has a more important role in the price impact, then the order size (Bouchaud, 2010a, b). Beside this it is also accepted, that the price impact is proportional to the bid-ask spread, and to the volatility per trade (Bouchaud, 2010b).

Finally, in Table 23 the literature on the virtual price impact function is reviewed briefly. Early research found that the virtual PIF can be estimated with a power-law function, where the exponent is significantly higher than in case of the empirical PIF. Weber és Rosenow (2005) identify a square-root function, but the authors state as well, that the virtual price impact is much bigger, than the empirical.

The authors state that it can be the consequence of the negative correlation between returns and limit orders.

Table 23: Shape of the virtual price impact function[38]

Authors	Examined stock exchange	Shape of PIF	Remarks, specialties
Challet & Stinchcombe (2001)	4 stocks 15 best bid and ask price level on Island ECN (NASDAQ)	The vPIF can be estimated with a power-law function. The exponent is between 1 and 3, depends on the day and on the stock.	The authors mainly talk about the static and dynamic properties of the limit order book, not about vPIF.
Maslov & Mills (2001)	NASDAQ Level II	The virtual PIF is a power-law function, where the exponent is between 1.7 and 2.2.	The authors state that the high exponent is the consequence that the virtual price impact differs from the empirical price impact.
Smith, Farmer, Gillemot & Krishnamurthy (2008)	London Stock Exchange	The virtual price impact function can be linear or concave, it depends on the parameters of the model.	The authors have built up a theoretical model, which was tested on an order book of the London Stock Exchange. They have found that the model gives back the statistical properties of the real data.
Weber & Rosenow (2005)	10 most frequently traded stocks on Island ECN (NASDAQ), aggregated for 5 minute interval, data of 2002	In case of the limit orders, the vPIF is a convex square-root function. In case of the market orders, the vPIF is a concave square-root function.	The virtual price impact is four times greater than the real one. They explain this difference with the negative correlation between the returns and the limit orders.

Source: proprietary

It is worth mentioning that I haven't found other studies on the shape of the virtual price impact besides those which are in Table 23. I think that this can be traced back to the fact, that the majority of the researchers looked for the reason for the price change, namely whether the price change is caused by the big order or by the lack of liquidity. Researchers can analyze this only on real transaction data, since they had to examine the real price changes.

[38] I haven't shown the period, since it was not available.

Tables 20-23 show that research done so far have found the shape of the price impact function can be mainly a power-law-, square-root-, concave function, or it can be linear. The concave shape is interesting, because it would encourage the market participants to give larger orders, since the price impact seems to be inversely proportionate with the size.

The literature gives two reasons for the concave shape of the empirical price impact function (Bouchaud et al. 2008). The first explanation can be related to Barclay and Warner (1993): the authors state, that the concave shape can be the consequence of the information content of the transactions. Namely if the small transaction have the same information content than the large transactions, than the price impact of large transactions won't be higher than that of small transactions. The second explanation was given by Bouchaud et al. (2008). These authors have explained the concave shape with the concept of selective liquidity. Selective liquidity means, that market participants' decision to submit an order or not will depend on the market liquidity. If they see, that there is liquidity on the market, they would give a large transaction otherwise they submit only small ones. Namely the market participants always try to give an order, which can be fulfilled on the best price level, and try to avoid that their orders delete a lot of levels from the limit order book.

It follows from the previous, that the shape of the empirical price impact function will be determined by the shape of the volumes on the best price level. Namely, price impact will occur, if the order deletes all the orders on the best price level. In this case the ePIF can be concave if $P(+|v)E(r)$ expression is concave, where $E(r)$ shows the price impact – the relative change of the mid price –, while $P(+|v)$ shows the probability of the price change at an order size of „*v*". It can happen only if $P(+|v)$ is concave, because of the non-negativity of E(r). If ϕ_b stands for the volume on the best price level on the opposite side of the book, which is a random variable from a $P_b(\phi_b)$ distribution, and independent from the order size „*v*", then a price impact will occur if $v \geq \phi_b$. In sum, the probability of the price impact will be the following (Bouchaud et al., 2008):

$$P(+|v) = \int_0^v P_b(\phi_b)\,d\phi_b \tag{50}$$

From the deduction, a connection can be shown between the virtual and empirical price impact, according to Equation 51. The left side of Equation 51 symbolizes the empirical price impact ($E(r|v)$), while on the right side the virtual price impact ($E(r)$) can be found, which is multiplied by the probability of occurrence of the price impact ($P(+|v)$).

$$E(r|v) = P(+|v)E(r) \tag{51}$$

1.5. Time-variation of price impact

The price impact's effect on a larger timescale was analyzed by Bouchaud (2010a, b), who emphasizes the permanent nature of the price impact, which is the consequence of the order flow's long memory. Several studies have analyzed the permanent and temporary nature of the price impact, from which I would like to highlight Bouchaud et al.'s (2008) and Almgren et al.'s (2005) work.

Bouchaud et al. (2008) have concluded that if single transactions are being analyzed, than the price impact function is concave, but the function becomes more linear if we aggregate the transactions. Based on this observation the authors have tested the effect of the price impact on a larger timescale, and concluded that it is worth discerning the PIF to a permanent PIF and to a temporary PIF, since the two functions behave much differently.

The researchers have tested for single transactions, the permanent and temporary proportion of the price impact, and whether these values have a fix or a variable value. They have tested also the effect of the order flow prior to the transaction.

Bouchaud et al. (2008) found, that the price impact disappears with the passage of time, and that the permanent price impact is asymmetric and depends on the past order flow. Asymmetry means, that every transaction has a price impact, but this price impact depends on the order flow in the past, and on the predictability of the transaction. The more it is predictable, the smaller the price impact will be.

Their viewpoint is, that the dynamics of the price formation, and the price impact, will depend on the dynamics of the order flow, and also on the information the liquidity provider has, and on the method the market players predict the future order flows (Bouchaud et al., 2008).

Almgren et al. (2005) have split the price impact also into a temporary and to a permanent part. Their opinion is that the permanent price impact reflects the information available for the market participants, and can be calculated from the imbalance of supply and demand. This effect is independent from the time of the transaction. In contrast, the temporary price impact is caused by the market participants' different short term notions of the price formation. Timing has a notable effect on the value of the price impact. In sum the realized (empirical) price impact, will be the result of the following two effects:

$$\text{Realized price impact} = \text{Permanent price impact} + \text{Temporary price impact} + \text{Noise} \tag{52}$$

1.6. Theoretical modeling of the price impact

In Subchapter 1.4 I covered the shape of the price impact function determined by real stock exchange data and also discussed the formal description of it. Simultaneously with empirical research, and sometimes in the same paper, many researchers try to model the evolution of price impact. The majority of these models try to capture the price impact by analyzing the behavior of rational agents and making assumptions about the order flow.

The classic model of Kyle (1985) presumes linear price impact. The models of Seppi (1990), Barclay & Warner (1993), and Keim & Madhavan (1996) suggest that the price impact is concave. The models of Zhang (1999) and Gabaix et al. (2003, 2006) are based on the optimal decisions of fund manager's resulting in a square-root function. In the popular model of Iori et al. (2003) market and limit orders are made randomly, the order flow is supposed to follow Poisson distribution. According to the authors, if the depth of the order book is increasing monotonically, then the price impact function is concave, and its shape is in line with empirical researches: $\Delta p \sim w^{\beta}$, where $\beta \leq 1$. The authors attribute the concavity of the function to the trading mechanism and market structure and not to optimal trading strategies based on rational decisions. The results of Iori et al. (2003) show a price impact function matching the shape of the real one, although the orders were randomly given in the model.

The paper of Bouchaud et al. (2004) models the evolution of the price impact in time by defining the price as the result of past transactions. An interesting attempt to model price impact is the neural network of Kempf & Korn (1999), the model of Challet & Stinchcombe (2001), in which the authors map the orders to particles and the paper of Rosenow (2008), where the author is using the popular spin model of physicists.

A part of the theoretical models sheds light on the factors determining the shape of the function. The majority of these models were created by the research divisions of market participants, e.g. Almgren et al. (2005) made their model within Citigroup. The primary goals of these models is to forecast the price impact of the future orders of the firm, to estimate the transaction costs of trading due to price impact and to design optimal trading strategies. According to e.g. Torre & Ferrari (1999) the size of the price impact (κ) is driven by six factors:

$$\kappa = F(V, \varepsilon, \sigma, \xi, \tau, \chi) \tag{53}$$

Table 24 contains the description of the parameters of the previous equation and their effect on price impact.

Table 24: Factors effecting price impact

Notation	Description of the factor	The effect of the growth of the factor on price impact	
V	The volume of traded stocks expressed in USD	V ↑	κ ↑
ε	Elasticity: the reaction of order flow to price impact	ε ↑	κ ↓
σ	The volatility of stock price	σ ↑	κ ↑
φ	Measure of intensity, describes the frequency of trading	φ ↑	κ ↓
Ξ	Shape indicator, describes the distribution of traded volumes	ξ ↑	κ ↓ (if the expected value increased) κ ↑ (if standard deviation increased)
τ	The indicator of the mood of the market, describes the price of liquidity	τ ↑	κ ↑
χ	The indicator of investor expertise	χ ↑	κ ↓

Source: based on Torre and Ferrari (1999)

Torre and Ferrari (1999) give a detailed explanation on how each of the above factors affects the size of the price impact. The main disadvantage of their study is that the shape of the function F remains hidden from the reader, it continues to be treated confidentially for competitive considerations.

I disregard abstain from the further demonstration of the theoretical models, since in my dissertation I am not supposed to build a theoretical model with respect to price impact functions. In chapter IV/2 of my dissertation I will show how to estimate a price impact function from the BLM database.

2. Empirical research: estimation and analysis of the price impact function

One of the explicit goals of my dissertation is to provide the market participants with a method that would enable them to estimate the price impact function easily without having to recourse to the data of the order book. Knowing the price impact function is important for the market participants, in order to be able to predict the price impact of trades in the future, and to estimate the additional trading costs related to the price impact, and also to be able to build an optimal trading algorithm based on the price impact function. Namely traders will submit their orders according to the time-variation of the virtual price impact function. In this chapter I show how a price impact function can be estimated based on the Budapest Liquidity Measure database. In other words I show the relationship between the PIF and the liquidity measures. In the course of the estimation I will define a virtual price impact function. The time series of the virtual price impact function can be analyzed by the market participants in order to establish a trading strategy. Namely the advantage of virtual price impact function as opposed to the empirical price impact function is, that it is suitable for time series analysis to be carried out on it. It is impossible to make a time series analysis on the empirical price impact function, since it gives the average value of the price impact for a longer period (e.g.: a year). The virtual price impact function on the other hand can be estimated for every second. In addition to the estimation of a virtual price impact function, I will make a time series analysis on the estimated database.

2.1. Research questions

I will analyze the time-variation of the price impact, and its basic statistical characteristics, in order to get a picture of the time series of the transaction cost that occur as a result of the lack of liquidity. During the analysis I will answer the following questions:

1. What are the basic statistics of the vPIF (average, standard deviation, minimum, maximum, skewness, curtosis and distribution)?
2. Is there a trend in the time series?

3. How does volatility change over time?
4. Are there outlier data, and are there structural breaks?
5. Can the time series data of vPIF be described as a mean reverting process?

My hypotheses based on the research questions will be the following:

H5: The dynamics of the virtual price impact function can be described by the following:

H5/a: symmetry,

H5/b: trend,

H5/c: cycles,

H5/d: mean reverting,

H5/e: shock resistance.

2.2. Research method

I will carry out the analysis of the virtual price impact function on the OTP stock's BLM database in Subchapter 2.4. The time series contains the BLM data from 1st January 2007 till 3rd June 2011. To be able to analyze the vPIF of OTP it is necessary to define the vPIF(q) function for every day from the BLM(q) function. As a first step I estimate the BLM(q) function, which I will do with a linear regression. I will describe the exact estimation of the BLM(q) and vPIF(q) in Subchapter 2.3.

After the estimation of vPIF I will analyze the time series of vPIF and the basic statistical characteristics of the function. I do this in order to get a closer picture of the behaviour of the transaction cost caused by the lack of liquidity in the past. The methods I have used during the analysis can be found in the next listing. In more details I will show the methods in Subchapter 2.4.

- Descriptive statistics: average, standard deviation, median, minimum, maximum, curtosis, skewness.
- Trend analysis: fitting of polynomial trend, calculating moving average.
- Symmetry of bid and ask side: correlation of the two sides.
- Shock resistance: testing the autocorrelation in the database with Breusch-Godfrey LM test (Breusch, 1979; Godfrey, 1978).

- Structural breaks: using Chow test (Chow, 1960), and Quandt-Andrews test (Andrews, 1993).
- Outlier data: analyzing boxplot figures.
- Mean reverting: using an extended Dickey-Fuller test (Dickey és Fuller, 1979).

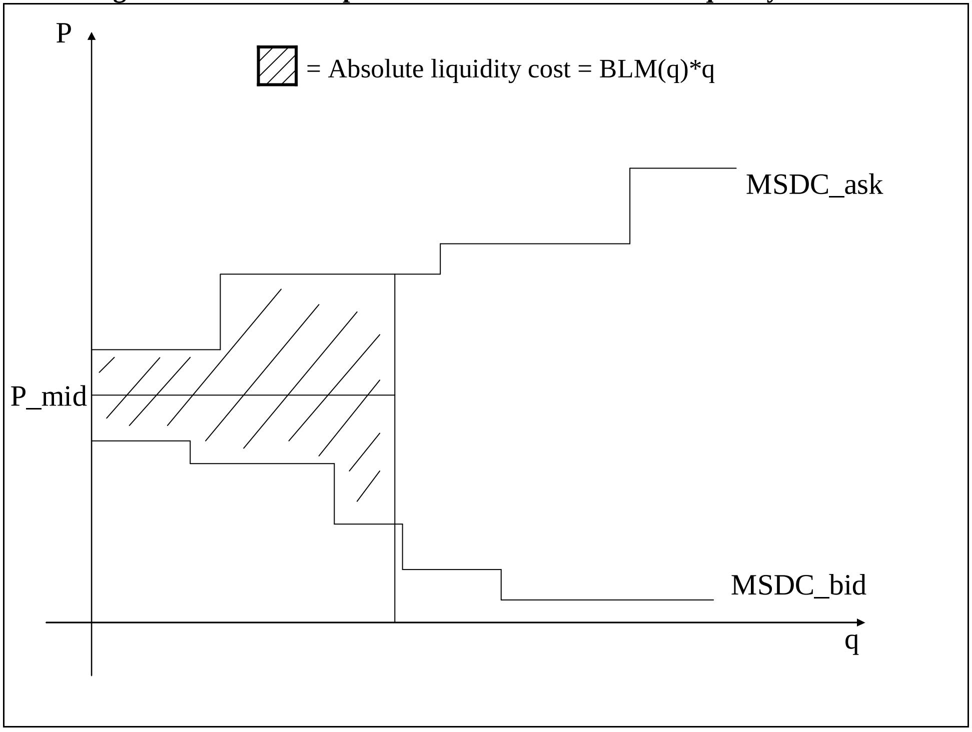
2.3. Estimating virtual price impact function

Market participants would be able to calculate the price impact from the order book. But the order book is not available for most of the participants, so they don't have precise information on market liquidity. This means that they cannot even define the MSDC(q) function, or the average price either, so they cannot estimate a price impact function. The only information they can read from the first few levels of the order book e.g. the bid-ask spread, or the volumes available on the first few levels of the book. Nevertheless a price impact function can be estimated not only from the order book, but from liquidity measures as well, as the liquidity measures are calculated from the order book data.

A liquidity measure, like the BLM(q) in itself is not a price impact function yet, as the BLM does not inform the trader about the new mid price realized after the transaction. Instead, the BLM measures the implicit cost of trading (in basispoints) stemming from the illiquidity of the markets. Since BLM's calculation is based on the order book, it is possible to estimate a marginal supply-demand curve (MSDC) (Acerbi, 2010), then to estimate the virtual price impact function. Namely in this chapter I will introduce a method which enables market participants to estimate a price impact function fast and easily without knowing the data in the whole order book.

In order to be able to estimate an MSDC(q) function from the BLM database, a relationship should be found between the two notions. This relation is shown in Figure 46. In the figure the implicit cost of trading can be seen, since the bid and ask side of the MSDC(q) function is shown in the figure. The area between the two sides of the MSDC(q) function is the implicit cost, which occurs in the absence of liquidity. The size of the area is equal to the BLM value, if we multiply the BLM with the total transaction size, *q*.

Figure 46: Relationship between the MSDC and the liquidity measure



Source: proprietary

In sum, the total transaction cost that occurs because one cannot trade on the mid price is shown by the banded area in Figure 46. So the total banded area shows that, what the transaction cost would be if one were to buy and sell immediately. Equation 54 shows how to calculate the size of the area, where *q* is the size of the transaction in Euros, while C_{total} shows the total implicit cost of trading.

$$C_{total}(q) = \int_0^q MSDC\_ask(x)dx - \int_0^q MSDC\_bid(x)dx \tag{54}$$

If we like to define the transaction cost only for the bid or the ask side, then it can be done by Equation 55 and 56,where C_{ask} shows the implicit cost during a buy order, while C_{bid} shows the implicit cost of a sell order.

$$C_{ask}(q) = \int_0^q MSDC(x)dx - P_{mid} * q \tag{55}$$

$$C_{bid}(q) = P_{mid} * q - \int_0^q MSDC(x)dx \tag{56}$$

According to Figure 46 and Equation 54, the value of BLM(q) – in the function of *q* – can be defined by Equation 57:

$$BLM(q) = \frac{\int_0^q MSDC\_ask(x)dx - \int_0^q MSDC\_bid(x)dx}{q} \tag{57}$$

To give estimation for the price impact function – with the aid of the MSDC – I had to define the value of the MSDC with the BLM.

The first step is to define the shape of the BLM(q) function. Based on a video made in Matlab about the time-variation of BLM(q) I have seen that the daily BLM(q) function can be estimated with a linear. The intraday BLM(q) function can have various shapes: linear, concave or convex. Since in my dissertation I am working with daily data, I have applied the assumption that the BLM(q) is linear, so it can be estimated with a linear regression. In this case the BLM(q) is defined by the Equation 58:

$$BLM(q) = a * q + b \tag{58}$$

If we model the BLM(q) function separately for the bid ask side of the limit order book, then we get for the buy side: BLM^b, and for the sell side BLM^a:

$$BLM = 2 * LP + APM_{bid} + APM_{ask}\,, \tag{59}$$

$$BLM^a = LP + APM_{ask}\,, \tag{60}$$

$$BLM^b = LP + APM_{bid} \tag{61}$$

In the equations LP is the liquidity premium, which is the half of the bid-ask spread, while the APM_{ask} is the adverse price movement on the ask side, and APM_{bid} is the adverse price movement on the bid side. The sum of LP and $APM_{bid/ask}$ will give Equation 60 and 61, since BLM^a, and BLM^b will represent the implicit trading cost on the ask and bid side, which contains the half of the spread and the adverse price movement.

The linear regressions for the ask and the bid side can be defined by Equation 62 and 63. This means, that when I estimate the vPIF for every day I have to estimate the parameters a_{ask}/a_{bid} and b_{ask}/b_{bid} separately for the two sides.

$$BLM^{a}(q) = a_{ask} * q + b_{ask} , \tag{62}$$

$$BLM^{b}(q) = a_{bid} * q + b_{bid} \tag{63}$$

The estimation of the MSDC by means of the BLM(q) function requires the following steps on the ask side:

1. step: Defining the total implicit cost on the ask side based on the BLM:

$$BLM^{a}(q) = \frac{\int_{0}^{q} MSDC\_ask(x)dx - q * P_{mid}}{q} \tag{64}$$

2. step: Rearrange the equation to MSDC(q):

$$\begin{gathered} BLM^{a}(q) * q = \int_{0}^{q} MSDC\_ask(x)dx - q * P_{mid} \rightarrow \\ dBLM^{a}(q) * q + BLM^{a}(q) = MSDC\_ask(q) - P_{mid} \rightarrow \\ dBLM^{a}(q) * q + BLM^{a}(q) + P_{mid} = MSDC\_ask(q) \end{gathered} \tag{65}$$

3. step: Substitute Equation 60 in the equation, and rearrange the equation:

$$\begin{gathered} a_{ask} * q + a_{ask} * q + b_{ask} + P_{mid} = MSDC\_ask(q) \rightarrow \\ \boldsymbol{2 * a_{ask} * q + b_{ask} + P_{mid} = MSDC\_ask(q)} \end{gathered} \tag{66}$$

The estimation of the MSDC by means of the BLM(q) function requires the following steps on the bid side, according to Equation 67:

$$\begin{aligned}
&\mathrm{BLM}^{b}(q)=\frac{q*P_{mid}-\int_{0}^{q}\mathrm{MSDC_bid}(x)dx}{q}\rightarrow\\
&\mathrm{BLM}^{b}(q)*q=q*P_{mid}-\int_{0}^{q}\mathrm{MSDC_bid}(x)dx\rightarrow\\
&\mathrm{dBLM}^{b}(q)*q+\mathrm{BLM}^{b}(q)=P_{mid}-\mathrm{MSDC_bid}(q)\rightarrow\\
&P_{mid}-(a_{bid}*q+a_{bid}*q+b_{bid})=\mathrm{MSDC_bid}(q)\rightarrow\\
&\boldsymbol{P_{mid}-(2*a_{bid}*q+b_{bid})=MSDC_bid(q)}
\end{aligned}\tag{67}$$

Finally, the virtual price impact function can be expressed in the function of MSDC(q), according to the Equation 48, which can be found in Subchapter IV/1.3.

$$\mathrm{vPIF}(q)=\frac{\mathrm{MSDC}(q)}{P_{mid}}-1\tag{68}$$

During the deduction I have assumed a linear BLM(q) function, and as a result the vPIF became linear as well.[39] Nevertheless I would have been able to estimate the function with any other shapes. I have three reasons why I have chosen the linear shape. Firstly, because in the literature – based on Subchapter IV/1.4 – the price impact function is linear in many cases (Almgren et al. 2005; Biais, Hillion & Spatt, 1994; Bouchaud et al., 2008; Cont, Kukanov and Stoikov, 2011; Hopman, 2007; Smith et al., 2008). Secondly, because I have tested statistically the shape of the BLM(q) function, and I have found that in case of fitting a linear regression for the BLM data, the R^2 value is around 0.95, so the linear approximation can be considered as good. Finally, I have chosen the linear shape, because I think that the relation between the BLM and the price impact function can be explained, understood and used most easily with the most simplest function-shape. In the next paragraph I shortly introduce the way to change the deduction if the BLM(q) is not linear.

Since we assumed the BLM(q) to be linear, the MSDC(q) and the price impact function became linear as well. If I would like to estimate a convex or concave PIF, BLM(q) should be non-linear. The difference in the deduction will be, that in the step

[39] I will show this in Subchapter 2.4.1 on Figure 47.

N3, i.e. when I substitute the BLM(q) function, the equation changes. For example if I would estimate the BLM(q) with a power-law function, then the BLM^a on the ask side would be the following:

$$BLM^{a}(q) = a_{ask} * q^{\alpha} + b_{ask} \tag{69}$$

The result is, that during the estimation another parameter should be estimated as well, namely the α. Another change, that in step N3, and the deduction changes as follows:

$$\begin{gathered} a_{ask} * q + a_{ask} * q^{\alpha} + b_{ask} + P_{mid} = MSDC\_ask(q) \rightarrow \\ \boldsymbol{a_{ask} * (q - q^{\alpha}) + b_{ask} + P_{mid} = MSDC\_ask(q)} \end{gathered} \tag{70}$$

However, concerning the daily data, estimation with a linear is proved to be enough. In further research it would worth estimating the BLM(q) with another shape, in order to compare the results with mine.

On the basis of the vPIF the empirical price impact function cannot be estimated, on the one hand because the BLM database does not provide information on the probability of the occurrence of the price impacts, on the other hand because the estimation of the ePIF depend on real transaction data, not on the order book. The ePIF can be estimated, for example, from the TAQ (*trades and quotes*) database (Margitai, 2009). Estimating the ePIF from the TAQ database is a time- and calculation consuming task. In my dissertation my main goal was to provide the market participants with a method that enables them to estimate the price impact function easily. The price impact function based on BLM is the result of an easy and quick calculation.

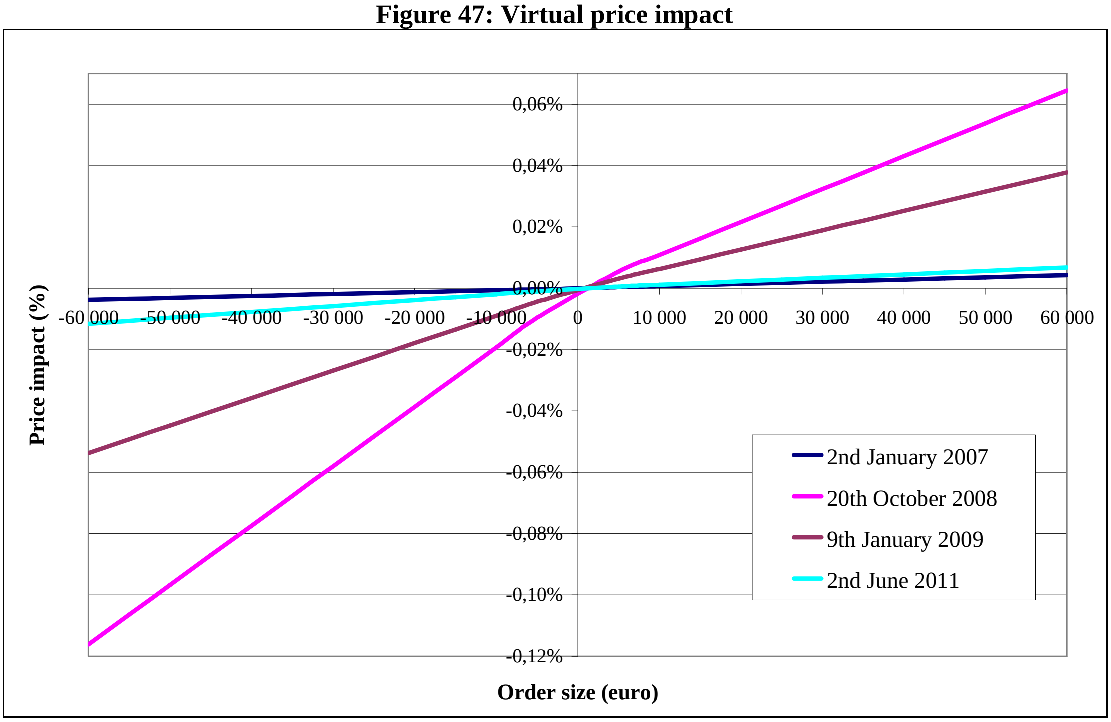
2.4. Analysis of the time series of the virtual price impact function

2.4.1. Descriptive statistics

The data I am going to analyze are based on the BLM data of OTP between 1st January 2007 and 3rd June 2011. I will estimate the virtual price impact function for every trading day with the method I have introduced in the previous chapter.

Figure 47 shows the virtual price impact function on the bid and on the ask side as well, for a few trading days. The four days have been chosen in order to show how the price impact is different in calm period (1st January 2007 and 2nd June 2011) and during crisis (20th October 2008 and 9th January 2009). On the figure it can be seen, that during a crisis the price impact function is steeper, which shows, that the transaction cost of trading is higher, because the markets are more illiquid, then during normal times.

Figure 47: Virtual price impact



Source: proprietary

In Figure 45, in Subchapter IV/1.4 the authors have estimated a virtual price impact function from order book data, and get the result, that the price impact function is nearly linear. In my dissertation I got a linear function, because I have estimated the BLM(q) to be linear, as I have mentioned in the previous chapter.

Before analyzing the time series of the virtual price impact function it is worth analyzing the descriptive statistics for a few order sizes, in order to get a full picture of the vPIF. The descriptive statistics can be seen in Table 25:

Table 25: Descriptive statistics

Descriptive statistics of the virtual price impact function for different order levels (bp)								
	BID				**ASK**			
	vPIF (-5e)	**vPIF (-20e)**	**vPIF (-40e)**	**vPIF (-50e)**	**vPIF (5e)**	**vPIF (20e)**	**vPIF (40e)**	**vPIF (50e)**
Average	-0.150	-0.606	-1.213	-1.517	0.143	0.568	1.134	1.417
Median	-0.082	-0.332	-0.665	-0.831	0.082	0.325	0.649	0.811
St.deviation	0.222	0.894	1.789	2.2360	0.198	0.788	1.574	1.967
Minimum	-2.048	-8.237	-16.489	-20.620	0.014	0.055	0.110	0.137
Maximum	-0.015	-0.061	-0.123	-0.153	2.043	8.123	16.230	20.284
Skewness	-3.955	-3.952	-3.952	-3.952	3.895	3.898	3.898	3.899
Curtosis	19.244	19.220	19.215	19.215	19.709	19.757	19.765	19.767

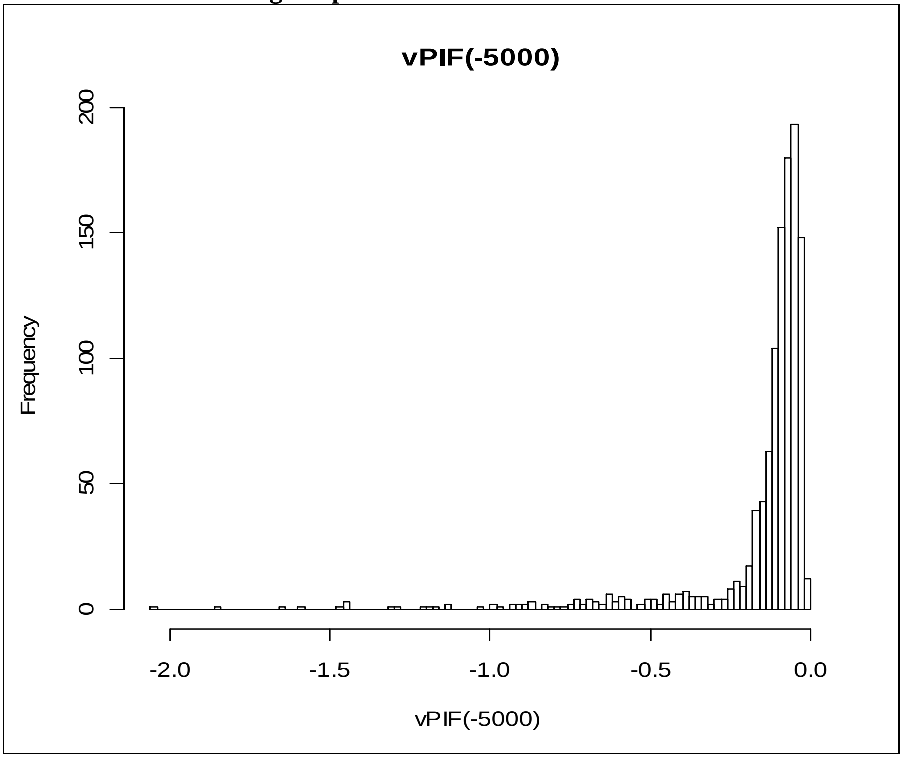
Source: proprietary

When a trader wants to sell on the market, the order will be fulfilled on the bid (buy) price, while in case he wants to buy, it will be fulfilled on the ask (sell) side of the book. Based on Table 25, the bid and ask side of the book have different characteristics. In case of the averages it can be seen, that on every order size level the average and the median have greater absolute value on the buy side of the function. I believe that the reason for that is the following: when investors buy stock, they don't do it at the same time, but, while when selling stock it is common to try to do it at the same time, for example maybe because of a panic on the market. In these cases they are willing to close their position even with higher transaction costs, causing a large price impact with this. *So the bid and the ask side of the vPIF can differ as a consequence of the so called herd effect. The market players want to sell at the same time, but buying stocks are more scattered.* This can be seen in the vPIF, which is based on the limit order book. The database I was analyzing contains the crisis of 2007/2008, and this is reflected in the dissimilarity of the two sides of the price impact function, since during crisis a few times there was a panic on the market, which was coupled with the lack of liquidity.

The other results show the same, as the averages. Namely the value of the standard deviation, the maximum and the minimum are greater in absolute value on the buy side than on the sell side.[40] I can conclude the same thing as I did in case of the averages, namely, that the limit order book reflects that the sell orders arrive at the market more concentrated than the buy orders.

The analysis of the skewness and curtosis – namely that the distribution differs from the normal distribution – is easier to carry out by making histograms (Figure 48 and 49). It can be seen, that on the bid side of the price impact function, the probability density function is skewed to the right, while the ask side's PIF is skewed to the left. Though the probability density function is more skewed on the bid side, which is because of the reasons mentioned before.

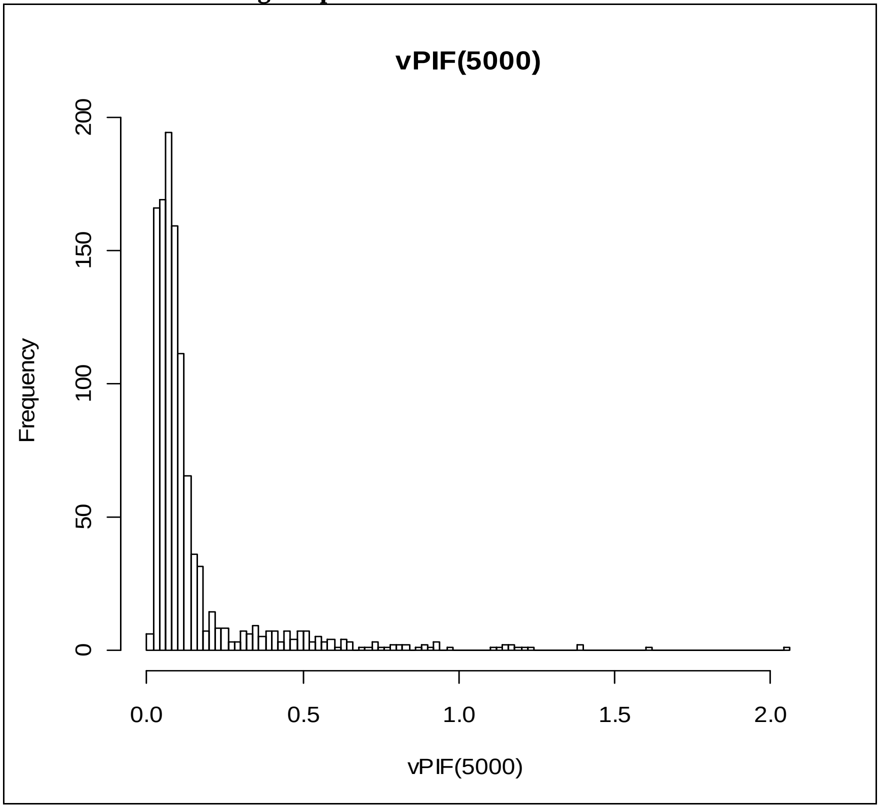
Figure 48: Density function of price impact value at buying EUR 5,000 of OTP during the period of 01.01.2007-02.06.2011.



Source: proprietary

[40] The buy side maximum/minimum value should be compared with the sell side minimum/maximum value.

Figure 49: Density function of price impact value at selling EUR 5,000 of OTP during the period of 01.01.2007-02.06.2011.

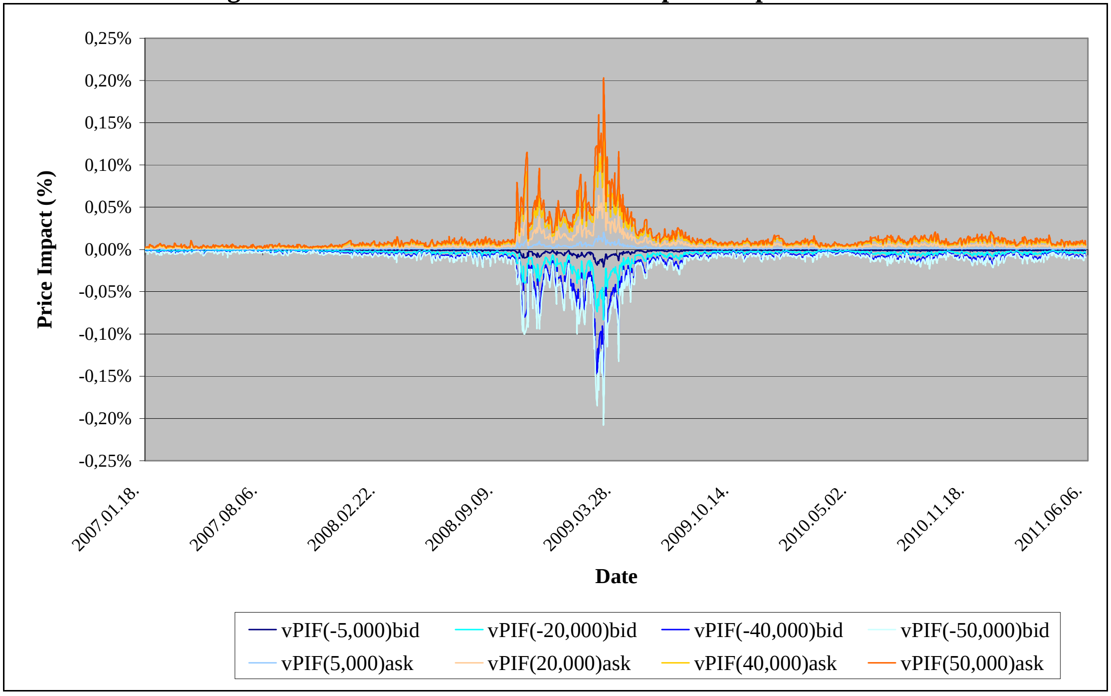


Source: proprietary

2.4.2. Trend

I will start the time series analysis with analyzing the trend of the time series. Knowing the trend is important, because it can help market participants to estimate when to open or close a position. Since knowing the trend one can forecast when the liquidity will increase or decrease. According to this it is worth plotting the time series vPIF values for a few order sizes, which is shown on the Figure 50.

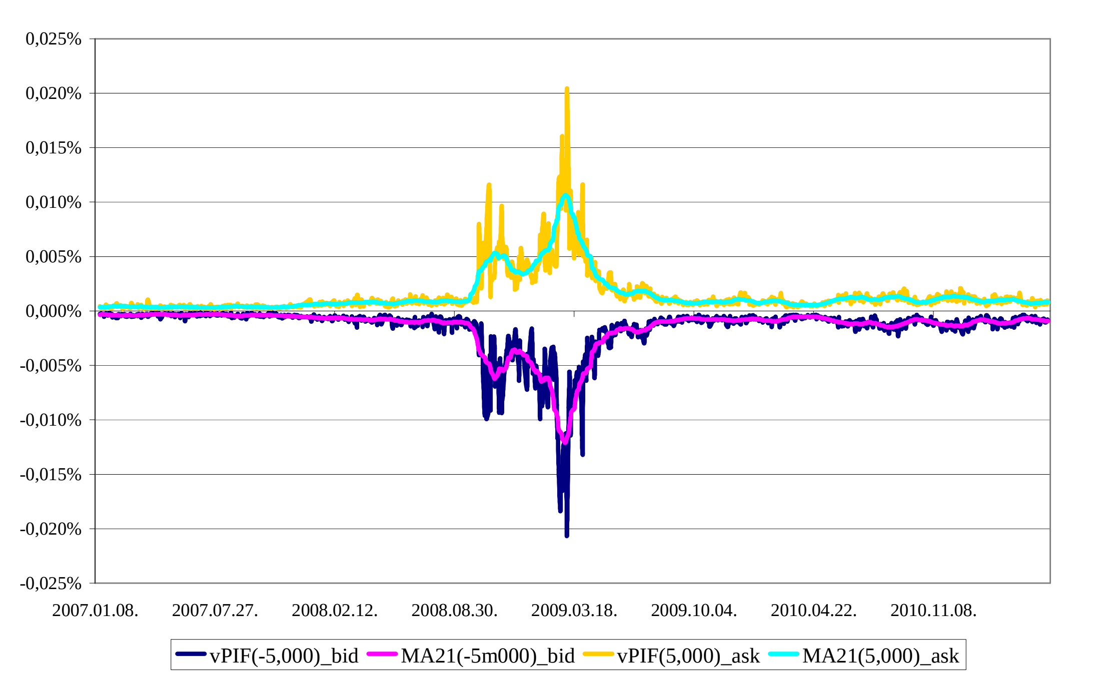
Figure 50: The time series of the virtual price impact function



Source: proprietary

The figure shows that there isn't a linear trend in the database on any order size level. This fact is logical, since if there would be a trend, then it would mean that the illiquidity is increasing or decreasing as a function of time. In a multi-year horizon it is hardly possible on the market that the liquidity is continuously increasing or decreasing. In order to clearly exclude the existence of a trend, I have made further analysis. Because of this I have analyzed whether there is a polynomial trend. I made the analysis for 5,000 Euros, and for 60,000 Euros. The R^2 values in case of a sixth degree polinom were very small: $R^2(5,000) = 0.419$, and $R^2(60,000) = 0.413$. In case of polinoms with smaller degrees, the R^2 were even smaller. This means that the polinoms haven't fitted well on the database, their explanatory power is small. Because of this I have tested another trend analysis method as well, the method of moving average. Figure 51 shows the 21 day moving average for 5,000 Euros for bid and for the ask side.

Figure 51: The virtual price impact and its 21 day moving average values

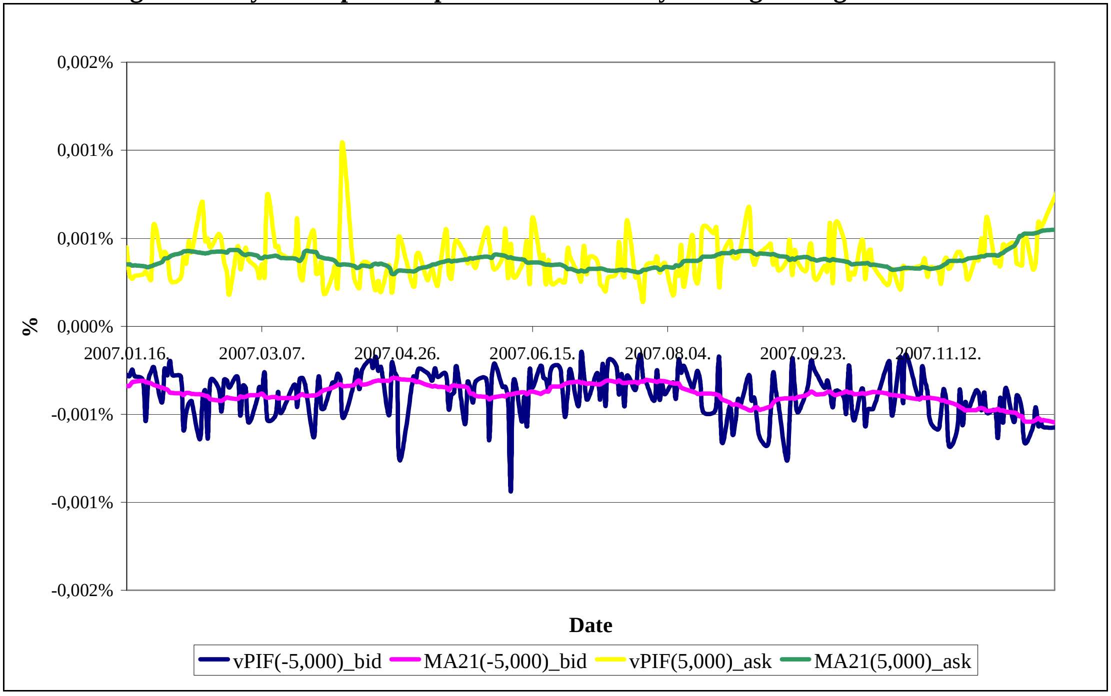


Source: proprietary

The figure shows that the price impact follows a strange trend, since there isn't a trend throughout the whole time series either on the bid or the ask side. The figure also shows, that before the crisis of 2008 the price impact was quite stable, then during the crisis it had increased, then at the end of the crises it decreased again, but never became as small as it was before the crisis. *So there isn't a trend in the database, but it seems that the price impact follows the economic cycles*. Because of this cyclical effect, I have split the database into three parts: before the crisis, crisis and after the crisis period. The splitting has been made according to the analysis of the 2.4.4 chapter, where I have defined the structural break points. For the analysis one year has been chosen from the before crisis period, and one year from the after crisis period.

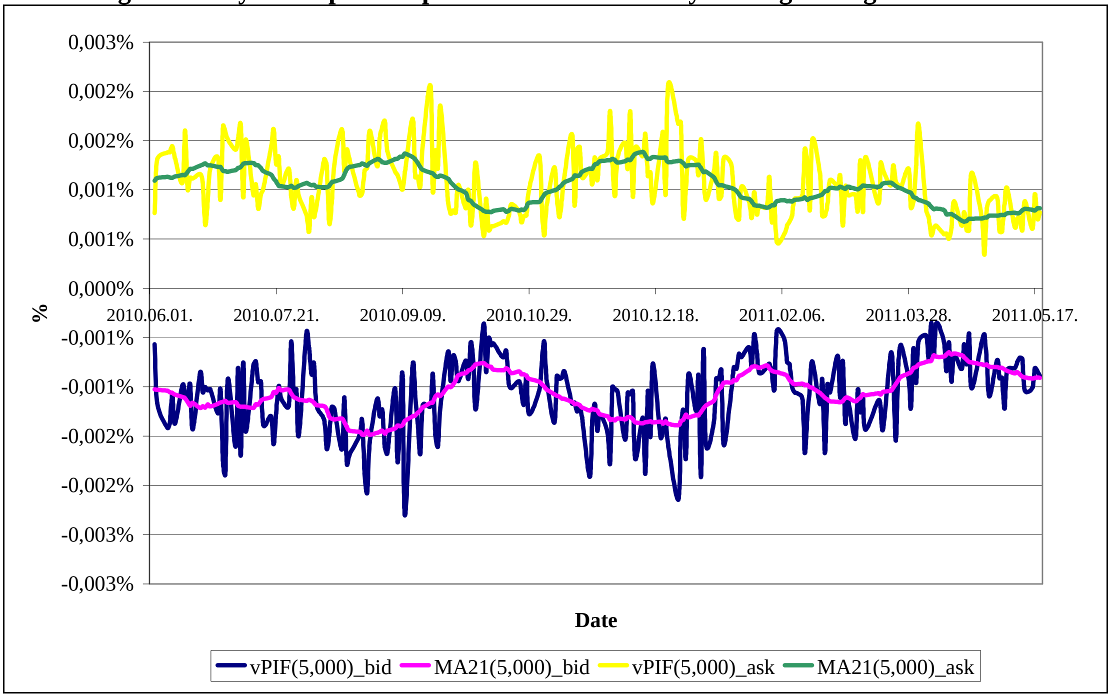
I have got the same results before and after the crisis. One of these is that the price impact develops the same way on the bid and on the ask side, which means that the liquidity of both sides are nearly the same. The other result is that though there isn't a trend in the database, but there is a cyclical effect in every quarter year. The results are shown in Figure 52 and 53:

Figure 52: Cycles of price impact based on 21 day moving average before crisis



Source: proprietary

Figure 53: Cycles of price impact based on the 21 day moving average after crisis



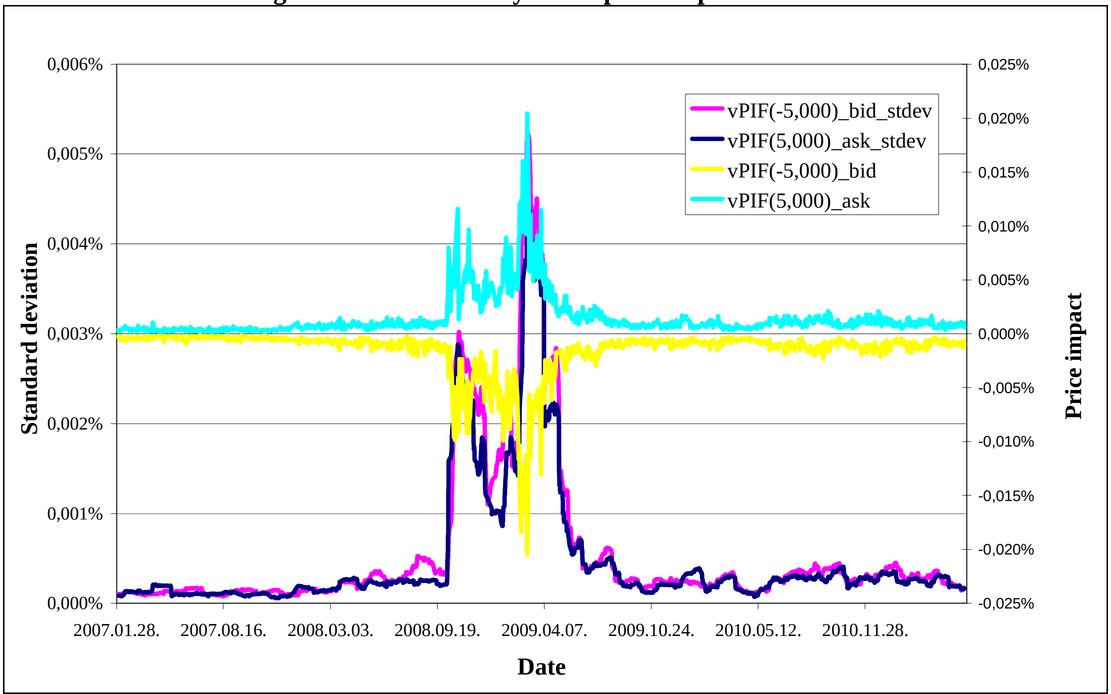
Source: proprietary

These cycles can be the result of the quarterly report in my point of view. On the day the quarterly reports are published, the investors' information asymmetry[41] regarding the operation of OTP is smaller, so they are more willing to trade with the paper, which results in more liquidity for the paper. On Figures 52 and 53 it can be seen that at the time of the quarterly reports (15th January, 15th April, 15th July, and 15th October) the price impact is the smallest, while their maximum values are halfway between two quarterly reports.

2.4.3. Volatility and correlation in the time series

I have analyzed the changing of the volatility of the price impact function on several order sizes. The results for the 5,000 euro order can be seen at Figure 54, where the volatility of a certain day is calculated from the price impact data of one month data prior to the day.

Figure 54: The volatility of the price impact over time



Source: proprietary

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[41] I ignore the detailed discription of information asymmetry, because it goes beyond the topic of my dissertation. More detailed on the information asymmetry in the international literature see e.g. Akerlof, 1970; Spence, 1973; Stiglitz, 1977; or in the Hungarian literature see e.g. Balla, 2006; Krénusz, 2007; Havran et al., 2010.

I have found that the volatility changes notably with the economic cycles. When the value of the price impact increases because of the lack of liquidity, then the volatility increases as well on both sides of the function.

It can be seen from the figure, that on 5,000 euro order level the correlation is high between the bid and the ask side. I have analyzed the correlation between the two sides of the function, and between the different order levels. This is important, because I have assumed before the analysis that if the limit order book shows low liquidity for example on the bid side this shouldn't mean that, that the liquidity is low on the other side of the book as well. If everyone would like to sell the stocks, it would be easy to buy, so the liquidity should be high as well on the ask side. But this is not the case according to the data. Table 26 summarizes the correlations: the correlation is nearly perfect in every case, which means, that the liquidity of the ask and bid side on every order size are strongly correlated, strongly moving together.

Table 26: Correlations

correlations	**vPIF(-5teur) _bid**	**vPIF(-20 teur) _bid**	**vPIF(-40 teur) _bid**	**vPIF(-50 teur) _bid**	**vPIF(-60 teur) _bid**
vPIF(5 teur) _ask	-0.9516	-0.9520	-0.9521	-0.9521	-0.9521
vPIF(20 teur) _ask	-0.9513	-0.9517	-0.9518	-0.9518	-0.9518
vPIF(40 teur) _ask	-0.9513	-0.9517	-0.9517	-0.9518	-0.9518
vPIF(50 teur) _ask	-0.9513	-0.9517	-0.9517	-0.9518	-0.9518
vPIF(60 teur) _ask	-0.9513	-0.9517	-0.9517	-0.9517	-0.9518

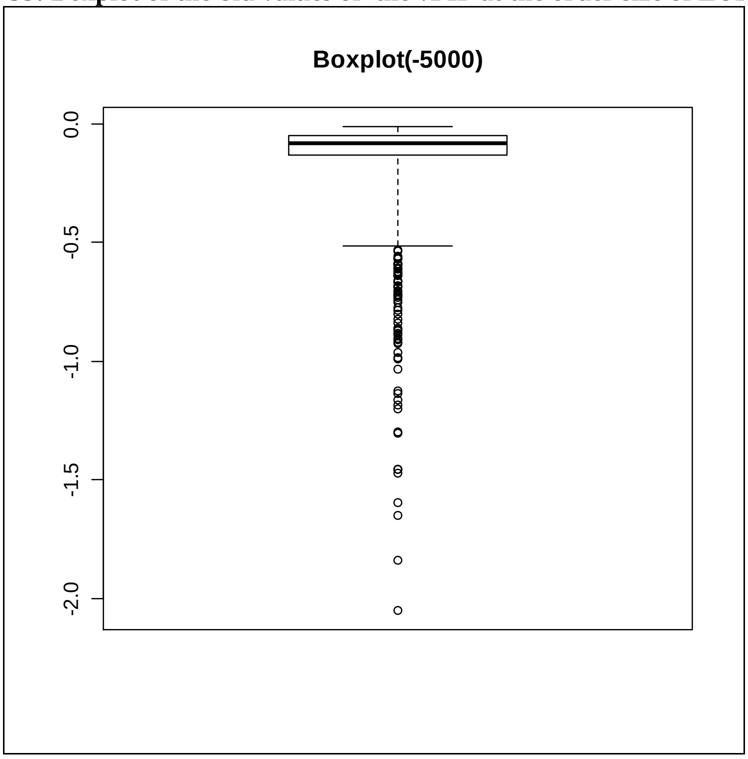
Source: proprietary

Based on Figure 54 it can be seen, that there is a relation between the price impact values of each trading days, since if one day the price impact is low/high, it is quite possible that the next day it will be low/high again. With statistical methods I have analyzed the relation between the days following each other, namely I was testing a first-order and a second-order autocorrelation. The usually used Durbin-Watson test for testing first-order autocorrelation cannot be applied in this case for two reasons (Darvas, 2004). On the one hand the residuals of the time series data of the price impact function on any order levels are non-normally distributed, and on the other hand it is quite possible that there is higher order autocorrelation in the time series as well. I will use instead the Breusch-Godfrey LM test, which has less restrictive assumptions. Based on the Breusch-Godfrey LM test, it is clear, that there is a positive autocorrelation in the time series on every order sizes. The test have rejected that the residuals are not autocorrelated. There can be detected, that there is a very high-order autocorrelation in the time series data. The tenth or even the twentieth-order autocorrelation were significant.

2.4.4. Outliers and structural breaks

On the basis of Figure 50 the absolute values of the virtual price impact function increase significantly in October-November 2008 and January-February 2009. The significant increase can be observed both on the bid and the ask side. With the aim of describing the turbulent period more properly and identifying the outliers I have prepared box plots (McGill et al., 1978). Box plots are based on quartiles, and represent a convenient way of graphically depicting the distribution of the values of the virtual price impact function belonging to various order sizes. Figure 55 shows the box plot of the bid values of the virtual price impact function belonging to contract sizes of EUR 5,000.

Figure 55: Boxplot of the bid values of the vPIF at the order size of EUR 5,000



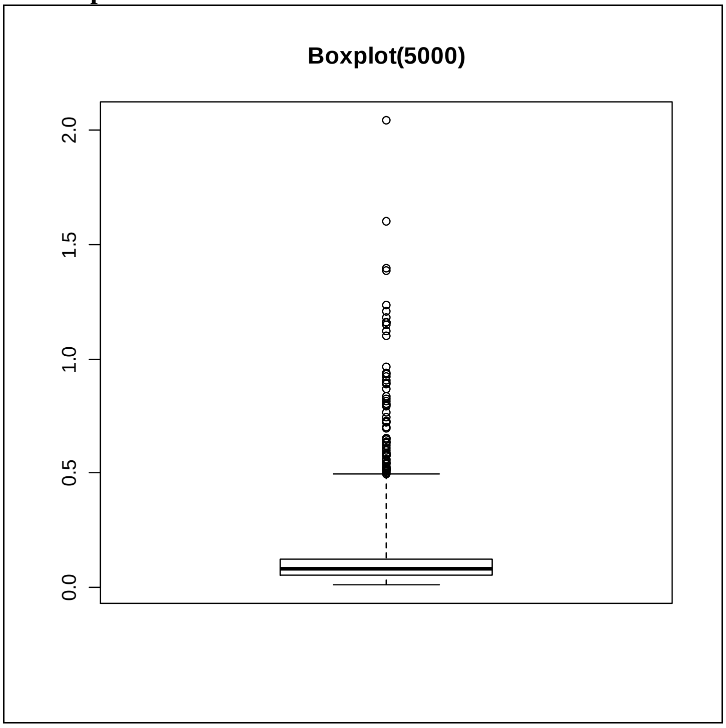
Source: proprietary

On Figure 55 the upper edge (hinge) of the box indicates the 75th percentile (Q3) of the data set, which currently equals -0.0522. The lower hinge of the box indicates the 25th percentile (Q1) of the underlying data, which has a value of -0.1312. In the literature the range of the middle two quartiles, that is, the difference between Q3 and Q1, is known as the inter-quartile range (IQR). The box itself contains the middle 50% of the values of the virtual price impact function. The line in the box indicates the median value (-0.0821) of the data. On the basis of Figure 55 the median line within the box is not equidistant from the hinges, which refers to the asymmetric nature of the data. (Note that the skewness of the data was also highlighted in Subchapter 2.4.1.).

The figure also contains the maximum (-0.0149, short horizontal line above the box) and the minimum (-2.0480, the circle situated at the bottom of the figure) of the bid values of the virtual price impact function belonging to contract sizes of EUR 5,000. The observations marked by circles represent those outliers that fall below the threshold calculated by the formula of $\mathrm{Q1} - 5 \cdot \mathrm{IQR}$. As the threshold calculated by the formula of $\mathrm{Q3} + 5 \cdot \mathrm{IQR}$ is higher than the maximum of the underlying values, this threshold is not shown in the figure. Instead, the maximum of the data set is shown in form of a short horizontal line situated right above the box. On the bid side the box plots belonging to various contract sizes look very similar to the one presented in Figure 55. The figures vary solely in the scaling of the y axis and in a couple of dates belonging to the outliers. (Note that for the sake of brevity the box plots belonging to various contract sizes are not shown.)

Figure 56 shows the box plot of the ask values of the virtual price impact function belonging to contract sizes of EUR 5,000. On the figure the lower hinge of the box represents the 75th percentile (Q3) of the data set, which currently equals 0.0532. The upper hinge of the box indicates the 25th percentile (Q1) of the underlying data, which has a value of 0.1269. The box itself contains the middle 50% of the values of the virtual price impact function. The values in the box fall between the borders of the inter-quartile range, that is, between 0.0532 and 0.1269. The line in the box indicates the median value (0.0819) of the data set. Figure 56 also contains the minimum (0.0142, short horizontal line below the box) and the maximum (2.043, the circle situated at the top of the figure) of the ask values of the virtual price impact function belonging to contract sizes of EUR 5,000. The observations marked by circles represent either those outliers that fall below the threshold calculated by the formula of $Q1 - 5 \cdot IQR$ or fall above the threshold calculated by the formula of $Q3 + 5 \cdot IQR$. As the threshold calculated by the formula of $Q1 - 5 \cdot IQR$ is lower than the minimum of the data set, this threshold is not shown in the figure. Similarly to the bid side, on the ask side the box plots belonging to various contract sizes look very similar to the one presented in Figure 56.

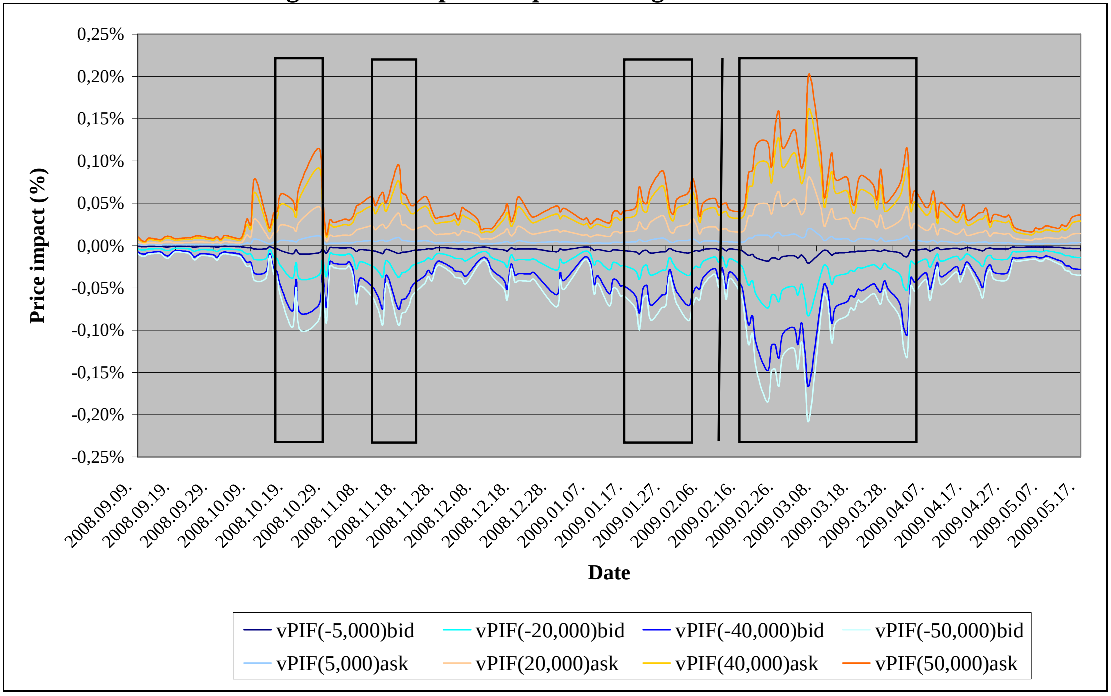
Figure 56: Box plot of the ask values of the vPIF at the order size of EUR 5,000



Source: proprietary

I have identified all the outliers marked by circles on the box plots for each contract size. As a next step I have looked up the dates of these outliers. Turbulent days were defined as days on which the value of the virtual price impact function at each contract size was identified as outlier. As a result, I have identified 52 turbulent days within the period under analysis. The turbulent days fall within one of the above five periods: working days between 17 and 27 October 2008, period between 10 and 20 November 2008, working days between 20 January and 4 February 2009, 12 February 2009, and period between 18 February and 3 April 2009. All of the periods can be found during the time of the global crisis of 2008, which evolved from the subprime crisis of 2007. The price impact values of these outlier periods are shown in Figure 57.

Figure 57: The price impact during turbulent times



Source: proprietary

On the basis of Figure 50 could observe structural breaks in the time series of the virtual price impact function. *Structural breaks exist both on the bid and on the ask side*. To show this I have used a formal statistical test. The Chow-test (Chow, 1960) is one of the most well-known tests to identify structural breaks. With this test the stability of two or three subsamples' model parameter can be analyzed. In this certain case I have split the database into three subsamples, by removing the period between October 2008 and April 2009. During the research 17 October 2008 was identified as the starting date of the crisis. This was the first day in the time series, when I have identified outliers by means of the box plot method at each order size under analysis. 3 April 2009 was considered as the end of the crisis. This was the last day in the time series, when I have identified outliers by means of the box plot method at each contract size under analysis. According to the test on every significance level (5%, 1%) I have found that there is a structural break in the time series. The Quandt-Andrews test (Andrews, 1993) has also indicated the existence of structural breaks. This test shows that there is a structural break in the database, but it is not necessary to give the date of the break in advance. Based on these results I can state that there is a structural break in the database in October 2008 and April 2009.

In general, the absolute values of the price impact function became higher after the turbulence of October-November 2008 and Spring 2009. Thus, a shift can be observed in the time series data under analysis. After the stock exchange's turbulence the values of the virtual price impact functions became on average 76% higher on the bid side. On the ask side the values became 86% higher in the post-crisis period in comparison to the pre-crisis period. This means, that after the crisis the market liquidity has decreased notably and as a consequence market participants had to face a significant increase – nearly twice as much – in the price impact, resulting in a higher transaction cost as well, than before the crisis.

2.4.5. Mean-reverting

On the basis of Figure 50 we might assume that the time series of the virtual price impact function do not follow a random walk. Instead, the values of the virtual price impact function can be characterized by mean reversion. I have tested the intuition of the mean reversion by the augmented Dickey-Fuller (ADF) test. While the simple Dickey-Fuller test cannot be used in case of autocorrelation in the residuals, the augmented Dickey-Fuller (ADF) test can also be used in the presence of autocorrelation (Darvas, 2004). In the ADF tests the lagged level of the series form part of the autoregressive process. The intuition behind the ADF test is that if the series is integrated, then the lagged level of the series will provide no relevant information in predicting the consecutive element of the time series. In that case the alternative hypothesis of having no unit root cannot be rejected. Thus, the time series sample can be characterized by a unit root, which refers to a random walk process.

If the autoregressive process has a unit root, than the asymptotic characteristics of the estimated parameter are different. The characteristics depend on the fact whether the estimated model has a drift and/or a time trend and whether the underlying process is a random walk with lag or without lag. During the research I have used lags of orders according to Schwert (1989) criteria in the ADF tests. Besides, based on my a priori knowledge of the time series of the virtual price impact function, I have assumed that the autoregressive model has a drift, but does not have any deterministic time trend.

On the ask side of the virtual price impact function the values of the ADF test statistics is around -2.65 for every order size, while on the bid side of the PIF it is around -2.6 for every order size. As the obtained ADF test statistics are lower than the reference values in the ADF tables at each confidence level, the null hypothesis of having a unit-root in the time series is rejected. Thus, in the time series of the virtual price impact function no unit-root can be found. The lack of the unit-root refers to the fact that *the values of the virtual price impact functions at a given contract size do not follow a random walk. Instead, they can be characterized by mean reversion.*

2.5. Conclusion

On illiquid markets, the participants have to carry out a dynamic portfolio optimization taking into account time, cost and transaction size. To be able to solve the task of dynamic optimization they have to have an assumption about the underlying stochastic process, namely the process of the transaction cost caused by illiquidity. I have introduced in this chapter how the Budapest Liquidity Measure, provided to the market participants by the Budapest Stock Exchange can contribute to this optimization process, since one can estimate a price impact function from the BLM without knowing the whole order book. I have shown a method with which one can estimate a price impact function fast and easily. After the estimation of the price impact function, I have made a time series analysis of the function. The analysis can help investors to forecast the future transactions' price impact, the transaction cost caused by the lack of liquidity and it can also help to build and optimal trading algorithm. I have based my fifth hypothesis on the time series analysis of the price impact function.

H5: The dynamics of the virtual price impact function can be described by the following:

- **H5/a: symmetry,**
- **H5/b: trend,**
- **H5/c: cycles,**
- **H5/d: mean reverting,**
- **H5/e: shock resistance.**

S1: The value of the descriptive statistics i.e. the mean, the median or the standard deviation have shown a higher value in every case on the bid side of the function than on the ask side. I have explained the phenomenon with the herd effect, namely that the virtual price impact reflects that usually traders buy stocks separately from each other, but selling stocks is often concentrated, for example because of a panic situation.

S2: The time series data of the virtual price impact function do not contain trends, however quarterly cyclicity can be discovered in the data.

S3: During the cycles the price impact values reach their minimum level in the time of quarterly reports, while their maximum values are halfway between two quarterly reports.

S4: By examining outlier data I have identified 52 turbulent days. All these days fall into the period of the 2008 crisis, since they can be found between 17 October 2008 and 9 April 2009.

S5: I have also identified a structural break in the time series with the aid of formalized statistical tests.

S6: There is a significant autocorrelation in the dataset, from which I draw the conclusion that the impact of an incidental shock prevails in the market data for a longer period of time.

S7: When liquidity ceases on one side of the order book, then liquidity will be lower on the other side of the book as well, i.e. the correlation between the buy and sell side price impact is very high.

S8: The vPIF process can be described as a mean reverting process.

Based on the result the acceptation of H5 is the following:

H5/a: I cannot reject the hypothesis that the price impact of the bid and ask side is symmetric.

H5/b: I reject the hypothesis that there is a trend in the vPIF time series.

H5/c: I cannot reject the hypothesis that there are cycles in the vPIF time series.

H5/d: I cannot reject the hypothesis that the vPIF is a mean reverting process.

H5/e: I cannot reject the hypothesis that effect of shocks on the price impact lasts longer.

Summary

The main goal of my research was to promote that (il)liquidity as a concept should be integrated into the daily practice of risk management. Within this, I have focused on three main issues: (1) on the one hand I examined the cross- and horizontal sectional statistical attributes of the BLM time series; (2) on the other hand I have shown how the BLM indicator can be integrated into a VaR-based risk management system; (3) finally I explained the relation between the BLM and the price impact function and I have examined the time series of the price impact function in order to form a view about the attributes of this important risk factor. Chapters II-IV contain my own findings; I hereby present the main statements as follows:

(1) In Chapter II. I gave an exhaustive view on the concept of market liquidity and the group of indicators with which market liquidity is measured by the market participants. I have observed how the average BLM value formed during the examined period; its relationship with the two liquidity indicators which are the most commonly used by market participants; furthermore I have observed the correlation between liquidity and volatility. I have examined whether market participants make a mistake if they – as an applied rule of thumb – only regard the bid-ask spread and turnover data as liquidity indicators. My most important findings were the followings:

<u>Ranking of stocks based on the liquidity indicators:</u>

- In the case of medium liquid and illiquid stocks, bid-ask spread does not give the same ranking as BLM, however the difference is not significant.
- In the case of liquid, medium liquid and illiquid stocks, turnover does not give the same ranking as BLM, however the difference is not significant.
- In a calm period i.e. before and after crisis, ranking differs less from the ranking provided by BLM based on turnover than from the one based on bid-ask spread.
- During a crisis, the ranking based on bid-ask spread differs less from the ranking provided by BLM than from the one based on turnover.
- During the crisis the rank-correlation has decreased between BLM and the spread and between BLM and the turnover.

- In the case of the medium liquid and illiquid stocks it would be worthwhile to take also the BLM into consideration as a liquidity indicator, because in their case the ranking in the wrong order is more significant. In respect of these stocks I have also shown during my analysis that there is a chance that a particular stock is sorted into a wrong liquidity category.

Change of liquidity indicators during crisis:

- In the case of liquid stocks, the values of BLM and the bid-ask spread returned to their pre-crisis level, while in the case of turnover it could only be observed in the case of OTP and MTelekom.
- In the case of medium liquid and illiquid stocks liquidity of some stocks did not return to the pre-crisis level according to the BLM and bid-ask spread, while it did not happen to any stocks according to turnover.

Relation between liquidity indicators:

- The correlation between bid-ask spread and BLM can be regarded as strongly positive, while the correlation of BLM and turnover shows a slightly negative relation.
- The less liquid a stock is, the lower the correlation between the liquidity indicators.
- The change of bid-ask spread has a strong explanatory power about BLM change in the case of a liquid stock, whilst in the case of medium liquid stocks this explanatory power is not significant. In the case of illiquid stocks, bid-ask spread change has very limited explanatory power, which cannot even be considered as significant before the crises.
- The turnover change cannot explain BLM change in the case of liquid and illiquid stocks, whilst it has also only a low explanatory power in the case of a medium liquid one.
- Turnover and liquidity do not co-move intradaily, for instance at the beginning of the day liquidity is low in every case regardless whether the turnover is big or small.
- BLM can be important for those market participants who invest in illiquid stocks or intraday.
- Each stock’s liquidity related to one another can significantly differ in the case of different liquidity indicators.

Relation between liquidity and volatility:

- On the Budapest Stock Exchange it has been justified, that there is a positive correlation between BLM and volatility, namely that the more volatile markets are, the transaction cost caused by the lack of liquidity is higher.
- The less liquid a stock is, the lower the correlation between liquidity and volatility tends to be.
- Before and during the crisis, the correlation between the true range and liquidity was stronger than the one between standard deviation and liquidity. However, after the crisis this has reversed.
- The crisis of 2008 can be regarded as a liquidity crisis based on the liquidity estimated from volatility, i.e. the estimated BLM value is lower than the actual BLM value.
- After the crisis, the estimated BLM value is typically higher than the actual value, i.e. liquidity is higher after the crises than it had been expected. The less liquid a stock is, the typically lower the correlation is between liquidity and volatility.

Therefore, I have pointed out that the rules of thumb applied by market participants do not lead to the appropriate investment decision regarding liquidity in every case. Namely, I have shown that BLM is a liquidity indicator which is able to measure the liquidity of the assets traded on the stock exchange along more dimensions, thus it provides a more reliable view on the current liquidity situation of the market than decisions based only on turnover data or only on bid-ask spread. In the case of medium liquid and illiquid stocks it would be essential to take also BLM into consideration as a liquidity indicator, because in their case it is more significant that they can be sorted into different liquidity categories based on bid-ask spread and on turnover. Furthermore, in the case of these stocks correlation between liquidity indicators cannot be considered as tight, which further decreases in the case of a crisis. Therefore, BLM can be important for investors who trade in illiquid stocks. However, during a crisis it is worthwhile to pay attention to the value-formation of the indicator also in the case of liquid stocks.

Based on the results of the analysis of the relation between the volatility and liquidity it can be said that the crisis of 2007/2008 was a liquidity crisis as well, which means that the cause of the increased implicit cost was not only the increased volatility. My results also prove the statement of Csávás and Erhart (2005), that the decrease of liquidity reflects the increase of unexpected volatility.

(2) In Chapter III. I have presented a theoretical model, in which I have described how the Value at Risk calculation can be supplemented with liquidity risk. In the first half of this part I have given a detailed description about the literature of liquidity adjusted VaR (LAVaR) models, while in the second half I presented my own model which was based on Giot and Gramming’s (2005) and Stange and Kaseres’s (2009b) work. My contribution to their work is that I set up the model on Hungarian database, because nobody had made tests based on it before, and I have calculated the VaR value also for liquid and illiquid stocks in the case of individual stocks and stock portfolios. My most important findings are the followings:

- I determined the net return, namely how return calculation changes if we take into consideration the cost that occur because the lack of liquidity. I have determined both for the individual stocks and for the volume and value weighted portfolios.
- Taking liquidity into consideration means a significant risk increase even in the case of the most liquid stocks both on the level of individual stocks and portfolios. Therefore it is not advisable to ignore this.
- In the case of portfolios, liquidity risk can be decreased by diversification; therefore it is worthwhile to hold various stocks in a portfolio, because thus not only the price risk, but also the liquidity risk decreases.

BLM and the method presented with the aid of it provide a simple and quick way to display liquidity in the capital requirement. Paying attention to the deficiencies and calculation problems of the index, the findings should be handled with precaution, but the presented model can appropriately reflect the essential empirical observations (e.g. OTP is the most liquid stock), thus in every case I recommend its integration into risk management systems.

(3) In Chapter IV. I presented how to estimate a virtual price impact function with the help of the BLM indicator, which nobody had done before me, i.e. the literature is typically concerned with the estimation and modeling of the empirical price impact function. As the estimation of the empirical price impact function is based on the average price impact of a long period, thus it cannot be a basis of a time series analysis. Hence I examined the price impact functions from a different approach, because in my opinion during trading it is important to know how the price impact evolves in time, since traders will base their trading strategy on it. The knowledge of the behavior of the price impact function in time helps market actors with timing their orders. When market actors decide whether to postpone a transaction in order to induce a lower price-shift effect on the market, then they have to have a notion on how the price impact function forms over time. However, the time series analysis of the price impact can only be carried out on the virtual price impact function, because in this case a sufficient amount of data is available. For this reason, after the estimation of the price impact function, I have made a time series analysis on the function, which had nobody had done in the literature before. My most important findings were the followings:

- Estimation of a virtual price impact function from the BLM database.
- The value of the descriptive statistics i.e. the mean, the median or the standard deviation have shown a higher value in every case on the bid side of the function than on the ask side. I have explained the phenomenon with the herd effect, namely that the virtual price impact reflects that usually traders buy stocks separately from each other, but selling stocks is often concentrated, for example because of a panic situation.
- The time series data of the virtual price impact function do not contain trends, however quarterly cyclicity can be discovered in the data.
- During the cycles the price impact values reach their minimum level in the time of quarterly reports, while their maximum values are halfway between two quarterly reports.
- By examining outlier data I have identified 52 turbulent days. All these days fall into the period of the 2008 crisis, since they can be found between 17 October 2008 and 9 April 2009.

- I have also identified a structural break in the time series with the aid of formalized statistical tests.
- There is a significant autocorrelation in the dataset, from which I draw the conclusion that the impact of an incidental shock prevails in the market data for a longer period of time.
- When liquidity ceases on one side of the order book, then liquidity will be lower on the other side of the book as well, i.e. the correlation between the buy and sell side price impact is very high.
- The vPIF process can be described as a mean reverting process. The time series data of the virtual price impact function do not contain trends, but quarter-year cyclicity can be discovered in the data.

The topic of my dissertation has evolved from the research activities I made in the past and from the series of interview series I did together with a few of my colleagues. My dissertation shows that the market participants use simple rules of thumb in order to be able to handle market liquidity easily, and they use simple indicators to measure its value. These indicators cannot capture market liquidity in full. In my dissertation I have used the Budapest Liquidity Measure – provided to me by the Budapest Stock Exchange – to show how this indicator can supplement the information other liquidity indicators provide about the liquidity of the market. Moreover I have shown methods that can reduce the liquidity risk market participants have to face, and methods that can help decision making. I think my achievement in addition to the my previous statements is, that I discuss the following in my dissertation in great detail: the importance of market liquidity; methods of liquidity risk management that already exist on the market, and also new, more complex ones; furthermore the possible research topics that could be undertaken in the future.

For further research directions a number of proposals have emerged during my examinations, of which I intend to highlight the most important ones:

- Concerning the set-up of the LAVaR model an important assumption was that the BLM(q) function can be estimated by a line. While in the case of daily data we can state that the BLM(q) function can be well approached by a line, on the other hand it is not valid for the intraday data anymore. Namely, for the modeling intradaily, we need to estimate the shape of the BLM(q) function for every moment of time, which is a complex task. The intraday BLM(q) function can take on any shape, it can either be convex, concave or even a line. Since the daily BLM-value is calculated as the average of the intraday values, as the consequence of this averaging the incidentally outlier values have sleeked into the average, which resulted in the fact that I could approach the daily BLM(q) function well with a line. For the estimation of the shape of intraday BLM(q) functions, the methods applied for the estimation of the yield curve can possibly provide a solution during subsequent research.

- The determination of the LAVaR values based on intraday data referring to the portfolio is a further direction, which can be an essential issue to e.g. the portfolio managers. However, the modeling of this is a complex task, since the BLM(q) function has to be estimated every second, which can have very different patterns during the day.
- During the LAVaR modeling a further assumption was that the order book is symmetric on the bid and the ask sides. In the future, BLM value could be divided directly into its components – the bid-ask spread and the bid and ask side adverse price movement – and after this separation the bid and ask side LAVaR values could be estimated.
- Concerning price impact functions as well, in the future it may be interesting to examine how this function develops intradaily.
- It would also be worthwhile to examine how the empirical and the virtual price impact functions are related to each other. With the comparison of the two functions it could be determined whether there is a need to estimate the empirical function at all, or it is sufficient to know the virtual price impact function during investment decisions. However, it makes it difficult to compare that the virtual price impact function can be estimated even for every moment, whilst the empirical price impact function can only be determined based on a relatively longer period, e.g. for a month based on real trades. Moreover, the empirical price impact function is not eligible to make a time series analysis thereon, thus it can only play a less important role in market actors' investment decisions than the virtual price impact function.
- Lastly it is worth to mention that it would be very important to use the nature of liquidity as transaction cost – which is quantified by BLM on the stock

exchange – for a comparison of each market. Comparing my findings with the results of other markets – in the absence of appropriate data – is for the time being not possible. Namely, the estimation of trade transaction costs presumes the knowledge of databases hardly or not at all available. Actually, thus I can only hope that in the future more and more databases and studies which allow comparative analysis will be at researchers' and market actors' disposal.

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