Doctoral School of Economics,
Business and Informatics

THESIS ON

Dóra Gréta Petróczy

Fairness and ranking:
Applications from the fields of economics and sports

Ph.D. dissertation

Supervisor:

László Csató, Ph.D.
associate professor

Budapest, 2022

THESIS ON

Dóra Gréta Petróczy

Fairness and ranking:
Applications from the fields of economics and sports

Ph.D. dissertation

Supervisor:

László Csató, Ph.D.
associate professor

Contents

List of Tables

List of Figures

Chapter 1

Introduction

This doctoral thesis addresses some problems of fairness and ranking in economics and sport. One of the most important questions of our time is what we call fairness. The dissertation deals with four particular issues on this ropic.

Structure of the thesis

In Chapter 2, we investigate the impact of possible further exits from the European Union after Brexit on the Council of the European Union using two well-known measures of power, the Shapley–Shubik and Banzhaf indices. Although the impact of Brexit has been quantified in a number of previous papers (Göllner, 2017; Grech, 2021; Kóczy, 2021; Szczypińska, 2018), further exits have not been addressed so far. We find the same pattern for the exit of all countries: population size and changes in decision-making power are closely correlated, with small countries showing the largest increases in the power index.

The chapter is based on Petróczy et al. (2019) and Petróczy et al. (2022). Our contribution resides in:

- performing the calculations (with Rogers Mark Francis); and
- participating in the editing of the articles, preparing the figures.

In Chapter 3, we propose a country ranking method that does not require an arbitrary choice of weights. Our approach is based on revealed preferences, with the underlying assumption that people migrate from their home country to countries they consider better. The simplicity of our approach makes it an alternative to, though not a complete substitute for, various composite indices. The results of this chapter are presented in Petróczy (2020) and Petróczy (2021a).

In Chapter 4, we present an alternative method of awarding prizes to Formula One teams. The approach uses pairwise comparisons and allows allocation based on a single parameter. This chapter is based on Petróczy (2021b) and Petróczy and Csató (2021). Our contribution is:

- to collect the data and perform the calculations;
- to participate in the editing of the co-authored paper.

In Chapter 5, we discuss the order of kicks in a penalty shootout in soccer. Teams of equal ability should have the same probability of winning, but the penalty kick rule may violate this requirement. Therefore, seven alternative mechanisms are compared using a mathematical model and empirical grounds. The results are presented in Csató and Petróczy (2019) and in the working paper Csató and Petróczy (2022). Our contribution is:

- the programming of ABBA|BAAB in model M1 and all rules in models M2 and M3;
- the example given in Table 5.1;
- participation in the editing of the articles.

Thanks are due to *László Csató*, father of my supervisor, for help in programming.

Chapter 2

Analysing the effects of exits from the European Union

The withdrawal of the United Kingdom (UK) from the European Union (EU), the Brexit, and its possible effects have become the subject of political debate in several countries like the Czech Republic, France, or Greece since the membership referendum in 2016. In 2021, the issue of a possible Polexit (Poland's exit) has emerged. Although numerous political and economic effects of an exit from the European Union might be worth inspecting, now we look at one particular aspect: how the power distribution changes in the Council of the European Union. The voting system of the Council of the European Union has long been the subject of academic interest.

Brams and Affuso (1976) have used the example of the Council to uncover a real-life occurrence of the new member paradox: Luxembourg has gained more voting power with the joining of Denmark, Ireland, and the United Kingdom in 1973. In the past, the voting weights have changed several times, most recently in 2014.

Göllner (2017); Grech (2021); Kóczy (2021) and Szczypińska (2018) have shown independently that Brexit mainly benefits large countries. Bertini et al. (2019) have examined the issue in the case of the European Parliament. We first try to explore whether the same result would hold if another country leaves after the UK. Secondly, we want to answer the question: what would have been the effect of Brexit if Croatia had not joined the EU?

The Council of the European Union, often referred to as the Council of Ministers, is an institution that represents the governments of the member states. It approves EU law and synchronizes the policy of the EU. Along with the European Parliament, the Council of the European Union is the main decision-making body of the EU. Every member state is represented by an individual. The difference in size among the member states appears in a weighted qualified majority voting system. Under the Treaty of Lisbon, voting is successful if

1. at least 55% of the member states (member quota), which
2. represent at least 65% of the habitants (population quota)

support the decision. Furthermore, any blocking minority should include at least four member states (blocking minority rule). Such creation of the weights enables us to calculate how the power distribution changes if any country leaves the European Union.

Several studies have addressed how voting power affects the overall likelihood of decision-making (Felsenthal and Machover, 1997, 2001). Therefore, it is an important question to measure how much power the countries have in the Council of the European Union.

Concerning our methodology, two well-known power indices are used: (1) the Shapley–Shubik index (Shapley and Shubik, 1954); and (2) the Banzhaf index (Banzhaf, 1965; Coleman, 1971; Penrose, 1946). These measures reflect the probabilities of the players to be instrumental in making decisions. As far as votes on the spending of the budget are concerned, the index value of a player reflects the probability of spending one (or a million) euro in the interest of that player. For several cases of departure, we show the change made by an exit until 2030, which can be called a 'farsighted' sense.

We find a pattern connected to a change in the number of states required to meet the 55% threshold. An exit that changes the absolute value of the member quota (for example, from 15 to 14) benefits the large, while an exit that does not cause such a change benefits the small countries. These results may suggest that a renegotiation of weights may become relevant.

Our results point in the direction that if the UK had left the European Union before the entry of Croatia, the effect would have been reversed as it would have favored the power of the small countries. According to the calculations, the exit of only one country from the EU 27 would be supported by the qualified majority of the Council, Poland.

2.1 Methodology

It is popular to study voting situations as simple cooperative games, where the players are the voters. The value of any coalition (a subset of the player set) is 1 if it can decide a question, or 0 if not. The most common measures of power in voting games are the *Shapley–Shubik* and the *Banzhaf indices*. They are used extensively for determining power in the Council of the European Union (Felsenthal and Machover, 2001; Herne and Nurmi, 1993; Kóczy, 2012; Widgrén, 1994).

Let N denote the set of players and $S \subseteq N$ be an arbitrary subset of N. We use the corresponding lower-case letters to denote the cardinality of sets, so that $s = |S|$ and $n = |N|$.

Definition 2.1. *Simple (voting) game*: A game $v : 2^N \to R$ is a simple game if it satisfies the relation

$$v(S) \in \{0, 1\} \text{ for all } S \subseteq N.$$

Coalitions S such that $v(S) = 1$ are called winning coalitions, while coalitions S with $v(S) = 0$ are the losing ones.

Definition 2.2. *Weighted voting game*: Let v be a game on the set of players N, which is defined by the tuple $(\mathbf{w} \in R_n^+; q \in R_+)$ as follows:

$$v(S) = \begin{cases} 1 & \text{if } \sum_{j \in S} w_j \geq q \\ 0 & \text{otherwise.} \end{cases}$$

This simple game represented by $(N, \mathbf{w}, q)$ is known as a *weighted voting game*.

The Shapley–Shubik index is an application of the Shapley value (Shapley, 1953) for voting games. Its principle can be described as follows: voters arrive in a random order, and when a coalition becomes winning, the full credit is given to the *pivotal* player arriving last. A player's power is specified by the proportion of orders in which it plays this role.

Definition 2.3. (*Shapley–Shubik index*) For any simple game v, the *Shapley–Shubik index* of player i is as follows:

$$\varphi_i(N, v) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} \left(v\left(S \cup \{i\}\right) - v(S) \right).$$

The Banzhaf index, which is the normalized Banzhaf value (Banzhaf, 1965; Coleman, 1971; Penrose, 1946), uses a different approach. A player is called *critical* if it can turn a winning coalition into a losing one. The index shows what is the probability that a player influences a decision.

Definition 2.4. Player i's *Banzhaf value* is:

$$\sum_{S \subseteq N \setminus \{i\}} \frac{1}{2^{n-1}} \left(v\left(S \cup \{i\}\right) - v(S) \right) = \frac{\eta_i(N, v)}{2^{n-1}},$$

where $\eta_i(v)$ is player i's Banzhaf score, the number of coalitions where i is critical.

Usually, its normalized value is reported as the measure of voting power.

Definition 2.5. The *Banzhaf index* is the normalized Banzhaf score:

$$\beta_i(N,v) = \frac{\eta_i(N,v)}{\sum_{j\in N}\eta_j(N,v)}.$$

Both indices somehow show the voter's expected relative share of the total payoff. When a country leaves, its payment to the EU budget is assumed to cease, therefore, the remaining countries do not share the same prize as before.[1] Taking this into account, we correct the power index by the following fraction:

$$\frac{\text{Original budget} - \text{the contribution of the leaving country}}{\text{Original budget}}. \tag{2.1}$$

We compute for every country and each exit the adjusted power index as a percentage of the pre-exit power index.

Adjusted power indices have not been normalized for the comparison. Thus, the change in the power index reflects two effects, a shift in power on the one hand and a reduction in the budget on the other hand.

2.2 Results

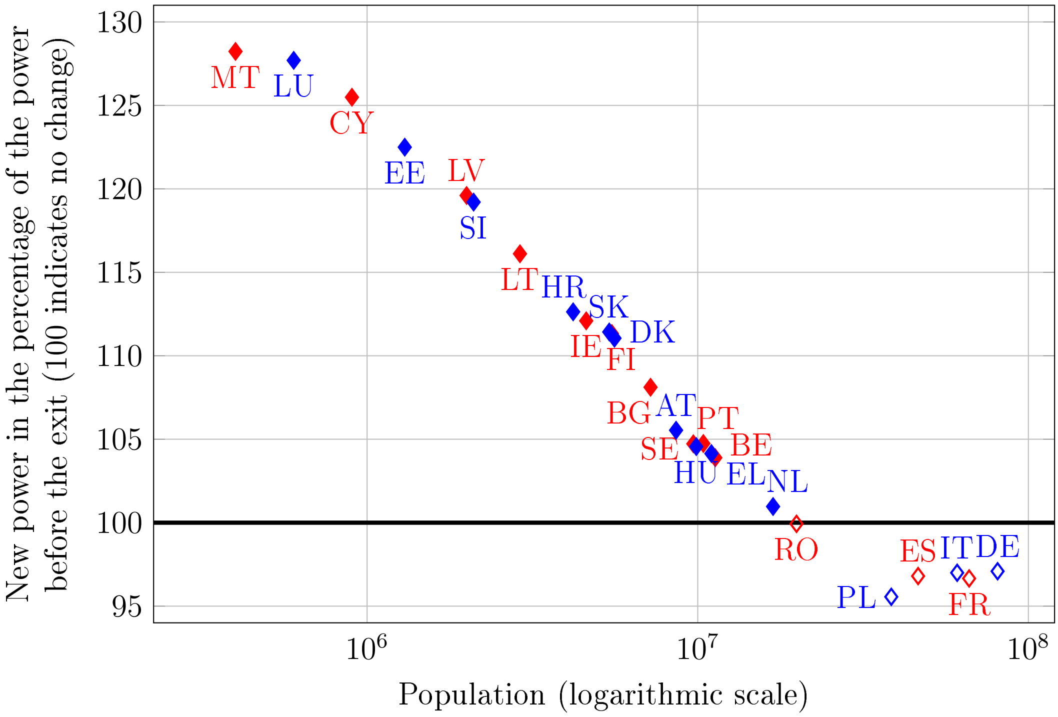
In this section, our findings are presented. We use population projections for 2015 and 2030 from Eurostat (Eurostat, 2017), and budget contribution data from the European Parliament (European Parliament, 2014). The software IOP-Indices of Power (Bräuninger and König, 2005) is used to calculate the Shapley–Shubik and Banzhaf indices.

Kóczy (2021) has shown that if the United Kingdom leaves the European Union, which has 28 member states, the smallest member states' power indices decrease. However, a further question arises: what happens if another member state leaves the EU? Here, we discuss the effects of the Czech Republic (Czexit) and Germany leaving the EU after Brexit. Secondly, building on our previous finding, we inspect what the effect of Brexit would have been on the power distribution of the EU had the United Kingdom left it before Croatia entered. Is Brexit in this sense a belated threat? Our results show that it is.

In the following, we will call a country large or small depending on its population size. We observe a pattern, which connects the change in the member state quota to a change in the power distribution: when the departure modifies this threshold, the power indices of the large countries increase. However, when the departure does not evoke such a change, the power indices of small countries increase.

[1] It is a simplification, as some non-EU member countries, like Norway, also contribute to the EU budget in a certain sense.

Figure 2.1: Effect of Czexit with populations for 2015, adjusted Shapley–Shubik index



The impact of additional departures to Brexit

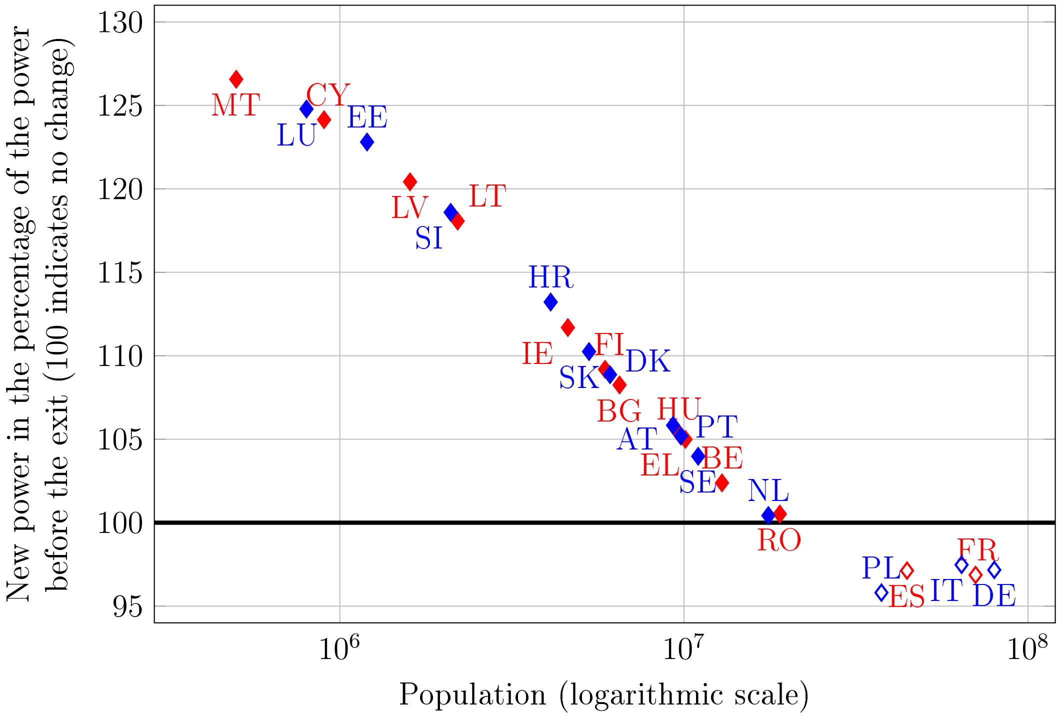
In the computations that investigate the results of an *additional departure to Brexit*, we base our calculations on the 27-member Union without the UK. As mentioned in before, it is also considered that the exit of a country decreases the budget. The example of the Czech Republic is presented first because the EU-skeptical sentiment has recently become stronger in this country. The budget correction ratio is 0.989 according to formula (2.1). Figure 2.1 shows the budget-adjusted change in power indices due to Czexit as a function of the population.

We find that in the case of Czexit, the power indices of small countries increase, and the power indices of large countries such as France, Germany, Italy, Poland, and Spain slightly decrease. Consequently, the main winners from Czexit are Cyprus, Estonia, Luxembourg, and Malta.

The same can be said if one investigates Czexit in a farsighted sense, meaning to repeat the analysis with population predictions for 2030. The only country whose power index change differs is Romania: from a slight decrease (see Figure 2.1), its power modestly increases (see Figure 2.2).

We get similar results for other departures from a 27-member EU, the power indices of small countries increase significantly. What has created more variation in these cases is the contribution of the particular country to the EU budget. To illustrate this point, let us look at the exit of Germany.

Figure 2.2: Effect of Czexit with population projections for 2030, adjusted Shapley–Shubik index



In the case of Germany's exit (Figure 2.3), the adjusted Shapley–Shubik indices of the smallest countries and Poland increase, while all the other countries lose power. This is because countries with large populations are also the ones that contribute the most, so the budget loss exceeds the power gains caused by the departure of Germany. The correction ratio (2.1) is 0.711.

The results concerning Poland are especially interesting. If one of the four large countries (Germany, France, Italy, or Spain) leaves, Poland is much better off than Romania or Spain, which are the closest countries in the size of the population. In all these four cases, its Shapley–Shubik index increases despite the power of the other remaining large countries decreases.

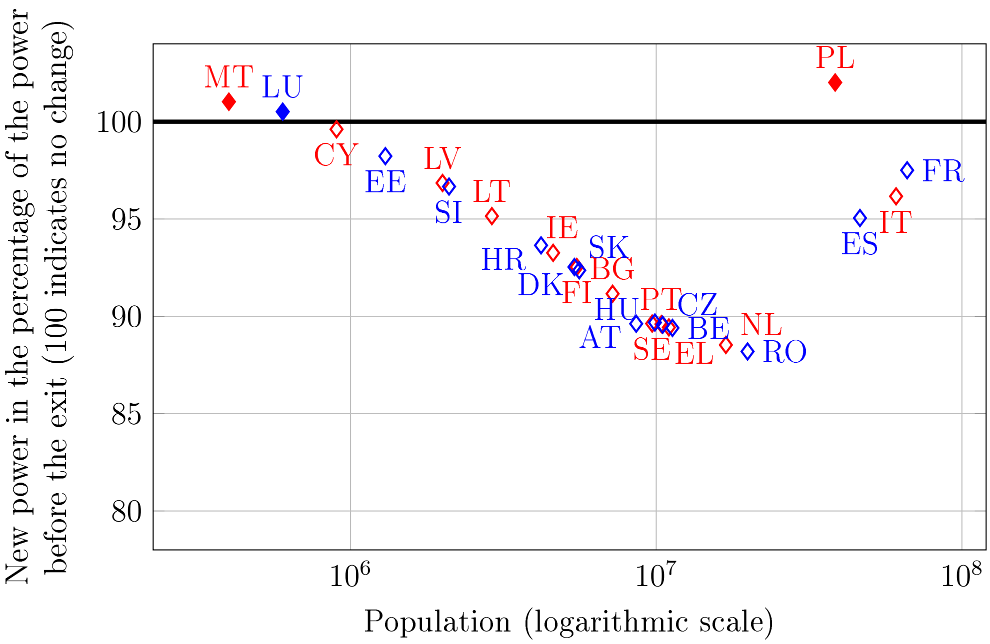
The simulations have been repeated with the other popular power measure, the Banzhaf index. We get the same results, the power of small countries increases. The most considerable difference is in the case of Germany. As one can see in Figure 2.3, with the use of the Banzhaf index, all countries — including Poland — lose power. As there is no significant difference, the Shapley–Shubik index is applied in the following.

Calculations for another country leaving the 26-member EU, for instance, if the Czech Republic leaves after Germany, show a similar pattern to Brexit (Figure 2.4). This can be elucidated by the fact that as the number of member states decreases from 26 to 25, the Council of the European Union's threshold for the number of supporting member states (determined by the member quota) decreases from 15 to

14. In this case, small countries would lose while the power of the large countries would increase.

Figure 2.3: Effect of the German exit with populations for 2015

(a) Adjusted Shapley–Shubik index



(b) Adjusted Banzhaf index

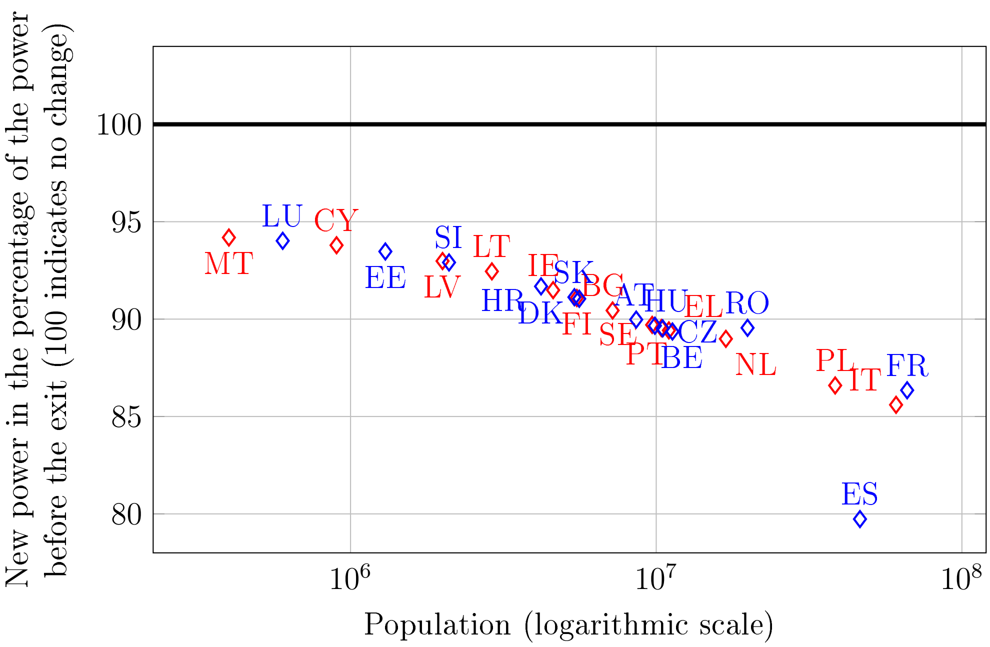
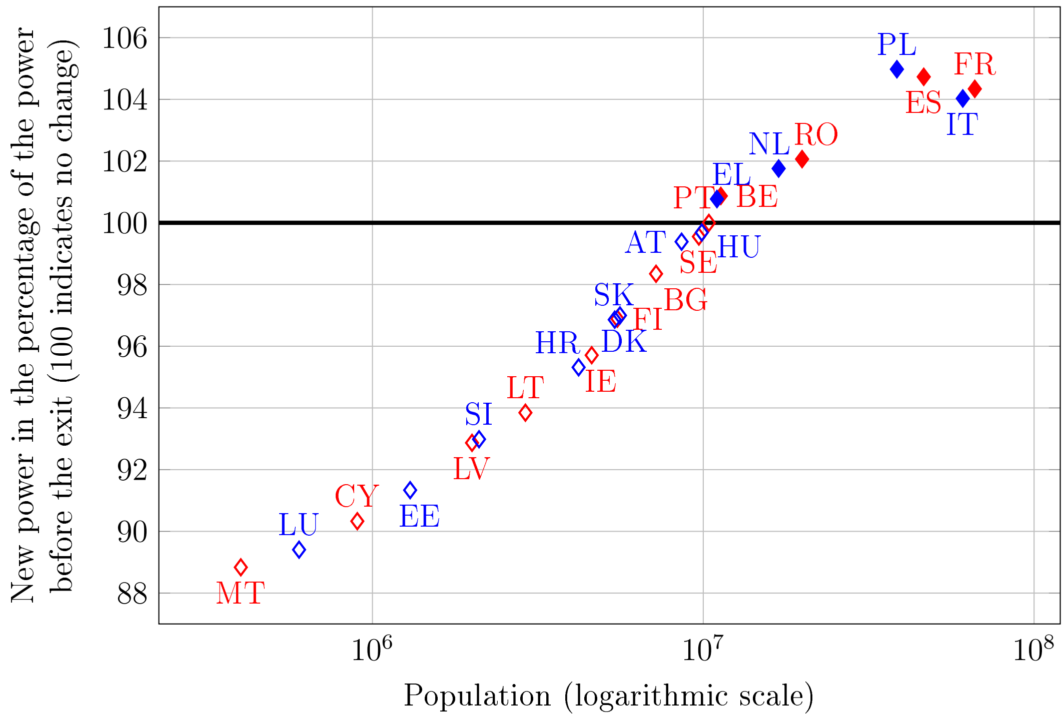


Figure 2.4: Change in power due to Czexit in the 26-member EU (after the exit of UK and Germany) with populations for 2015, adjusted Shapley–Shubik index



The effect of Brexit before the accession of Croatia

Since our findings on an additional departure show an impact that is the inverse of Brexit's (Kóczy, 2021), Brexit might have had a different impact before the accession of Croatia compared to the exit from the 28-member EU.

This has significance because if Brexit had decreased the power of large countries such as France and Germany, the impact of the potential Brexit would have been calculated differently by these states that usually dominate the policy of the EU: Brexit would have been a greater risk for them. In other words, if Brexit would have had the reverse impact before Croatia joined, it could be seen as a belated threat.

We find that Brexit before the accession of Croatia would have favored smaller countries (Figure 2.5). In this case, the power of larger countries slightly increased, but not nearly as much as what Kóczy (2021) found after the enlargement of EU. The results are similar not only for Brexit but for the case of an exit of any other member state from the EU without Croatia.

2.3 Conclusions

Note that an additional departure to Brexit has an inverted impact compared to Brexit's impact from the 28-member EU, but it is similar to the potential effect of Brexit if it would have happened before Croatia's membership. Results for a depar-

Figure 2.5: Effect of Brexit before Croatia joined the EU
with populations for 2015, adjusted Shapley–Shubik index

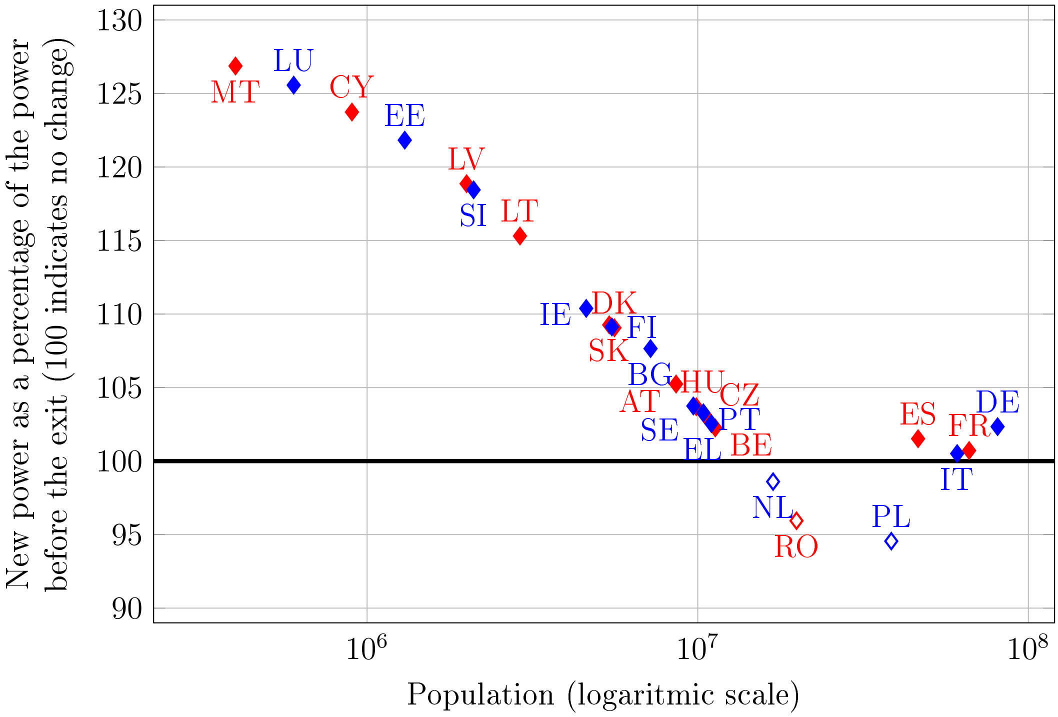
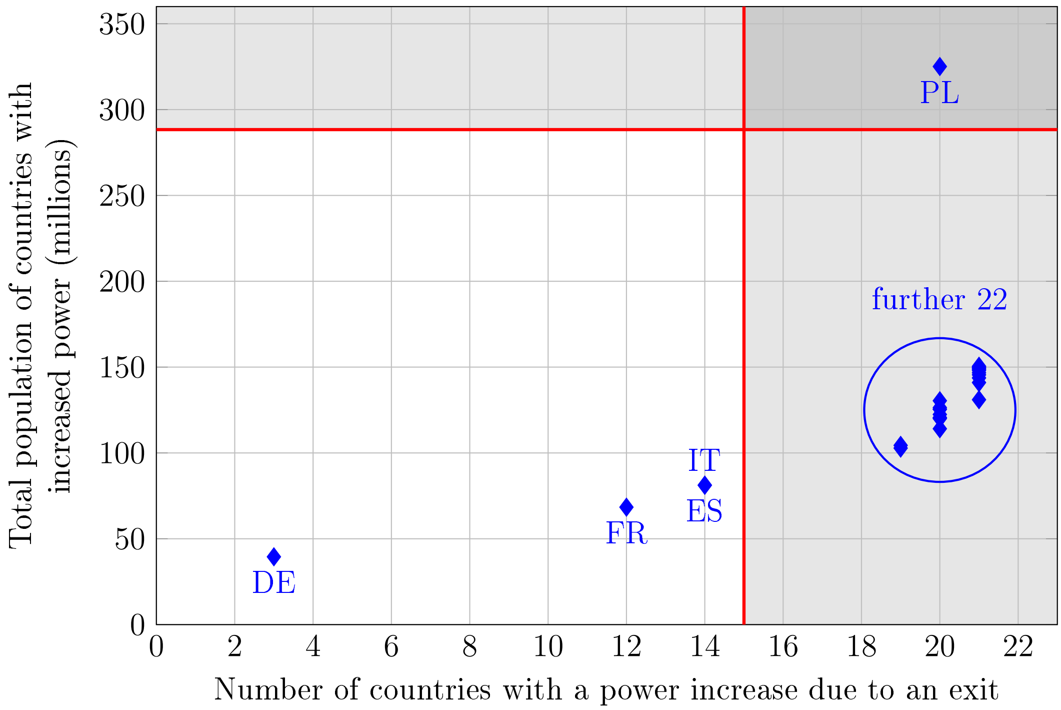


Figure 2.6: Effect of a departure from the EU after Brexit with populations for 2015, adjusted Shapley–Shubik index



ture from the hypothetical 26-member European Union have a strong resemblance to the consequences of Brexit. The inverted impact of an additional departure to Brexit is due to the fact that 15 countries are necessary to make the vote successful in the case of both 26 and 27 members. However, the population threshold decreases after an additional exit.

The voting rule states two main requirements: the support of a given number of countries and a certain percentage of the population. A country will turn a losing coalition into a winning one if (a) the coalition just misses a member state to pass the threshold; and/or (b) if the coalition has the required participation, but the supporting countries are too small to reach the population quota.

With Czexit after Brexit, the population threshold decreases while the member state threshold remains the same, so coalitions with smaller countries become winning, which shifts power from the large to the small member states. This pattern is quite prevalent, we find similar results using population projections for 2030 (Figure 2.2).

It seems to be a pattern that an exit triggering a decrease in the member quota benefits more the large, while an exit not triggering such a change benefits the small member states. Since the adjustment is only a vertical downward shift, the direction of the results, meaning which countries are the largest beneficiaries, remains unchanged even for unadjusted indices.

Any exit induces three types of effects: (1) The increase/decrease in the relative share of the (rounded) numerical quota may increase/decrease the equality among countries of different sizes; (2) in the presence of the double quota, there is a complementarity/substitution effect such that an exit benefits similar countries, finally (3) there is a complex packaging issue with an ambiguous effect. Any of these three can dominate in a given numerical problem.

In the case of 27 member states, voting is successful if at least 15 countries, having together at least a population of 288 million vote in favor. We have examined the number of countries whose power increases if a particular country leaves, which can be considered as a yes vote for the exit of the departing country. Figure 2.6 presents the number of countries and their total population with an increasing power. Most of the countries would get a positive vote for leaving from 20 or 21 countries, but without the required population. However, in the case of Poland, both thresholds are met, because the power of small and large countries increases, and merely some medium countries (Belgium, Czech Republic, Greece, the Netherlands, Portugal, Romania) lose power. If we ignore the correction for the budget change, the result is unanimous: all countries would increase their influence in the Council in the case of Polexit.

Chapter 3

An alternative quality of life ranking on the basis of remittances

Researchers have maintained an interest in measuring the quality of life in various geographic areas since the 1930s, when President Hoover's Committee on Social Trends issued its report "Recent Social Trends in the US" (Wish, 1986). Country rankings seem to be increasingly popular in economics and can often have a considerable impact on politicians and government strategies. In recent decades, scientists have recommended several alternative approaches to define and measure the quality of life, see Diener and Suh (1997) for a summary. However, most of them are composite indices, a construction that remains highly controversial due to the arbitrary selection of criteria and ad hoc choice of component weights (Ravallion, 2012). While robustness check may provide a kind of remedy (Foster et al., 2013), there exists an alternative solution, that is, to apply a parameter-free algorithm on an appropriate dataset.

It is widely argued that people and their capabilities should be the ultimate criteria for assessing a country's development, not economic indicators alone (Sen, 1985, 1992). One way to measure the perceptions of people is to observe their decisions on important questions of life such as working abroad. Although migration emanates from the desire to improve livelihood, it requires a certain development level, so it is not the poorest who migrate (De Haas, 2005). Besides economic migration, another approach prevalent in the literature is lifestyle migration (Benson and O'Reilly, 2009, 2016; Saar and Saar, 2020). Evidence suggests that the two motivations cannot be wholly separated (Bobek, 2020).

To summarise, people often choose with their foot between countries. Someone migrates from country A to country B if the latter is judged to be a better place. Data on international migration is highly unreliable, most countries do not collect data on their leaving citizens. Because of their importance to labour-exporting countries, the most visible aspect of international migration is the amount of remittances received (Adams, 2003). A remittance is a transfer of money by a foreign worker to an individual in their home country. It constitutes a significant part of international capital flows, especially for labour-exporting countries. That is why we use these data to proxy the choice of people between countries. Using personal remittances to rank countries is not unique (see, for example, IndexMundi).

Our dataset contains estimates of bilateral remittances by the World Bank, based on migrant stocks, host country incomes, and origin country incomes (World Bank, 2017). They are not officially reported data since bilateral remittance flows are not registered appropriately. The estimation uses the methodology of Ratha and Shaw (2007) who have suggested a simple formula to allocate the recorded remittances received by each country to the source countries. This applies a remittance function assuming that the amount sent by an average worker increases with the migrant's income but at a decreasing rate. Furthermore, in the case of migration to a country where the per capita income is lower than in the host country, the transfer is supposed to be at least as much as the per capita income of the origin country.

Our work contributes to the literature on alternative quality of life country indexes, surveyed in Somarriba and Pena (2009), but offers a new approach using pairwise comparisons.

3.1 Methodology

We consider one unit of money transferred from a country to another country such that the former is preferred over the latter by one voter, and use techniques from social choice theory to evaluate these "votes". Let us introduce the matrix $\mathbf{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$ of bilateral remittances among n countries, where a_{ij} is the sum of transfers from country i to country j in the given period. It immediately determines the skew-symmetric *results matrix* $\mathbf{R} = \mathbf{A} - \mathbf{A}^\top$ and the symmetric *matches matrix* $\mathbf{M} = \mathbf{A} + \mathbf{A}^\top$.

Perhaps the simplest measure is to calculate the net remittances (the difference of total outflow and total inflow), denoted by $s_i = \sum_{j=1}^{n} r_{ij}$ for each country i. Dividing these amounts by the total remittance flow (the sum of total outflow and total inflow) $\sum_{j=1}^{n} m_{ij}$ of the country leads to p_i.

Finally, the least squares method adjusts net remittances by taking the whole network of flows into account. The least squares weight q_i of country i can be obtained as the solution of the following optimization problem:

$$\min_{q \in \mathbb{R}^n} \sum_{1 \leq i,j \leq n} m_{ij} \left(\frac{r_{ij}}{m_{ij}} - q_i + q_j \right)^2 . \tag{3.1}$$

The first order conditions of optimality lead to a linear equation for each country

i:

$$\left(\sum_{j=1}^{n} m_{ij}\right) q_i - \sum_{j=1}^{n} m_{ij} q_j = s_i = \sum_{j=1}^{n} r_{ij}. \tag{3.2}$$

After normalizing the weights by $\sum_{i=1}^{n} q_i = 0$, the solution of this system becomes unique if the countries are connected at least indirectly by transfers, that is, the multigraph of bilateral remittances is connected (Čaklović and Kurdija, 2017). Our dataset has satisfied this condition in every year.

All of the weight vectors above determine a ranking $\succeq$ of the countries. For example, the ranking from the least squares method is given by $q_i \geq q_j \iff i \succeq j$.

In order to highlight the characteristics of the three methods, two axiomatic properties have been considered. First, the ranking is required to be *invariant to country sizes*, that is, countries i and j should get the same rank if country j has a fixed proportion of transfers to and from every third country as country i. This axiom has been introduced in Csató and Tóth (2020) under the name *size invariance*.

Second, it should be *independent of bridge country*, namely, in a hypothetical world consisting of two set of countries connected only by a particular country called bridge country, the relative rankings within each set of countries should be independent of the remittances among the countries in the other set (González-Díaz et al., 2014).

Ranking by net remittances violates both properties. The ranking derived from vector $\mathbf{p}$ is invariant to country sizes but does not meet bridge country independence. The least squares method satisfies both axioms, therefore, we suggest applying this procedure.

Note that the least squares method is equivalent to the Potential Method (Čaklović and Kurdija, 2017), to the EKS (Éltető–Köves–Szulc) method used for international price comparisons by the OECD (Éltető and Köves, 1964; Szulc, 1964), and to the Logarithmic Least Squares Method defined in the framework of (incomplete) multiplicative pairwise comparison matrices (Bozóki et al., 2010).

3.2 Results

In the following, the results of our calculations with the methodology suggested in Section 3.1 are presented.

3.2.1 The ranking in 2015

Due to the lack of reliable data for some developing countries, we have restricted the investigation to Europe. To select European countries, the United Nations geoscheme has been used. However, we have omitted Andorra, Liechtenstein,

Table 3.1: Ranking of European countries on the basis of remittances in 2015

Country	$\mathbf{s}(\mathbf{A})$	$\mathbf{p}(\mathbf{A})$	$\mathbf{q}(\mathbf{A})$	Country	$\mathbf{s}(\mathbf{A})$	$\mathbf{p}(\mathbf{A})$	$\mathbf{q}(\mathbf{A})$
AL	25	37	35	IS	18	21	19
AT	10	13	16	IT	5	9	9
BA	32	41	41	LT	28	39	31
BE	39	24	22	LU	23	19	21
BG	30	40	39	LV	27	34	28
BY	14	14	13	MD	29	35	33
CH	4	3	2	ME	22	38	40
CY	13	8	6	MK	20	28	30
CZ	24	20	25	MT	19	27	23
DE	3	11	12	NL	7	2	5
DK	12	12	10	NO	8	4	4
EE	21	29	27	PL	40	33	29
ES	6	10	11	PT	34	26	24
FI	16	15	14	RO	36	36	34
FR	37	18	17	RS	35	30	37
GB	1	1	1	RU	2	7	8
GR	11	6	7	SE	15	16	15
HR	26	25	38	SI	17	17	26
HU	38	32	32	SK	31	31	36
IE	9	5	3	UA	33	22	20
				Other	41	23	18

Monaco, San Marino, and Vatican City because lack of data, but we have enrolled Cyprus due to its membership of the European Union. Table 3.1 presents the ranking of the countries with the three methods based on the transfers in 2015 such that all non-European countries are regarded as one entity. ISO 3166-1 standard alpha-2 codes are used to abbreviate countries. Lighter colour indicates a worse rank.

The highest difference between the ranking from $\mathbf{s}$ and $\mathbf{q}$ is in the case of France. Net remittances place it to the 37th position, while the least squares method gives the 17th rank. The reason is the size effect: France is one of the largest countries in Europe, hence it is natural that both inflow and outflow are huge (more than 20 billion USD), which implicates that the difference is also great (-2482 million USD). In this case, even Albania overtakes France with net remittances of -852 million USD, but with almost five times higher inflow than outflow.

It can be realized from formula (3.2) that q_i is close to the size-invariant ratio $p_i = s_i / (\sum_{j=1}^{n} m_{ij})$ if the weights of the countries connected to country i by remittances are close to the average weight of 0. On the other hand, q_i becomes higher (lower) than this ratio if country i is mainly connected to higher (lower) ranked countries by the transfers. This adjustment is the most substantial in the case of Croatia, Lithuania, and Slovenia. The main destinations for Croatia are Germany and Serbia, while it receives workers from Bosnia and Herzegovina, Serbia, and Slovenia, therefore Croatia is mainly connected to lower ranked countries. On the other hand, Lithuania (LT) is predominantly connected to some higher ranked countries (the United Kingdom, Russia), which implicates its better rank with q_i.

3.2.2 The dynamics of country rankings in recent years

According to Table 3.2, the least squares ranking is relatively robust across the years and does not yield many unexpected results. For example, the four members of the Visegrád Group are around the 30th place, only the Czech Republic shows some improvement in the years 2013 and 2014.

However, there are some counterintuitive findings. Data problems are responsible for the decline in the performance of Iceland and Sweden. The top position of Cyprus can be probably explained by the significant role of its banks in international finance. The United Kingdom leads the ranking in certain years partly due to its liberal migration policy. Russia gains from retaining connections of the Soviet era, as well as from the huge regional inequalities caused by the agglomerations of Moscow and Saint Petersburg.

3.2.3 Comparison with the HDI

The United Nation's *Human Development Index* (HDI) is perhaps the best known and probably the most researched measure of human development. It is a composite index of three dimensions: health (measured by life expectancy at birth), education (mean of years of schooling for adults aged 25 years and more, as well as the expected years of schooling for children of school entering age), and standard of living (gross national income per capita, previously GDP).

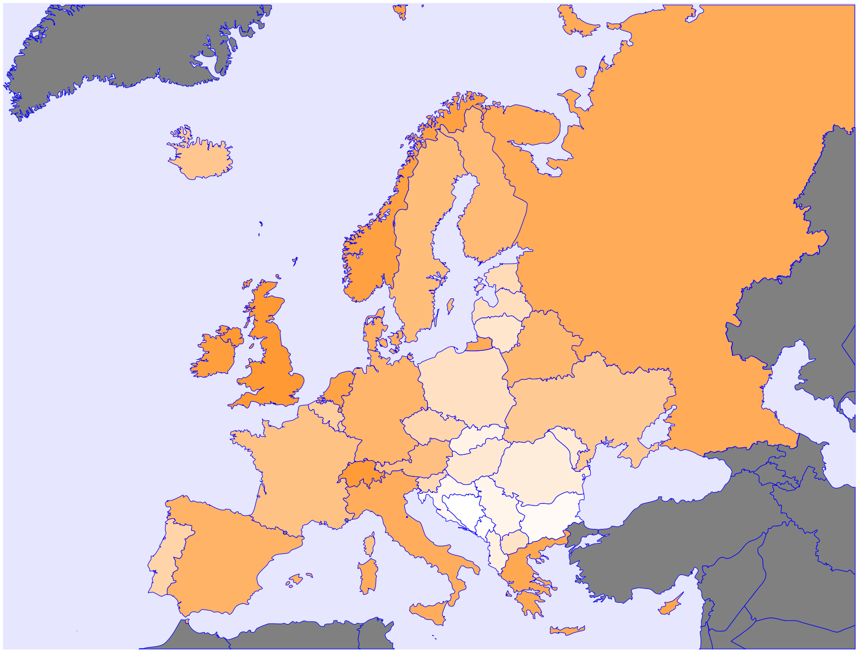
The HDI is compared with our alternative quality of life ranking in Figure 3.1. A darker colour indicates a higher rank. The results are similar for most countries. While the HDI places Norway at the top, the United Kingdom remains in a leading position according to the proposed measure. Belarus, Russia, and Ukraine get a significantly better rank with our methodology than shown by the HDI. Since a possible reason is that we have restricted our analysis to the European countries, and handle all others as one entity, this bias requires further investigation.

Therefore, we have merged Armenia, Azerbaijan, Georgia, Kazakhstan, the Kyrgyz Republic, Tajikistan, Turkmenistan, and Uzbekistan into a so-called "post-Soviet" or CIS (Commonwealth of Independent States) entity, and repeated the computations.

Table 3.3 compares the new rankings by the least squares method to the original one. Red arrow indicates a negative, green signs positive a change, while circle means

Figure 3.1: Comparison of the least squares and HDI rankings in 2015

(a) Least squares ranking from remittances



(b) HDI ranking

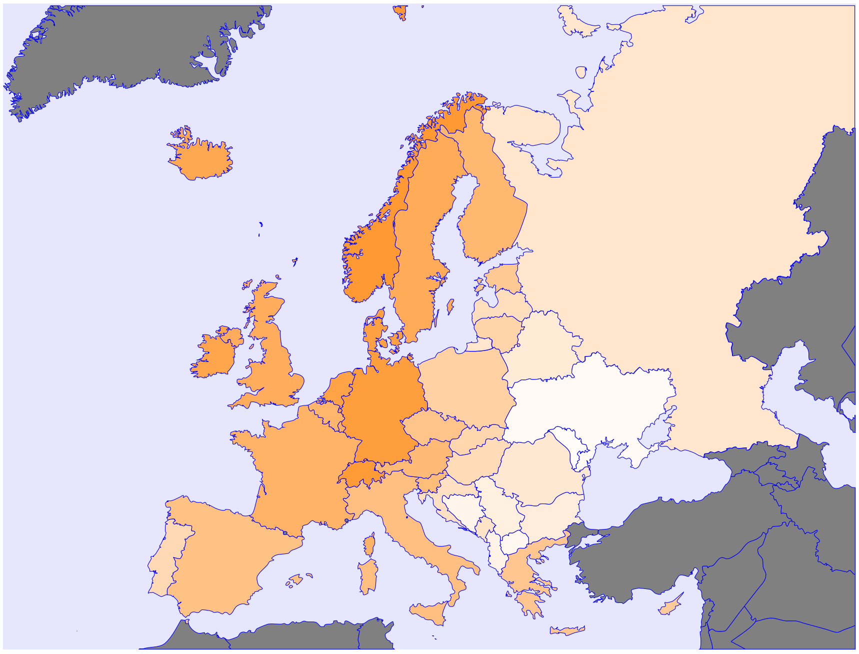


Table 3.2: Quality of life rankings by the least squares method

Country	2010	2011	2012	2013	2014	2015
AL	38	38	37	37	36	35
AT	18	18	20	12	13	16
BA	41	41	41	41	41	41
BE	22	22	22	26	25	22
BG	39	39	39	39	39	39
BY	15	15	15	15	14	13
CH	8	8	8	3	2	2
CY	5	4	4	2	7	6
CZ	27	26	26	21	20	25
DE	10	11	11	11	11	12
DK	12	12	13	10	10	10
EE	26	27	27	25	27	27
ES	9	9	9	9	9	11
FI	17	17	17	16	18	14
FR	13	14	16	17	17	17
GB	4	5	5	1	1	1
GR	21	20	14	13	12	7
HR	29	29	29	30	29	38
HU	31	30	30	34	32	32
IE	2	2	1	4	4	3
IS	1	1	2	19	21	19
IT	11	10	12	8	8	9
LT	36	36	35	32	31	31
LU	23	23	23	23	23	21
LV	24	24	25	20	22	28
MD	34	34	34	31	30	33
ME	37	37	38	40	40	40
MK	32	32	31	36	35	30
MT	19	19	18	29	34	23
NL	7	7	6	5	3	5
NO	6	6	7	7	5	4
PL	28	28	28	28	28	29
PT	20	21	21	27	24	24
RO	35	35	36	35	33	34
RS	40	40	40	38	38	37
RU	14	13	10	6	6	8
SE	3	3	3	14	15	15
SI	30	31	32	22	26	26
SK	33	33	33	33	37	36
UA	25	25	24	24	19	20
Other	16	16	19	18	16	18

no change.

According to the new ranking, Belarus, Latvia, Lithuania, Moldova, Russia, and

Table 3.3: Quality of life rankings by the least squares method with and without the CIS in 2015

Country	R	R (CIS)	Country	R	R (CIS)
AL	35	33 ↑	IS	19	18 ↑
AT	16	14 ↑	IT	9	8 ↑
BA	41	42 ↓	LT	31	37 ↓
BE	22	21 ↑	LU	21	19 ↑
BG	39	39 ●	LV	28	34 ↓
BY	13	20 ↓	MD	33	40 ↓
CH	2	2 ●	ME	40	41 ↓
CY	6	6 ●	MK	30	30 ●
CZ	25	25 ●	MT	23	22 ↑
DE	12	11 ↑	NL	5	4 ↑
DK	10	10 ●	NO	4	5 ↓
EE	27	27 ●	PL	29	28 ↑
ES	11	9 ↑	PT	24	23 ↑
FI	14	12 ↑	RO	34	32 ↑
FR	17	15 ↑	RS	37	36 ↑
GR	7	7 ●	RU	8	17 ↓
GB	1	1 ●	SE	15	13 ↑
HR	38	38 ●	SI	26	24 ↑
HU	32	31 ↑	SK	36	35 ↑
IE	3	3 ●	UA	20	26 ↓
Other	18	16 ↓	CIS	—	29

The column with the header "R" shows the original rank of the countries according to the least squares method, and the columns with the header "R (CIS)" corresponds to the case when the CIS (Commonwealth of Independent States) entity is introduced.

Ukraine obtain a substantially worse position, but there are no visible improvements. The reason is that original "Other" entity is in the middle of the ranking, however, the CIS is low-ranked entity, thus the countries are rearranged. Consequently, the use of the least squares method to country ranking may be sensitive to aggregation, and it requires properly disaggregated data. But this is not an inherent flaw of our proposal: Csató (2019) shows via an impossibility theorem that any reasonable ranking should depend on the level of aggregation.

3.3 Conclusions

This chapter has considered remittances as quantification of preferences revealed by people working abroad, underlying a ranking of countries around the world. Nonetheless, the use of remittances can be criticised from various aspects because: (1) the data on migration in various destination countries are incomplete; (2) the incomes of migrants and the costs of living are proxied by per capita incomes in PPP terms; and (3) there is no way to capture remittances flowing through informal, unrecorded channels.

These caveats somewhat limit the validity of our results. On the other hand, the proposed methodology has some advantages, illustrated by its independence of arbitrary parameter choices and favourable axiomatic properties. Its similarity to the Human Development Index indicates that we are able to capture at least some aspects of "quality of life". It can help evaluate a country's performance, even more finding a role model for emerging regions. For instance, among Post-Yugoslav states North Macedonia stands out, while in the Baltic region of the former Soviet Union, Estonia performs slightly better than Latvia and Lithuania. Finally, the suggested ranking can serve as a baseline for other composite indices.

While our approach cannot immediately substitute other rankings, it may become an alternative to various composite indices as it is straightforward to calculate. Furthermore, it only requires data on remittances. Hopefully, the research will contribute to a better understanding of economic and social development.

Chapter 4

Revenue allocation in Formula One

Professional sports leagues and championships generate billions of euros in common revenue. However, its allocation among the participants is often burdened with serious legal disputes centred around unequal shares and the possible violation of competition laws. Consequently, constructing allocation rules that depend only on a few arbitrary variables, and are relatively simple, robust, and understandable for all participants, poses an important topic of academic research.

Formula One (Formula 1, or simply F1) is the highest class of single-seater car racing. A Formula One season consists of several races taking place around the world. The drivers and constructors accumulate points on each race to obtain the annual World Championships, one for drivers and one for constructors. The distribution of Formula One prize money (1,004 million US dollars in 2019 (Rencken and Collantine, 2019)) can substantially affect competitive balance and the uncertainty around the expected outcome of races.

The current prize money allocation of Formula One is presented in Table 4.1. Column 1 corresponds to the revenue distributed equally among the teams which have finished in the top ten in at least two of the past three seasons. Column 2 corresponds to the performance-based payment, determined by the team's finishing position in the previous season. Ferrari has a special Long-standing Team payment as being the only team that competes since the beginning of the championship. Column 4 is paid to the previous champions, and three other teams receive bonus payments.

It is worth noting that only the third of the money pot is allocated strictly on the basis of performance in the previous season and bottom teams are underrepresented. Therefore, the new owner of the company controlling Formula One since January 2017 (Liberty Media), plans to reform the revenue allocation of the championship, mainly to increase the competitiveness of smaller teams (Collantine and Rencken, 2018).

Nonetheless, since the money allocation is based on the ranking of the construc-

Table 4.1: Formula One prize money allocation, 2019 (in million US dollars)

Team	Column 1	Column 2	Long-standing team	Championship bonus	Other	**Sum**
Ferrari	35	56	73	41	—	**205**
Mercedes	35	66	—	41	35	**177**
Red Bull	35	46	—	36	35	**152**
McLaren	35	32	—	33	—	**100**
Renault	35	38	—	—	—	**73**
Haas	35	35	—	—	—	**70**
Williams	35	15	—	—	10	**60**
Racing Point	35	24	—	—	—	**59**
Sauber	35	21	—	—	—	**56**
Toro Rosso	35	17	—	—	—	**52**
Sum	350	350	73	151	80	**1004**

Source: Rencken and Collantine (2019)

tors, it is important to apply a robust and reliable ranking procedure—but the current points scoring system fails to satisfy this requirement.

The long list of Formula One World Championship points scoring systems highlights the arbitrariness of the designs, and suggests that the relative importance of the different positions in a race remains unclear. The criticism in Haigh (2009, Section 2) is also worth studying.

We aim to outline a formal model that can be used to share Formula One prize money among the teams in a meaningful way. The proposal is based on pairwise comparisons and has strong links to the Analytic Hierarchy Process (AHP), a famous decision-making framework. In particular, we construct a multiplicative pairwise comparison matrix from the race results. Contrary to the Condorcet-like methods (Soares de Mello et al., 2015), it is not only said that a team is preferred to another if it has better results in the majority of the races, but the intensity of these pairwise preferences are also taken into account. Two popular weighting methods, the eigenvector method and the row geometric mean, are considered to compute the revenue share of each team.

The main contribution of our study is providing an alternative solution in the Formula One industry, which means innovation in reforming revenue allocation and at the same time applicable in other settings. Our approach has the following features. The derivation of the pairwise comparison matrix from the race results depends on a single variable, which regulates the inequality of the distribution. The user can choose it by taking into account preferences on how much inequality is desirable. It allows for the use of any weight deriving methods of the AHP literature and except for its sole parameter, the methodology is not influenced by any ad hoc decision such as the scores used in the official points system of Formula One. It supports the reliable performance of the bottom teams, which seldom score points, therefore the current system awards if they achieve unexpected results, mainly due to extreme events in some races. Besides that, a reasonable axiom in our setting is introduced for weighting methods, and its violation by the eigenvector method is presented in real data.

4.1 Theoretical background

In this section, the main components of the model will be presented: the multiplicative pairwise comparison matrix, its derivation from the race results, a straightforward axiom in our setting, and a basic measure of inequality.

4.1.1 Multiplicative pairwise comparison matrices

Consider a set of alternatives $N = \{1, 2, \dots, n\}$ such that their pairwise comparisons are known: a_{ij} shows how many times alternative i is better than alternative j.

The sets of positive (with all elements greater than zero) vectors of size n and matrices of size $n \times n$ are denoted by $\mathbb{R}^n_+$ and $\mathbb{R}^{n \times n}_+$, respectively.

The pairwise comparisons are collected into a matrix satisfying the reciprocity condition, hence any entry below the diagonal equals the reciprocal of the corresponding entry above the diagonal.

Definition 4.1. *Multiplicative pairwise comparison matrix*: Matrix $\mathbf{A} = [a_{ij}] \in \mathbb{R}^{n \times n}_+$ is a *multiplicative pairwise comparison matrix* if $a_{ji} = 1/a_{ij}$ holds for all $1 \leq i, j \leq n$.

We will regularly omit the word "multiplicative" for the sake of simplicity.

Let $\mathcal{A}^{n \times n}$ be the set of pairwise comparison matrices with n alternatives.

Pairwise comparisons are usually used to obtain an approximation of the relative priorities of the alternatives.

Definition 4.2. *Weight vector*: Vector $\mathbf{w} = [w_i] \in \mathbb{R}^n_+$ is a *weight vector* if $\sum_{i=1}^n w_i = 1$.

Let $\mathcal{R}^n$ be the set of weight vectors of size n.

Definition 4.3. *Weighting method*: Mapping $f : \mathcal{A}^{n \times n} \to \mathcal{R}^n$ is a *weighting method*.

The weight of alternative i in the pairwise comparison matrix $\mathbf{A} \in \mathcal{A}^{n \times n}$ according to the weighting method f is denoted by $f_i(\mathbf{A})$.

There exist several methods to derive a weight vector. The most popular procedures are the *eigenvector method* (Saaty, 1977, 1980) and the *row geometric mean (logarithmic least squares) method* (Crawford and Williams, 1985; De Graan, 1980; de Jong, 1984; Rabinowitz, 1976; Williams and Crawford, 1980).

Definition 4.4. *Eigenvector method* (EM): The weight vector $\mathbf{w}^{EM}(\mathbf{A}) \in \mathcal{R}^n$ provided by the *eigenvector method* is the solution of the following system of linear equations for any pairwise comparison matrix $\mathbf{A} \in \mathcal{A}^{n \times n}$:

$$\mathbf{A}\mathbf{w}^{EM}(\mathbf{A}) = \lambda_{\max}\mathbf{w}^{EM}(\mathbf{A}),$$

where $\lambda_{\max}$ denotes the maximal eigenvalue, also known as the principal or Perron eigenvalue, of the (positive) matrix $\mathbf{A}$.

Definition 4.5. *Row geometric mean method* (RGM): The *row geometric mean method* is the function $\mathbf{A} \to \mathbf{w}^{RGM}(\mathbf{A})$ such that the weight vector $\mathbf{w}^{RGM}(\mathbf{A})$ is given by

$$w_i^{RGM}(\mathbf{A}) = \frac{\prod_{j=1}^n a_{ij}^{1/n}}{\sum_{k=1}^n \prod_{j=1}^n a_{kj}^{1/n}}.$$

The row geometric mean method is sometimes called the *Logarithmic Least Squares Method* ($LLSM$) because it is the solution to the following optimisation problem:

$$\min_{\mathbf{w} \in \mathcal{R}^n} \sum_{i=1}^n \sum_{j=1}^n \left[\log a_{ij} - \log \left(\frac{w_i}{w_j} \right) \right]^2.$$

4.1.2 From race results to a pairwise comparison matrix

A Formula One season consists of a series of races, contested by two cars/drivers of each constructor/team. We say that team i has scored one goal against team j if a given car of team i is ahead of a given car of team j in a race. Thus, if there are no incomparable cars, then:

- Team i (j) has scored four (zero) goals against team j (i) if both cars of team i have finished above both cars of team j;
- Team i (j) has scored three (one) goal(s) against team j (i) if one car of team i has finished above both cars of team j, and the other car of team i has finished above one car of team j;
- Team i (j) has scored two (two) goals against team j (i) if one car of team i has finished above both cars of team j but both cars of team j have finished above the other car of team i.[2]

[2] We follow the official definition in classifying a driver as finished if he completed over 90% of the race distance.

The goals scored by the constructors in a race are aggregated over the whole season without weighting, similar to the official points scoring system.[3]

Consequently, the maximum number of goals that a team can score against another is four times the number of races. Since a car might not finish a race, each finishing car is assumed to be better than another which fails to finish. Nonetheless, two cars may be incomparable if both of them fail to finish the race. In this case, no goal is scored. The goals of the constructors are collected into the $n \times n$ *goals matrix*.

The pairwise comparison matrix $\mathbf{A} = [a_{ij}]$ is obtained from the goals matrix: if constructor i has scored g_{ij} goals against constructor j, while constructor j has scored g_{ji} goals against constructor i, then $a_{ij} = g_{ij}/g_{ji}$ and $a_{ji} = g_{ji}/g_{ij}$ to guarantee the reciprocity condition.

As we have mentioned, two weighting methods, the EM and the RGM will be used to derive a weight vector, which directly provides an allocation of the available amount.

The presented procedure does not contain any variable, thus it might lead to an allocation that cannot be tolerated by the decision-maker because of its (in)equality. Hence, the definition of the pairwise comparison matrix is modified such that:

$$a_{ij} = \left(\frac{g_{ij}}{g_{ji}} \right)^{\alpha} \qquad \text{and} \qquad a_{ji} = \left(\frac{g_{ji}}{g_{ij}} \right)^{\alpha} \qquad \text{for all } i, j \in N,$$

where $\alpha \geq 0$ is a parameter. If α is small, then $\mathbf{A}$ is close to the unit matrix, the weights are almost the same, and the shares remain roughly equal. The effect of α will be further investigated in Section 4.2.

4.1.3 A natural axiom for weighting methods

Axiom 4.1. *Scale invariance*: Let $\mathbf{A} = [a_{ij}] \in \mathcal{A}^{n \times n}$ be any pairwise comparison matrix and $\alpha > 0$ be a (positive) parameter. Let $\mathbf{A}^{(\alpha)} = \left[a_{ij}^{(\alpha)} \right] \in \mathcal{A}^{n \times n}$ be the pairwise comparison matrix defined by $a_{ij}^{(\alpha)} = a_{ij}^{\alpha}$. Weighting method $f : \mathcal{A}^{n \times n} \to \mathcal{R}^n$ is called *scale invariant* if for all $1 \leq i, j \leq n$:

$$f_i(\mathbf{A}) \geq f_j(\mathbf{A}) \iff f_i\left(\mathbf{A}^{(\alpha)}\right) \geq f_j\left(\mathbf{A}^{(\alpha)}\right).$$

Scale invariance implies that the ranking of the alternatives does not change if a different scale is used for pairwise comparisons.

In the setting of Section 4.1.2, scale invariance does not allow the ranking of the teams to depend on the parameter α, which seems to be reasonable because the

[3] Except for the 2014 Abu Dhabi Grand Prix, which awarded double points as the last race of the season.

underlying data (the goals matrix) are fixed. In other words, if constructor i receives more money than constructor j under any value of α, then it should receive more money for all $\alpha > 0$.

Lemma 4.1. *The eigenvector method does not satisfy scale invariance.*

Proof. It is sufficient to provide a counterexample. See Genest et al. (1993, Example 2.1) and Genest et al. (1993, Figure 1). □

The following result has already been mentioned in Genest et al. (1993, p. 581-582).

Lemma 4.2. *The row geometric mean method satisfies scale invariance.*

Proof. Note that $w_i^{RGM} \geq w_j^{RGM} \iff \prod_{k=1}^n a_{ik} \geq \prod_{k=1}^n a_{jk} \iff \prod_{k=1}^n a_{ik}^{(\alpha)} \geq \prod_{k=1}^n a_{jk}^{(\alpha)}$, which immediately verifies the statement. □

4.1.4 Measuring inequality

A basic indicator of competition among firms is the Herfindahl–Hirschman index Herfindahl (1950); Hirschman (1945, 1964).

Definition 4.6. *Herfindahl–Hirschman index* (HHI): Let $\mathbf{w} \in \mathcal{R}^n$ be a weight vector. Its *Herfindahl–Hirschman index* is:

$$HHI(\mathbf{w}) = \sum_{i=1}^{n} w_i^2.$$

The maximum of HHI is one when one constructor receives the whole amount. However, its minimum of $1/n$ (reached when all constructors receive the same amount) is influenced by the number of constructors, therefore, it is worth considering a normalised version of the HHI.

Definition 4.7. *Normalised Herfindahl–Hirschman index* (HHI^*): Let $\mathbf{w} \in \mathcal{R}^n$ be a weight vector. Its *normalised Herfindahl–Hirschman index* is:

$$HHI^*(\mathbf{w}) = \frac{HHI(\mathbf{w}) - 1/n}{1 - 1/n} = \frac{\sum_{i=1}^n w_i^2 - 1/n}{1 - 1/n}.$$

The value of HHI^* is always between 0 (equal shares) and 1 (maximal inequality).

Since the HHI better reflects market concentration, while the normalised Herfindahl–Hirschman index quantifies the equality of distributions, we will use the latter.

Table 4.2: Goals matrix, 2018

	Mercedes	Ferrari	Red Bull	Renault	Haas	McLaren	Racing Point	Sauber	Toro Rosso	Williams
Mercedes	—	47	59	78	78	77	79	78	78	79
Ferrari	37	—	49	73	74	73	73	74	74	74
Red Bull	23	31	—	60	61	59	59	62	60	60
Renault	4	8	19	—	45	52	45	55	55	63
Haas	6	9	18	35	—	49	39	39	47	55
McLaren	7	9	20	29	31	—	30	39	46	57
Racing Point	5	9	21	35	41	51	—	54	56	65
Sauber	6	8	19	23	38	42	28	—	50	54
Toro Rosso	4	8	22	25	32	35	24	30	—	44
Williams	5	8	20	20	26	26	18	29	38	—

4.2 Results

To illustrate the proposed allocation scheme, the 2018 Formula One season will be investigated in detail. Table 4.2 shows the goals matrix, where the teams are ranked according to the official championship result. For example, a car of Mercedes was better than a car of Ferrari on 47 occasions, while a car of Ferrari was better than a car of Mercedes on 37 occasions.

The corresponding pairwise comparison is presented in Table 4.3 with the choice $\alpha = 1$. It can be seen that all entries above the diagonal are higher than the corresponding element below the diagonal, thus a team, which has scored more points in the official ranking, is almost always preferred to a team with a lower number of points by pairwise comparisons. An exception is Racing Point, but unlike the official ranking, we have treated Racing Point/Force India as one team.

Figure 4.1 plots the shares of the top five competing teams with our methodology. Due to the same underlying pairwise comparison matrix, results given by the eigenvector and row geometric mean methods do not differ substantially.

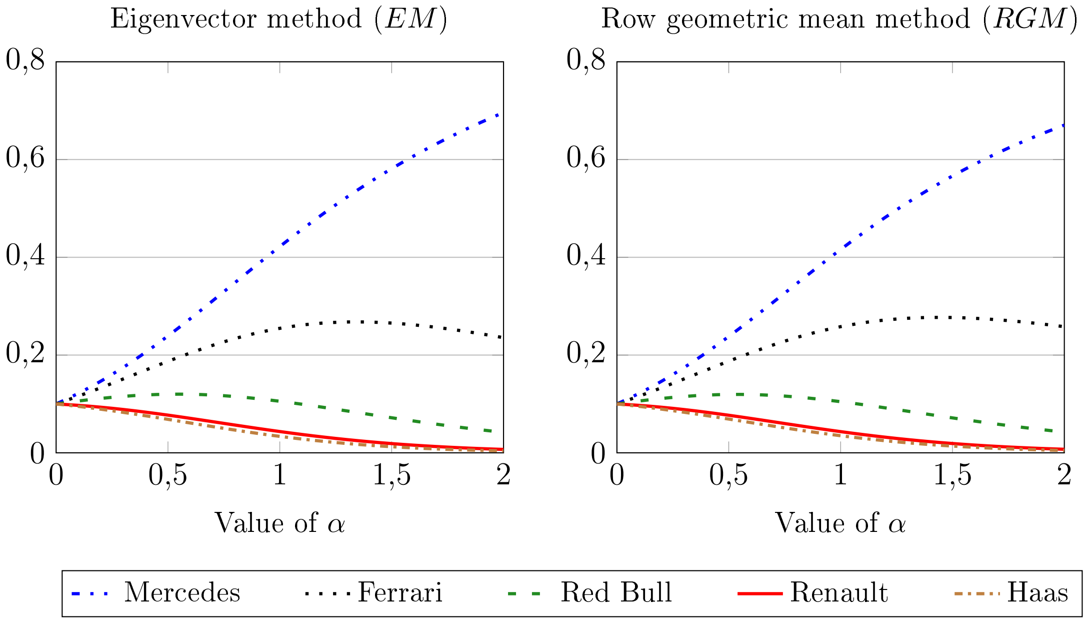
Mercedes is the dominant team according to both methods, with an ever increasing share as the function of parameter α. There are two teams (Ferrari, RedBull), which receive more money than the equal share in the case of small α. On the other hand, other teams receive less if parameter α increases.

Figure 4.2 depicts the value of the inequality measure HHI^* as the function of parameter α in the five seasons between 2014 and 2018. This can be especially relevant for a decision-maker who should fix the rules before the start of a season with having in mind a maximal level of inequality. For instance, choosing $\alpha = 1$

Table 4.3: Pairwise comparison matrix, $\alpha = 1$, 2018

	Mercedes	Ferrari	Red Bull	Renault	Haas	McLaren	Racing Point	Sauber	Toro Rosso	Williams
Mercedes	1	1,27	2,57	19,50	13,00	11,00	15,80	13,00	19,50	15,80
Ferrari	0,79	1	1,58	9,13	8,22	8,11	8,11	9,25	9,25	9,25
Red Bull	0,39	0,63	1	3,16	3,39	2,95	2,81	3,26	2,73	3,00
Renault	0,05	0,11	0,32	1	1,29	1,79	1,29	2,39	2,20	3,15
Haas	0,08	0,12	0,30	0,78	1	1,58	0,95	1,03	1,47	2,12
McLaren	0,09	0,12	0,34	0,56	0,63	1	0,59	0,93	1,31	2,19
Racing Point	0,06	0,12	0,36	0,78	1,05	1,70	1	1,93	2,33	3,61
Sauber	0,08	0,11	0,31	0,42	0,97	1,08	0,52	1	1,67	1,86
Toro Rosso	0,05	0,11	0,37	0,45	0,68	0,76	0,43	0,60	1	1,16
Williams	0,06	0,11	0,33	0,32	0,47	0,46	0,28	0,54	0,86	1

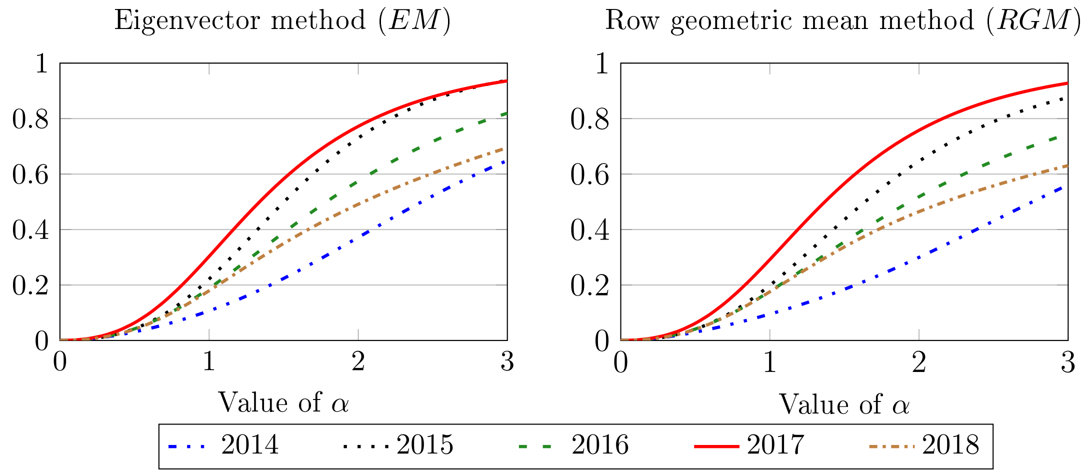
Figure 4.1: Revenue shares, top five teams, 2018



provides that the normalised Herfindahl–Hirschman index will not exceed 0.31 if the given season remains more balanced than the 2017 season.

As our intuition suggests, a higher α results in a more unequal distribution. It is also worth noting that HHI^* is consistently smaller in the case of row geometric mean than for the eigenvector method. In other words, the eigenvector method allows for bigger differences between the ratios of the weights, which is—according to Definition 4.6—unfavourable for HHI, thus for HHI^*, too. Furthermore, while the order of the seasons by the normalised Herfindahl–Hirschman index for a given α is relatively robust, the shape of the five lines varies.

Figure 4.2: The normalised Herfindahl–Hirschman index (HHI^*), 2014–2018



Lemma 4.2 guarantees that the ranking of the constructors according to their share of the revenue remains unchanged as the function of α if the row geometric mean method is applied. On the contrary, the eigenvector method can lead to a rank reversal: Figure 4.3 shows that McLaren will receive a higher share than Williams if α is bigger than 1.5.

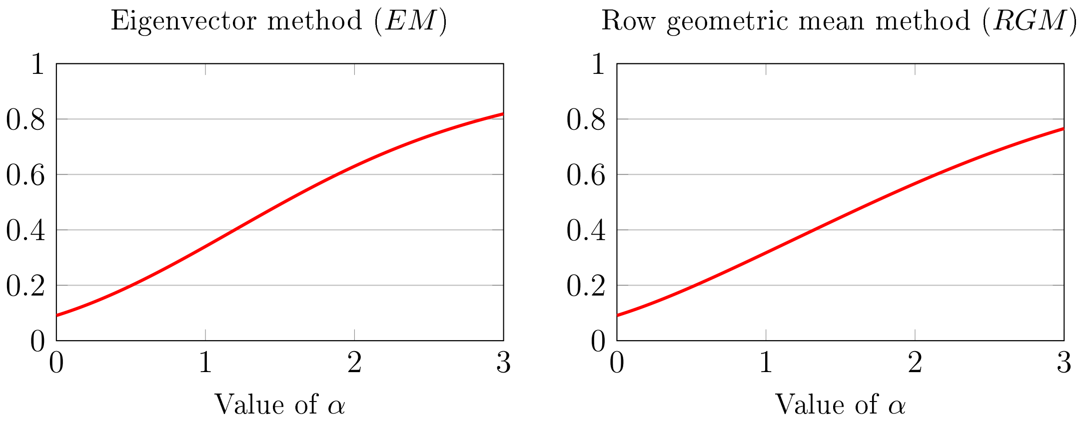
4.3 Conclusions

We have presented a model to share the revenue of an industry through the example of the Formula One World Constructors' Championship. The methodology is based on multiplicative pairwise comparison matrices and makes possible to tune the inequality of the allocation by its single parameter. Since the choice of the weighting method has only a marginal effect in this particular application, we recommend using the row geometric mean, which has favourable theoretical properties. The proposed technique has an important advantage over the official points scoring system of Formula One as it is independent of the somewhat arbitrary valuation given to the race prizes.

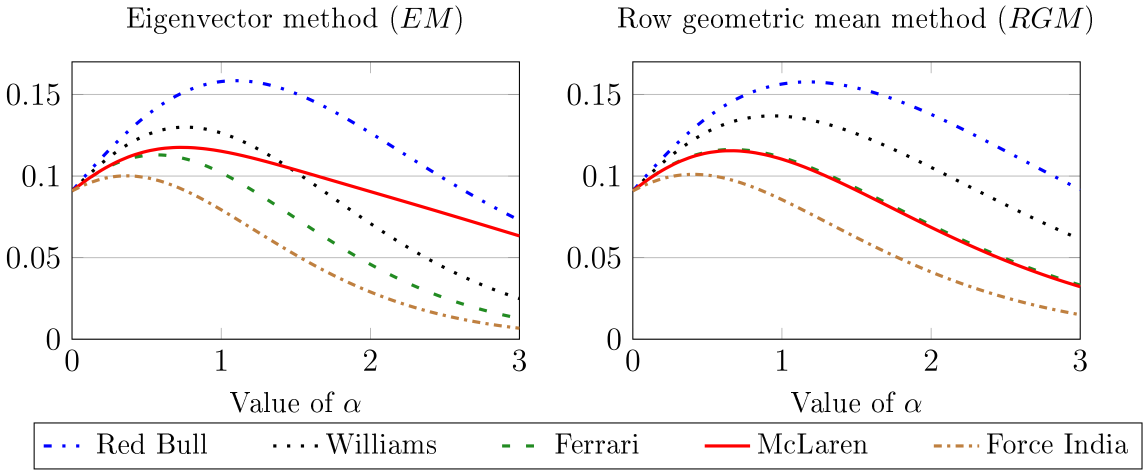
Besides offering a novel way for sharing Formula One prize money among the constructors, our methodology can be applied in any area where resources/revenues should be allocated among groups whose members are ranked several times. Potential examples include further racing competitions such as Grand Prix motorcycle racing, combined events in athletics like decathlon and heptathlon, or even workplaces where individual contributions on various projects are ranked.

Figure 4.3: Revenue shares, 2014

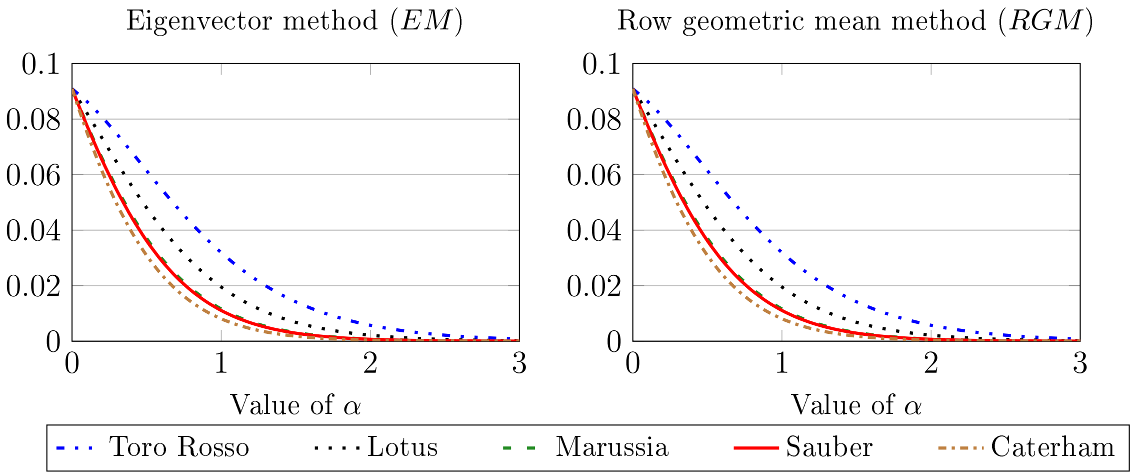
(a) The top team: Mercedes



(b) The five middle teams



(c) The five bottom teams



Chapter 5

Fairness in penalty shootouts

The order of actions in a sequential contest can generate various psychological effects that may change the ex-ante winning probabilities of the contestants. Soccer penalty shootouts, used to decide a tied match in a knockout tournament, offer a natural laboratory to test whether teams with equally skilled players have the same probability to win. Since the team favoured by a coin toss can decide to kick the first or the second penalty in all rounds (before June 2003, this team was automatically the first-mover), psychological pressure may mean that the second-mover has significantly less than 50% chance to win.

Previous research has shown mixed evidence and resulted in an intensive debate. While most authors find that the starting team enjoys an advantage (Apesteguia and Palacios-Huerta, 2010; Palacios-Huerta, 2014; Da Silva et al., 2018), some papers do not report such a problem (Kocher et al., 2012; Arrondel et al., 2019). Vandebroek et al. (2018) argue that the disagreement is mainly due to inadequate sample sizes, thus the natural solution would be to design and implement an appropriate field experiment—but this would take years.

Nonetheless, almost all stakeholders recognise that penalty shootouts are potentially *unfair*. According to a survey, more than 90% of coaches and players choose to increase the psychological pressure on the other team by kicking the first penalties (Apesteguia and Palacios-Huerta, 2010). A straightforward alternative, the Alternating ($ABBA$) rule, has been trialled in some matches (Csató, 2021). However, the 133rd Annual Business Meeting (ABM) of the IFAB (International Football Association Board, the body that determines the Laws of the Game of soccer) has decided to stop the experiment due to "*the absence of strong support, mainly because the procedure is complex*" (FIFA, 2018).

Even though the Alternating ($ABBA$) sequence has been found to provide no advantage for any player in a tennis tiebreak (Cohen-Zada et al., 2018), this is not the only proposal to ensure fairness. Palacios-Huerta (2012) has argued to follow the Prouhet-Thue-Morse sequence, where the first n moves are mirrored in the next n.

Recent suggestions include the Catch-up rule (Brams and Ismail, 2018), its variant called the Adjusted Catch-up rule (Csató, 2021), and the Behind-first rule (Anbarcı et al., 2021).

We contribute to the topic by evaluating all these mechanisms in three different probabilistic models that reflect the potential advantage of the team kicking the first penalty. From the plethora of interesting findings, the following results are worth underlining. The Catch-up rule turns out to be inferior compared to the straightforward Alternating ($ABBA$) rule as it does not yield any gain in fairness but it is more complex. On the other hand, the Behind-first mechanism—that alternates the shooting order but guarantees the first penalty to the team lagging behind in each round—somewhat overperforms them concerning fairness, which can be improved further by compensating the second-mover in the sudden death based on an idea of Csató (2021).

5.1 Mechanisms and models of psychological pressure

Denote the team that kicks the first penalty by A and the other team by B. The penalty shootout consists of five rounds in its regular phase. In each round, both teams kick one penalty. The shooting order in a round can be (1) independent of the outcomes in the previous rounds (*static* rule); (2) influenced by the results of preceding penalties (*dynamic* rule). The scores are aggregated after the five rounds, and the team that has scored more goals than the other wins the match. If the scores are level, the *sudden death* stage starts and continues until one team scores a goal more than the other from the same number of penalties.

Penalty shootout designs

We examine three static procedures:

- *Standard ($ABAB$)* rule: team A kicks the first and team B kicks the second penalty in each round. This is the official soccer penalty shootout design.

- *Alternating ($ABBA$)* rule: the order of the teams alternates, the second round (BA) mirrors the first (AB), and this sequence continues without any change, even in the possible sudden death phase.

- $ABBA|BAAB$ rule: the order in the first two rounds is $ABBA$, which is mirrored in the next two ($BAAB$), and this sequence is repeated.

The "double alternating" $ABBA|BAAB$ mechanism is considered because it takes us one step closer to the Prouhet–Thue–Morse sequence than the Alternating ($ABBA$) design. In our opinion, it is unlikely that the administrators want to move further along this line.

There are two dynamic designs, both of them having two variants:

- *Catch-up* rule (Brams and Ismail, 2018): the first kicking team alternates but the shooting order does not change if the first kicker missed and the second succeeded in the previous round.
- *Adjusted Catch-up* rule (Csató, 2021): the first five rounds are designed according to the Catch-up rule, however, team B kicks the first penalty in the sudden death stage (sixth round) regardless of the outcome in the previous round.
- *Behind-first* rule (Anbarcı et al., 2021): the team having less score after some rounds kicks the first penalty in the next round, and the order of the previous round is mirrored if the score is tied.
- *Adjusted Behind-first* rule: the first five rounds are designed according to the Behind-first rule, however, team B kicks the first penalty in the sudden death stage (sixth round) regardless of the outcome in the previous round.

The Adjusted Behind-first mechanism applies the idea underlying the Adjusted Catch-up rule, introduced in Csató (2021), for the Behind-first design.

Table 5.1 illustrates the eight penalty shootout designs. Since the scores are 2-2 after five penalties, the shootout goes to sudden death, where both teams succeed in the sixth round. However, in the seventh round only team A scores, hence it wins the match.

Note that the four dynamic mechanisms lead to different shooting orders. Team B kicks the first penalty in the third round under the Catch-up rule because both teams score in the second round where team A is the first-mover. On the other hand, the Behind-first rule favours team A in the third round as it is lagging in the number of goals. Both designs give the first penalty in the sixth round to team A since team B is the first-mover in the fifth round. However, the Adjusted Catch-up and Behind-first rules compensate team B by kicking first in the sudden death for being disadvantaged in the first round of the shootout.

Models of psychological pressure

According to Apesteguia and Palacios-Huerta (2010, Figure 2A), the first kicking team scores its penalties with a higher probability in all rounds. A possible reason

Table 5.1: An illustration of the penalty shootout mechanisms

		Penalty kicks in the regular phase										Sudden death			
Mechanism	Team	1	2	3	4	5	6	7	8	9	10	11	12	13	14
ABAB	A	✗		✓		✗		✓		✗		✓		✓	
	B		✓		✓		✗		✗		✗		✓		✗
ABBA	A	✗			✓	✗			✓	✗			✓	✓	
	B		✓	✓			✗	✗			✗	✓			✗
ABBA|*BAAB*	A	✗			✓		✗	✓		✗			✓		✓
	B		✓	✓		✗			✗		✗	✓		✗	
Catch-up	A	✗		✓			✗	✓			✗	✓			✓
	B		✓		✓	✗			✗	✗			✓	✗	
Adj. Catch-up	A	✗		✓			✗	✓			✗		✓	✓	
	B		✓		✓	✗			✗	✗		✓			✗
Behind-first	A	✗		✓		✗		✓			✗	✓			✓
	B		✓		✓		✗		✗	✗			✓	✗	
Adj. Behind-first	A	✗		✓		✗		✓			✗		✓	✓	
	B		✓		✓		✗		✗	✗		✓			✗

is that most soccer penalties are successful, thus the player taking the second kick usually faces greater mental pressure. Therefore, following Brams and Ismail (2018) and Csató (2021), our first model M1 assumes that the first kicker has a probability p of scoring, while the second kicker has a probability q of scoring, where $p \geq q$.

However, this anxiety may be missing if the first kicker fails. The second model M2 assumes that each player has a probability p of scoring, except for the second shooter after a successful penalty, who scores with probability q, where $p \geq q$.

Finally, the psychological pressure can come from lagging in the number of goals as Apesteguia and Palacios-Huerta (2010) and Vandebroek et al. (2018) argue. Consequently, the third model M3 is defined such that each player has a probability p of scoring, except for the kicker from the team having fewer scores, who succeeds with probability q, where $p \geq q$. This assumption is used by Lambers and Spieksma (2021, Section 4).

All of the above models are mainly consistent with the previous empirical works analysing penalty shootout datasets, even though the exact values of parameters p and q might be different.

Naturally, the proposed framework is an idealised version of reality because each player may have a different skill level. It is also important to note that the introduction of two consecutive penalties by the same team may lead to other types of psychological pressure. For example, if the goalkeeper defends a penalty, it may influence the probability of scoring the next penalty by the same team since there is no time to "calm down". In the absence of adequate experiments with alternative shootout designs, it makes no sense to consider such effects.

5.2 Results

The probability of winning can be accurately determined by a computer code for any values of p and q to gain an insight into the fairness of the presented penalty shootout mechanisms. It is carried out by brute force, calculating the probability of all the 2^{2n} cases where n is the number of rounds before the sudden death stage. Even though the computation can be stopped if one team has scored more goals than the other could score, this trick is not worth implementing because the running time is moderated for any reasonable length of the regular phase.

Brams and Ismail (2018) quantify the unfairness of the Standard ($ABAB$) and Catch-up rules under model M1. Csató (2021) extends this study by considering the Alternating ($ABBA$) and Adjusted Catch-up designs. Vandebroek et al. (2018) derive the winning probabilities for the Standard ($ABAB$) and Alternating ($ABBA$) mechanisms, as well as for the Prouhet–Thue–Morse-sequence under model M3. Analogously, Lambers and Spieksma (2021) show how to find the least unfair static sequence under model M3, and empirically compute these for relevant values of p and q if $n = 5$ and for various n-s if $p = 3/4$ and $q = 2/3$. However, the previous literature has addressed neither model M2, nor the Behind-first rule and the Adjusted variants.

First, similar to recent papers (Brams and Ismail, 2018; Csató, 2021; Lambers and Spieksma, 2021), the parameters $p = 3/4$ and $q = 2/3$ are studied as they are close to the empirical scoring probabilities. Table 5.2 reveals the probability of winning for team A which kicks the first penalty. The Standard ($ABAB$) rule is the most unfair, and it deviates from equality even further as the number of rounds in the regular stage grows, except for model M3.

Some rules favour the second-mover team B in models M1 and M2 but not in M3. Remarkably, the $ABBA|BAAB$ mechanism gives an advantage for the second-mover if the number of rounds in the regular stage is six or seven: in the former case, the sudden death is started by team B, and in the latter, team B kicks four penalties first in the regular phase. The Alternating ($ABBA$), (Adjusted) Catch-up, and (Adjusted) Behind-first designs exhibit a moderate odd-even effect, they are less fair when the number of rounds is odd, which restricts the opportunities to balance the advantage enjoyed by team A. There are similar cycles with the length of four under the $ABBA|BAAB$ rule.

The three static rules do not differ in the regular phase when there is only one round there, but they imply a different shooting order in the sudden death. On the other hand, the (Adjusted) Catch-up and the (Adjusted) Behind-first mechanisms coincide unless the regular stage consists of at least three rounds.

Table 5.2: The probability that A wins including sudden death ($p = 3/4$ and $q = 2/3$)

a Model M1: the second player has a scoring probability q

Model M1	Number of rounds							
Mechanism	1	2	3	4	**5**	6	7	8
$ABAB$	0.600	0.608	0.618	0.628	**0.637**	0.645	0.653	0.661
$ABBA$	0.526	0.511	0.519	0.508	**0.515**	0.507	0.513	0.506
$ABBA\|BAAB$	0.513	0.494	0.489	0.504	**0.509**	0.497	0.492	0.503
Catch-up	0.526	0.516	0.518	0.513	**0.514**	0.512	0.512	0.511
Adjusted Catch-up	0.526	0.495	0.515	0.501	**0.509**	0.504	0.507	0.504
Behind-first	0.526	0.516	0.516	0.512	**0.512**	0.510	0.510	0.508
Adj. Behind-first	0.526	0.495	0.512	0.500	**0.506**	0.501	0.503	0.501

b Model M2: the second player has a scoring probability q if the first player succeeds

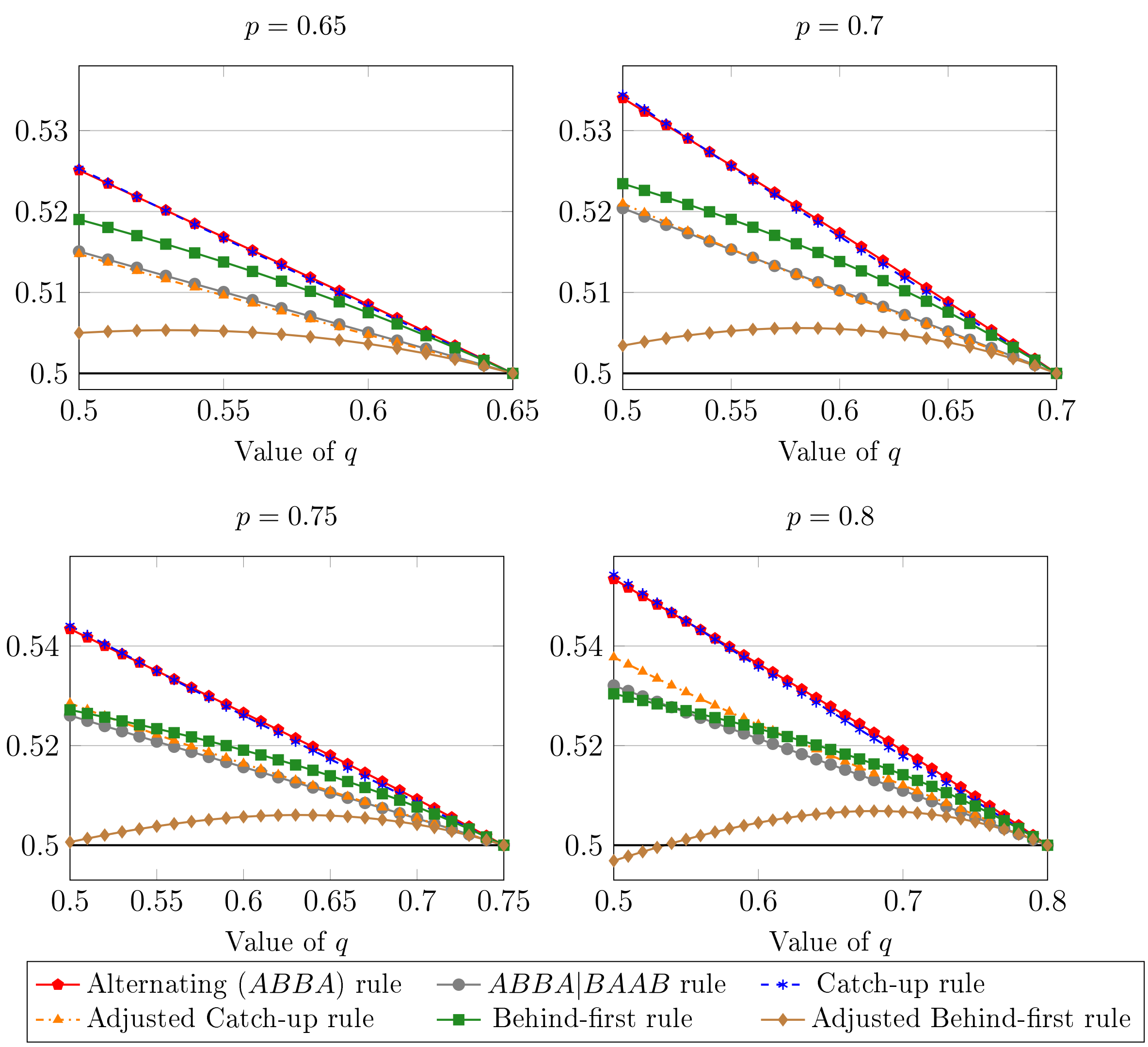
Model M2	Number of rounds							
Mechanism	1	2	3	4	**5**	6	7	8
$ABAB$	0.571	0.578	0.586	0.593	**0.600**	0.606	0.612	0.618
$ABBA$	0.520	0.508	0.514	0.506	**0.511**	0.505	0.509	0.504
$ABBA\|BAAB$	0.510	0.496	0.492	0.503	**0.506**	0.497	0.494	0.502
Catch-up	0.520	0.513	0.514	0.510	**0.511**	0.509	0.509	0.508
Adjusted Catch-up	0.520	0.497	0.510	0.502	**0.506**	0.503	0.505	0.503
Behind-first	0.520	0.513	0.513	0.510	**0.509**	0.508	0.507	0.507
Adj. Behind-first	0.520	0.497	0.509	0.501	**0.505**	0.502	0.503	0.501

c Model M3: the team lagging behind has a scoring probability q

Model M3	Number of rounds							
Mechanism	1	2	3	4	**5**	6	7	8
$ABAB$	0.571	0.571	0.571	0.571	**0.571**	0.571	0.571	0.571
$ABBA$	0.520	0.516	0.514	0.514	**0.512**	0.512	0.511	0.511
$ABBA\|BAAB$	0.510	0.504	0.507	0.510	**0.507**	0.504	0.507	0.508
Catch-up	0.520	0.515	0.515	0.513	**0.513**	0.512	0.511	0.511
Adjusted Catch-up	0.520	0.501	0.513	0.506	**0.510**	0.507	0.508	0.508
Behind-first	0.520	0.515	0.515	0.513	**0.513**	0.512	0.511	0.511
Adj. Behind-first	0.520	0.501	0.513	0.506	**0.510**	0.507	0.508	0.508

Except for the Standard ($ABAB$) rule and its adjusted variant, increasing the number of rounds in the regular stage would improve fairness in general as there remains more scope to balance the advantages between the two teams. Naturally, organising more rounds requires more time and the number of players is also limited. On the other hand, it is an almost obvious policy implication to choose an even number of rounds for the regular phase, when both teams can be the first-mover in the same number of regular rounds.

Figure 5.1: The probability that team A wins a penalty shootout over five rounds including sudden death, model M1

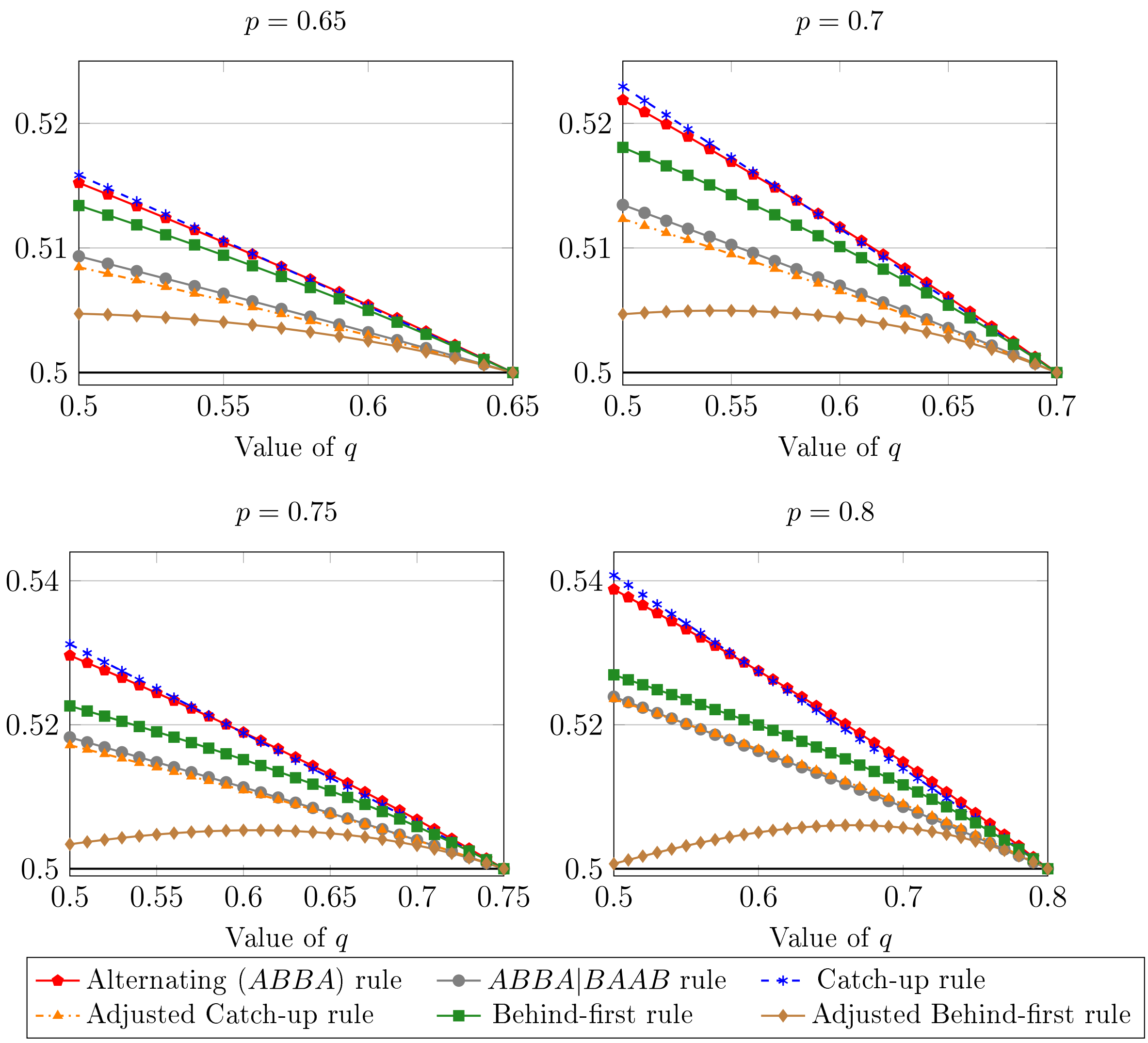


The Alternating ($ABBA$) mechanism outperforms the Standard ($ABAB$) mechanism in models M1 and M2 (Echenique, 2017)—but this is not proven in model M3 yet. The fairness of the Standard ($ABAB$) rule is not influenced by the number of rounds in model M3.

The Catch-up rule is not considerably fairer than the already tried Alternating ($ABBA$) rule, and the Behind-first rule does not perform substantially better than them. However, the minor amendment proposed by Csató (2021) in the first round of the sudden death consistently makes the dynamic designs fairer. Thus they become a competitive alternative to the static $ABBA|BAAB$ mechanism that aims to implement the Prouhet–True–Morse sequence, especially in model M1.

In order to extend these findings, Figures 5.1–5.3 plot the winning probability of team A as the function of parameter q—that has a different meaning in each model—for four values of p, the scoring probability of the advantaged team. The

Figure 5.2: The probability that team A wins a penalty shootout over five rounds including sudden death, model M2

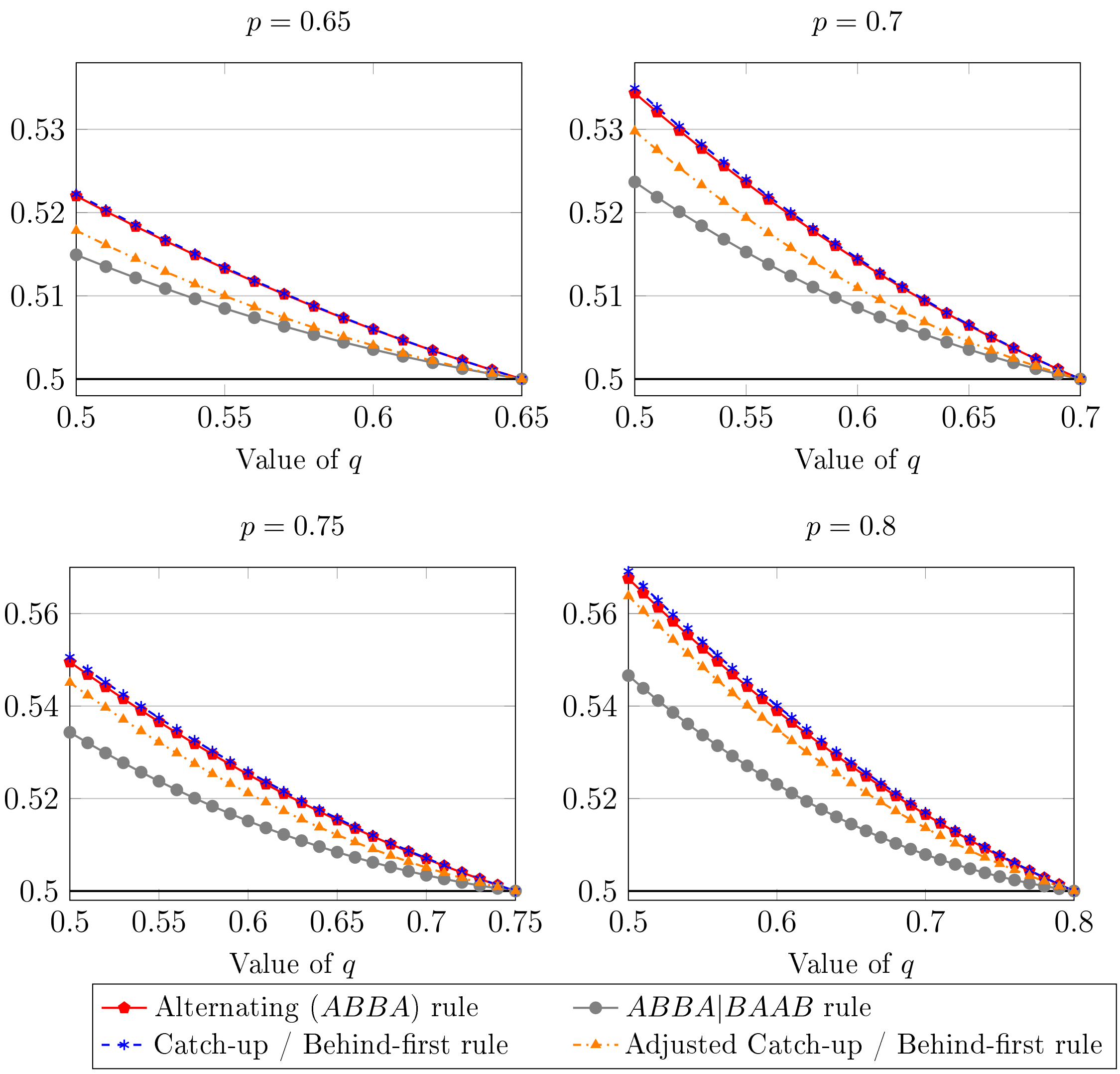


Standard ($ABAB$) rule are not depicted due to its high level of unfairness, which makes it impossible to visualise this mechanism together with the other six.

The main message can be summarised as follows:

- All mechanisms are closer to fairness in model M2 than in model M1 because the former punishes the second player only if the first player scores.
- Fairness is the most difficult to achieve in model M3, where the disadvantage of team B cannot be balanced by providing it with a higher scoring probability as the first kicker in a round. On the other hand, the Standard ($ABAB$) rule is the least unfair in model M3.
- The straightforward Alternating ($ABBA$) rule is not worse than the dynamic Catch-up rule, the use of the latter cannot be justified by the need

Figure 5.3: The probability that team A wins a penalty shootout over five rounds including sudden death, model M3



to improve fairness. This finding considerably decreases the value of the contribution by Brams and Ismail (2018).

- The adjustment of the dynamic designs, suggested by Csató (2021), robustly improves fairness. It is worth guaranteeing the first penalty of the sudden death stage for team B.

- The $ABBA|BAAB$ rule remains competitive with the Adjusted Catch-up rule in models M1 and M2, while it is the closest to fairness among all designs considered here in model M3. If the Alternating ($ABBA$) rule is judged inadequate, and the use of dynamic mechanisms should be avoided, further steps towards the Prouhet–True–Morse sequence can be an effective measure to increase fairness.

- The Adjusted Behind-first rule outperforms all other mechanisms if the first-mover advantage originates exclusively from the shooting order, that is, model M1 or model M2 is valid. The compensation of team B in the sudden death is so powerful that the winning probability of team A becomes non-monotonic as the difference between p and q grows. However, the psychological pressure is unlikely to reach this level in practice, thus the observation remains rather a theoretical curiosity. Nevertheless, it is an important implication that the Adjusted Behind-first rule would be especially desirable relative to other mechanisms if model M1 or M2 holds and there is substantial psychological pressure.

5.3 Conclusions

We have analysed seven soccer penalty shootout rules under three different mathematical models. The Standard ($ABAB$) mechanism is the simplest but robustly unfair design, at least compared to the others. The Alternating ($ABBA$) design remains a straightforward and relatively fair solution, which is already used in tennis tiebreaks. The $ABBA|BAAB$ mechanism is a static rule approaching the Prouhet–Thue–Morse sequence that further improves fairness and outperforms all other designs in model M3 when the team lagging behind has a lower scoring probability.

Two dynamic mechanisms have also been evaluated together with their slightly modified variants. The Catch-up rule yields no gain in fairness compared to the Alternating ($ABBA$) rule, thus it remains a poor choice to try. Both the Catch-up and the Behind-first designs can be adjusted according to the proposal of Csató (2021), by compensating team B—the second-mover in the first round—in the sudden death phase. These versions improve fairness at the price of marginally increasing complexity.

Nonetheless, in the case of stationary scoring probabilities considered here, it remains sufficient to use static rules in order to improve fairness. On the other hand, compensating the second-mover by making it first-mover in the sudden death stage seems to be a reasonable modification.

Bibliography

Adams, Jr., R. H. (2003). International migration, remittances, and the brain drain: A study of 24 labor-exporting countries. World Bank Policy Research Working Paper 3069.

Anbarcı, N., Sun, C.-J., and Ünver, M. U. (2021). Designing practical and fair sequential team contests: The case of penalty shootouts. *Games and Economic Behavior*, 130:25–43. DOI: 10.1016/j.geb.2021.07.004.

Apesteguia, J. and Palacios-Huerta, I. (2010). Psychological pressure in competitive environments: Evidence from a randomized natural experiment. *American Economic Review*, 100(5):2548–2564. DOI: 10.1257/aer.100.5.2548.

Arrondel, L., Duhautois, R., and Laslier, J.-F. (2019). Decision under psychological pressure: The shooter's anxiety at the penalty kick. *Journal of Economic Psychology*, 70:22–35. DOI: 10.1016/j.joep.2018.10.008.

Banzhaf, J. F. (1964-1965). Weighted voting doesn't work: A mathematical analysis. *Rutgers Law Review*, 19(2):317–343.

Benson, M. and O'Reilly, K. (2009). Migration and the search for a better way of life: A critical exploration of lifestyle migration. *The Sociological Review*, 57(4):608–625. DOI: 10.1111/j.1467-954X.2009.01864.x.

Benson, M. and O'Reilly, K. (2016). From lifestyle migration to lifestyle in migration: Categories, concepts and ways of thinking. *Migration Studies*, 4(1):20–37. DOI: 10.1093/migration/mnv015.

Bertini, C., Gambarelli, G., Stach, I., and Zibetti, G. (2019). Seat apportionment by population and contribution in European Parliament after Brexit. In Nguyen, N. T., Kowalczyk, R., Mercik, J., and Motylska-Kuźma, A., editors, *Transactions on Computational Collective Intelligence XXXIV*, pages 109–126. Springer, Berlin, Heidelberg. DOI: 10.1007/978-3-662-60555-4.

Bobek, A. (2020). Leaving for the money, staying for the 'quality of life'. Case study of young Polish migrants living in Dublin. *Geoforum*, 109:24–34. DOI: 10.1016/j.geoforum.2019.12.006.

Bozóki, S., Fülöp, J., and Rónyai, L. (2010). On optimal completion of incomplete pairwise comparison matrices. *Mathematical and Computer Modelling*, 52(1-2):318–333. DOI: 10.1016/j.mcm.2010.02.047.

Brams, S. J. and Affuso, P. J. (1976). Power and size: A new paradox. *Theory and Decision*, 7(1-2):29–56. DOI: 10.1007/BF00141101.

Brams, S. J. and Ismail, M. S. (2018). Making the rules of sports fairer. *SIAM Review*, 60(1):181–202. DOI: 10.1137/16M1074540.

Bräuninger, T. and König, T. (2005). Indices of Power IOP 2.0. http://www.tbraeuninger.de/download.

Čaklović, L. and Kurdija, A. S. (2017). A universal voting system based on the Potential Method. *European Journal of Operational Research*, 259(2):677–688. DOI: 10.1016/j.ejor.2016.10.032.

Cohen-Zada, D., Krumer, A., and Shapir, O. M. (2018). Testing the effect of serve order in tennis tiebreak. *Journal of Economic Behavior & Organization*, 146:106–115. DOI: 10.1016/j.jebo.2017.12.012.

Coleman, J. S. (1971). Control of collectivities and the power of a collectivity to act. In Lieberman, B., editor, *Social Choice*, pages 192–225. Gordon and Breach, New York.

Collantine, K. and Rencken, D. (2018). Revealed: The winners and losers under Liberty's 2021 F1 prize money plan. https://www.racefans.net/2018/04/11/revealed-the-winners-and-losers-under-libertys-2021-f1-prize-money-plan/.

Crawford, G. and Williams, C. (1985). A note on the analysis of subjective judgment matrices. *Journal of Mathematical Psychology*, 29(4):387–405. DOI: 10.1016/0022-2496(85)90002-1.

Csató, L. (2019). Journal ranking should depend on the level of aggregation. *Journal of Informetrics*, 13(4):100975. DOI: 10.1016/j.joi.2019.100975.

Csató, L. (2021). A comparison of penalty shootout designs in soccer. *4OR*, 19:183–198. DOI: 10.1007/s10288-020-00439-w.

Csató, L. and Petróczy, D. G. (2019). Hogyan tehető igazságosabbá a labdarúgó mérkőzéseket követő büntetőpárbaj? *Statisztikai Szemle*, 97(8):779–798. DOI: 10.20311/stat2019.8.hu0779.

Csató, L. and Petróczy, D. G. (2022). Fairness in penalty shootouts: Is it worth using dynamic sequences? Manuscript. arXiv: 2004.09225.

Csató, L. and Tóth, Cs. (2020). University rankings from the revealed preferences of the applicants. *European Journal of Operational Research*, 286(1):309–320. DOI: 10.1016/j.ejor.2020.03.008.

Da Silva, S., Mioranza, D., and Matsushita, R. (2018). FIFA is right: the penalty shootout should adopt the tennis tiebreak format. *Open Access Library Journal*, 5(3):1–23. DOI: 10.4236/oalib.1104427.

De Graan, J. G. (1980). Extensions of the multiple criteria analysis method of T. L. Saaty. Report, National Institute for Water Supply, Voorburg.

De Haas, H. (2005). International migration, remittances and development: myths and facts. *Third World Quarterly*, 26(8):1269–1284. DOI: 10.1080/01436590500336757.

de Jong, P. (1984). A statistical approach to Saaty's scaling method for priorities. *Journal of Mathematical Psychology*, 28(4):467–478. DOI: 10.1016/0022-2496(84)90013-0.

Diener, E. and Suh, E. (1997). Measuring quality of life: Economic, social, and subjective indicators. *Social Indicators Research*, 40(1-2):189–216. DOI: 10.1023/A:1006859511756.

Echenique, F. (2017). ABAB or ABBA? The arithmetics of penalty shootouts in soccer. Manuscript. `https://pdfs.semanticscholar.org/85d0/3edc04470d5670266c075f7860c441a17bce.pdf`.

Éltető, Ö. and Köves, P. (1964). Egy nemzetközi összehasonlításoknál fellépő indexszámítási problémáról. *Statisztikai Szemle*, 42(5):507–518.

European Parliament (2014). EU budget explained: expenditure and contribution by member state. Accessed on 20 January 2017. `http://web.archive.org/web/20190429033152/http://www.europarl.europa.eu/news/en/headlines/eu-affairs/20141202IFG82334/eu-budget-explained-expenditure-and-contribution-by-member-state`.

Eurostat (2017). EUROPOP 2013 – Population projections at national level. Accessed on 6 January 2017. http://ec.europa.eu/eurostat/documents/276524/8602151/Eurostat_Table_tps00002+Population+projections.xls/4dcc0e4e-09ab-408a-a659-6d4b2ed122a4.

Felsenthal, D. S. and Machover, M. (1997). The weighted voting rule in the EU's Council of Ministers, 1958–1995: Intentions and outcomes. *Electoral Studies*, 16(1):33–47. DOI: 10.1016/S0261-3794(96)00055-8.

Felsenthal, D. S. and Machover, M. (2001). The Treaty of Nice and qualified majority voting. *Social Choice and Welfare*, 18(3):431–464. DOI: 10.1007/s003550100137.

FIFA (2018). IFAB's 133rd Annual Business Meeting recommends fine-tuning Laws for the benefit of the game. 22. November http://web.archive.org/web/20200829060851/https://www.fifa.com/who-we-are/news/ifab-s-133rd-annual-business-meeting-recommends-fine-tuning-laws-for-the-benefit.

Foster, J. E., McGillivray, M., and Seth, S. (2013). Composite indices: Rank robustness, statistical association, and redundancy. *Econometric Reviews*, 32(1):35–56. DOI: 10.1080/07474938.2012.690647.

Genest, C., Lapointe, F., and Drury, S. W. (1993). On a proposal of Jensen for the analysis of ordinal pairwise preferences using Saaty's eigenvector scaling method. *Journal of Mathematical Psychology*, 37(4):575–610. DOI: 10.1006/jmps.1993.1035.

Göllner, R. (2017). The Visegrád Group – A rising star post-Brexit? Changing distribution of power in the European Council. *Open Political Science*, 1(1):1–6. DOI: 10.1515/openps-2017-0001.

González-Díaz, J., Hendrickx, R., and Lohmann, E. (2014). Paired comparisons analysis: an axiomatic approach to ranking methods. *Social Choice and Welfare*, 42(1):139–169. DOI: 10.1007/s00355-013-0726-2.

Grech, P. D. (2021). Power in the Council of the EU: organizing theory, a new index, and Brexit. *Social Choice and Welfare*, 56(2):223–258. DOI: 10.1007/s00355-020-01273-z.

Haigh, J. (2009). Uses and limitations of mathematics in sport. *IMA Journal of Management Mathematics*, 20(2):97–108. DOI: 10.1093/imaman/dpn024.

Herfindahl, O. C. (1950). *Concentration in the steel industry*. PhD thesis, Columbia University, New York.

Herne, K. and Nurmi, H. (1993). The distribution of a priori voting power in the EC Council of Ministers and the European Parliament. *Scandinavian Political Studies*, 16(3):269–284. DOI: 10.1111/j.1467-9477.1993.tb00041.x.

Hirschman, A. O. (1945). *National Power and the Structure of Foreign Trade*. University of California Press, Berkeley and Los Angeles.

Hirschman, A. O. (1964). The paternity of an index. *American Economic Review*, 54(5):761–762.

Kocher, M. G., Lenz, M. V., and Sutter, M. (2012). Psychological pressure in competitive environments: New evidence from randomized natural experiments. *Management Science*, 58(8):1585–1591. DOI: 10.1287/mnsc.1120.1516.

Kóczy, L. Á. (2012). Beyond Lisbon: Demographic trends and voting power in the European Union Council of Ministers. *Mathematical Social Sciences*, 63(2):152–158. DOI: 10.1016/j.mathsocsci.2011.08.005.

Kóczy, L. Á. (2021). Brexit and power in the Council of the European Union. *Games*, 12(2):51. DOI: 10.3390/g12020051.

Lambers, R. and Spieksma, F. C. R. (2021). A mathematical analysis of fairness in shootouts. *IMA Journal of Management Mathematics*, 32(4):411–424. DOI: 10.1093/imaman/dpaa023.

Palacios-Huerta, I. (2012). Tournaments, fairness and the Prouhet-Thue-Morse sequence. *Economic Inquiry*, 50(3):848–849. DOI: 10.1111/j.1465-7295.2011.00435.x.

Palacios-Huerta, I. (2014). *Beautiful Game Theory: How Soccer Can Help Economics*. Princeton University Press, Princeton, New York.

Penrose, L. S. (1946). The elementary statistics of majority voting. *Journal of the Royal Statistical Society*, 109(1):53–57. DOI: 10.1111/j.2397-2335.1946.tb04638.x.

Petróczy, D. G. (2020). Egy életminőség-rangsor a hazautalások alapján. *Szigma*, 51(2):169–184.

Petróczy, D. G. (2021a). An alternative quality of life ranking on the basis of remittances. *Socio-Economic Planning Sciences*, 78:101042. DOI: 10.1016/j.seps.2021.101042.

Petróczy, D. G. (2021b). Teljesítményalapú pénzfelosztás a Forma–1-ben páros összehasonlításokkal. *Szigma*, 52(1):61–74.

Petróczy, D. G. and Csató, L. (2021). Revenue allocation in Formula One: a pairwise comparison approach. *International Journal of General Systems*, 50(3):243–261. DOI: 10.1080/03081079.2020.1870224.

Petróczy, D. G., Rogers, M. F., and Kóczy, L. Á. (2019). Tagkilépések és a magyar befolyás változása az Európai Unió Tanácsában. *Alkalmazott Matematikai Lapok*, 36(1):65–81.

Petróczy, D. G., Rogers, M. F., and Kóczy, L. Á. (2022). Exits from the European Union and their effect on power distribution in the Council. *Games*, 13(1):18. DOI: 10.3390/g13010018.

Rabinowitz, G. (1976). Some comments on measuring world influence. *Conflict Management and Peace Science*, 2(1):49–55. DOI: 10.1177/073889427600200104.

Ratha, D. and Shaw, W. (2007). *South-South migration and remittances.* The World Bank.

Ravallion, M. (2012). Mashup indices of development. *The World Bank Research Observer*, 27(1):1–32. DOI: 10.1093/wbro/lkr009.

Rencken, D. and Collantine, K. (2019). Formula 1 teams' prize money payments for 2019 revealed. `https://www.racefans.net/2019/03/03/formula-1-teams-prize-money-payments-for-2019-revealed/`.

Saar, M. and Saar, E. (2020). Can the concept of lifestyle migration be applied to return migration? The case of Estonians in the UK. *International Migration*, 58(2):52–66. DOI: 10.1111/imig.12601.

Saaty, T. L. (1977). A scaling method for priorities in hierarchical structures. *Journal of Mathematical Psychology*, 15(3):234–281. DOI: 10.1016/0022-2496(77)90033-5.

Saaty, T. L. (1980). *The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation.* McGraw-Hill, New York.

Sen, A. (1985). *Commodities and Capabilities.* Amsterdam:North Holland.

Sen, A. (1992). *Inequality Reexamined.* Oxford University Press.

Shapley, L. S. (1953). A value for n-person games. In Kuhn, H. W. and Tucker, A. W., editors, *Contributions to the Theory of Games Volume II*, volume 28 of *Annals of Mathematical Studies*, pages 307–317. Princeton University Press, Princeton, New Jersey. DOI: 10.1515/9781400881970-018.

Shapley, L. S. and Shubik, M. (1954). A method for evaluating the distribution of power in a committee system. *American Political Science Review*, 48(3):787–792. DOI: 10.2307/1951053.

Soares de Mello, J. C. C. B., Gomes Júnior, S. F., Angulo-Meza, L., and Mourão, C. L. d. O. (2015). Condorcet method with weakly rational decision makers: A case study in the Formula 1 Constructors' Championship. *Procedia Computer Science*, 55:493–502. DOI: 10.1016/j.procs.2015.07.024.

Somarriba, N. and Pena, B. (2009). Synthetic indicators of quality of life in Europe. *Social Indicators Research*, 94(1):115–133. DOI: 10.1007/s11205-008-9356-y.

Szczypińska, A. (2018). Who gains more power in the EU after Brexit? *Finance a Uver*, 68(1):18–33.

Szulc, B. (1964). Indeksy dla porównań wieloregionalnych. *Przeglad Statystyczny*, 3:239–254.

Vandebroek, T. P., McCann, B. T., and Vroom, G. (2018). Modeling the effects of psychological pressure on first-mover advantage in competitive interactions: The case of penalty shoot-outs. *Journal of Sports Economics*, 19(5):725–754. DOI: 10.1177/1527002516672060.

Widgrén, M. (1994). Voting power in the EC decision making and the consequences of two different enlargements. *European Economic Review*, 38(5):1153–1170. DOI: 10.1016/0014-2921(94)90042-6.

Williams, C. and Crawford, G. (1980). Analysis of subjective judgment matrices. Interim report R-2572-AF, Rand Corporation, Santa Monica.

Wish, N. B. (1986). Are we really measuring the quality of life? Well-being has subjective dimensions, as well as objective ones. *American Journal of Economics and Sociology*, 45(1):93–99. DOI: 10.1111/j.1536-7150.1986.tb01906.x.

World Bank (2017). Brief: Migration and Remittances Data. November 16, 2017. `http://www.worldbank.org/en/topic/migrationremittancesdiasporaissues/brief/migration-remittances-data`.

List of publications

Publications in English

Scientific articles

1) Csató, L. and Petróczy, D. G. (2021a). On the monotonicity of the eigenvector method. *European Journal of Operational Research*, 292(1):230–237. DOI: 10.1016/j.ejor.2020.10.020.

2) Petróczy, D. G. (2021a). An alternative quality of life ranking on the basis of remittances. *Socio-Economic Planning Sciences*, 78:101042. DOI: 10.1016/j.seps.2021.101042.

3) Petróczy, D. G. and Csató, L. (2021). Revenue allocation in Formula One: a pairwise comparison approach. *International Journal of General Systems*, 50(3):243–261. DOI: 10.1080/03081079.2020.18702.

4) Petróczy, D. G., Rogers, M. F., and Kóczy, L. Á. (2022). Exits from the European Union and their effect on power distribution in the Council. *Games*, 13(1):18. DOI: 10.3390/g13010018.

Working paper

5) Csató, L. and Petróczy, D. G. (2022). Fairness in penalty shootouts: Is it worth using dynamic sequences? Working paper. arXiv: 2004.09225.

Other

6) Petróczy, D. G. and Németh-Durkó, E. (2019). Using real options for real estates In Dömötör, B. and Váradi, K. (eds.): *Selected Chapters of Corporate Finance and Risk Management*, pages 66–75. Corvinus University of Budapest, Budapest.

Publications in Hungarian

Scientific articles

7) Petróczy, D. G. (2012). Az adócsalás egy ágensalapú modellje *Köz-gazdaság*, 7(4):199–210.

8) Petróczy, D. G. (2015). Stabil párosítások a gyakorlatban : Alvin E. Roth: Who Gets What - and Why. The New Economics of Matchmaking and Market Design. Houghton Mifflin Harcourt, New York, 2015, 260 oldal *Competitio*, 14(2):83–87. DOI: 10.21845/comp/2015/2/5.

9) Petróczy, D. G. (2017). Korrupciós kapcsolatok - Egy ágensalapú megközelítés *Köz-gazdaság*, 12(3):111–128.

10) Petróczy, D. G. and Németh, Bálint (2017). Korrupciós kapcsolatok *Köz-gazdaság*, 12(4):241–255.

11) Csató, L. and Petróczy, D. G. (2018). Néhány gondolat a labdarúgás rangsorolási szabályairól a 2018. évi labdarúgó-világbajnokság európai selejtezője kapcsán. *Közgazdasági Szemle*, LXV(6):632–649. DOI: 10.18414/KSZ.2018.6.632.

12) Csató, L. and Petróczy, D. G. (2019). Hogyan tehető igazságosabbá a labdarúgó mérkőzéseket követő büntetőpárbaj? *Statisztikai Szemle*, 97(8):779–798. DOI: 10.20311/stat2019.8.hu0779.

13) Petróczy, D. G., Rogers, M. F., and Kóczy, L. Á. (2019). Tagkilépések és a magyar befolyás változása az Európai Unió Tanácsában. *Alkalmazott Matematikai Lapok*, 36(1):65–81.

14) Petróczy, D. G. (2020). Egy életminőség-rangsor a hazautalások alapján. *Szigma*, LI(2):169–184.

15) Csató, L. and Petróczy, D. G. (2020). Miért igazságtalan a 2020-as labdarúgó-Európa-bajnokság kvalifikációja? *Közgazdasági Szemle*, LXVII(7–8):734–747. DOI: 10.18414/KSZ.2020.7-8.734.

16) Petróczy, D. G. (2021b). Teljesítményalapú pénzfelosztás a Forma–1-ben páros összehasonlításokkal. *Szigma*, LII(1):61–74.

Conference paper

17) Petróczy, D. G. (2018). Egy életminőség-rangsor a hazautalások alapján. In Temesi, J. (ed.): *XV. Gazdaságmodellezési Szakértői Konferencia : Elõadások* Gazdaságmodellezési Társaság, Budapest. pages 97–106. o.

Other

18) Németh-Durkó, E. and Petróczy, D. G. (2019). Reálopció–Napelem telepítés. In: Walter, Gy. (ed.): *Pénzügyi, Vállalati Esetek és Döntések*, pages 40–47. Corvinus University of Budapest, Budapest.