



**DOCTORAL  
SCHOOL OF  
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## **THESIS ABSTRACT**

**Péter Vakhal**

**Network analysis of global value chains**

**Supervisor:**

**Dr. Erzsébet Kovács, CSc**  
university professor

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**DEPARTMENT OF OPERATIONAL RESEARCH AND ACTUARY  
SCIENCES**

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## **I. Research background**

The phenomena of suppliers' interdependency in the international labour share and the lack of robustness of the global economy has been already known before 2020 (Cerina et al., 2015; Donato et al., 2015; Garlaschelli & Loffredo, 2005), however when the Chinese government locked down one province only, the global trade collapsed as a house of cards. The countries, industries and companies are directly or indirectly connected, which results in an interdependent, multicollinear world economic system. In that landscape, events such as the coronavirus pandemic in 2020 can easily generate chaos just as Lorenz poetically described the butterfly effect (Lorenz, 1963).

In the past decades the organisation of companies (and indirectly countries) into production networks has restructured the dependencies, and thus one cannot clearly conclude whether reliance goes from a smaller country to a larger one or the other way around. In such systems, the role of companies or industries in the network has become one of the principal research questions. Most scholars study the links, development paths, and outlooks of modern supply chains.

Between 1990 and 2018, the volume of the global export of goods at constant price was tripled. Nowadays, all the countries are part of the global trade apart from a few exceptions (countries under embargo). The global spread of multi- or transnational companies, the change in production and inventory management processes, outsourcing, free trade agreements, organisation into value chains, free movement of capital, low interest rates, and liquidity on the money markets contributed to the severe increase in global trade volume. The growth policy of the USA and China built on the increase in consumption provided sufficient demand until the global economic crisis in 2009.

Following the financial crisis of 2009, the extremely fast processes called hyperglobalisation (Rodrik, 2012) were stalling, and a slow-down occurred, which was called 'slowbalisation' by Timmer et al. (2016) or 'deglobalisation' by Antràs (2020). Since 2010, it is evident that the global trade became fragmented and regional blocks began to emerge because of protectionist trade policies. They replaced the global production structure (Baldwin & Lopez-Gonzalez, 2015), and consequently, the value chains became shorter (Miroudot & Nordström, 2019), individual companies started to repatriate their production to their own countries or at least

closer to the final consumer (Ancarani et al., 2019; Backer et al., 2018). Consequently, the global business environment has been rapidly changing without interruption which also affects the performance of national economies.

The research of global value chains (GVCs) is hindered by two factors. First, the theory of GVCs is not clear, and recent studies have underlined the disadvantages of integration to the GVCs (McGrath, 2013; Stringer & Michailova, 2018) and shortcomings of current theories. Second, the official statistics cannot cope with the development of globalisation.

The latter one is extremely problematic. One of the main challenges in GVC accounting is the inaccurate definition of resident companies that consider subsidiaries of trans-national companies as separate entities (Rassier, 2017), which can cause severe biases in macroeconomic statistics like GDP, GNI, FDI, IIP and some items of the balance of payments (Dridi & Zieschang, 2004; Mead, 2014; Nakamura et al., 2015). New forms of trade emerged and these pose further challenged to the statistical agencies, because it is almost impossible to trace the transactions within the GVCs. The international input-output (IO) tables, by the fact that they consider the global economy as a closed system, seemingly provide an alternative solution; however, these databases are also based on official statistics and thus conserve these biases that were a priori included in the statistics.

Albeit, the topic is widely researched, no universal definition of the GVCs exists. There are a few definitions applied parallel in the literature (Kaplinsky (2000), Koopman et al. (2014), Porter (1998), however most of them mix the value chains and the production network. These two must be separated and in this the study of Sturgeon (2001) is the guiding principle:

**1. Table: The definition of value chain and production network**

Name	Definition	Metrics	Other names
<b>Value chain</b>	The sequence of productive activities leading to and supporting end use.	Bundles of activities that various actors do, or do not, engage in.	supply chain, production chain, activities chain
<b>Production network</b>	A set of inter-firm relationships that bind a group of firms into a larger economic unit.	The character and extent of inter-firm relationships.	value network, supply base

Source: Sturgeon (2001)

Therefore, it is possible to evaluate and rank the importance of the companies, company groups, and countries (taking part in the GVC) in the value chain. This enables the connection between business and national competitiveness and GVCs. There are only two ways for a company to stay in the competition. Either it establishes a stable position in the strong competition at the current level or it does the upgrade and uses the competitive advantage of innovation as a newcomer. In the context of that it must be stressed that the relation of GVCs to competitiveness is not self-evident. Of course, every export success can be interpreted as a success of competitiveness upon one condition, namely the exporter must produce higher value-added. However, this is also ambiguous, because the level of value-added content depends on many factors and industries producing high value-added still can be less competitive or vice versa. The position in the GVCs, and the successes achieved on that stage are only a part of the national competitiveness, because other factors like institutional background, human development or healthcare are also necessary and essential (Palócz & Vakhai, 2018).

It has been a considerable milestone in value chain research to include the concept of value-added, as it is a measurable value on micro and macro levels. On the basis of the produced value-added, a relation can be determined between the production sequences. All this has considerably softened the definition of the GVC (Koopman et al., 2010): GVCs '*... are a system of value-added sources at different locations in a globally integrated production network*'. Therefore, the latest definition does not cover product types, distribution channels, companies, or companies organised in a chain but organisation into networks is the central focus.

The analysis of IO tables using graph theory is not a new approach, because the structure of the transaction matrices ( $\mathbb{R}_{\geq 0}^{n \times n}$ ) are similar to an adjacency matrix, and many scholars have already investigated their applicability (Alves et al., 2018, 2019; Amador & Cabral, 2017; Ferrarini, 2013; Barigozzi et al., 2011). Still, the literature is enriched by descriptive statistics only. Heretofore, no study in the pertinent literature has been published that deals specifically with the system of value chains in the field of graph theory. One reason behind this could be the fact that the adjacency matrices of IO tables are very special, and from an economic perspective, they have characteristics that make them difficult to analyse using network theory:

1. The edges of the IO adjacency matrix are weighted and directed. At the same time, these weights have a high correlation with the size of nodes representing the size of the economy<sup>1</sup>. Generally, the weight of a random  $V_i$  vertex can be derived from  $W(V_i) = \sum_{j=1}^n a_{ij}$  (where  $a_{ij}$  is the weights of the edges), which results in the following:  $cor(W(V_i), \rho_i) > 0$ , where  $\rho_i$  is degree of node  $i$ . In other words, the weight of the node correlates with the number of edges. In the special-case IO matrices, this relation does not hold, because the network is complete (i.e. every node is connected to every other node); thus, the weights of the vertices depend on the size of the country it represents, consequently interpreting the edge weights as distances are inaccurate.
2. The expected values of gross value added produced at different production stages are not equal, that is  $E(x \in S) \neq E(x \in S')$ , where  $x$  is the production sequence,  $S$  and  $S'$  are the different stages of production (Sturgeon et al., 2013). If they were approximately equal, one could apply the classical segmentation algorithms. These methods cannot be utilised directly because the procedures rely on distance metrics, which are not satisfied (see the first point).
3. There are no clearly distinct production processes because there are no producers in the world who would not use any imported value added from a foreign country<sup>2</sup>. Thus, one cannot build a network flow because there is no  $t_0$  point of source. Nevertheless, the end points are known, because once the final good is made in the production process, there will be no more transformation. Knowing only the final points (sinks), the flows cannot be interpreted as a whole but by stages only.
4. The path of value-added flow cannot be simplified using different tools of combinatorics (there is no ‘shortest path’), which is a consequence of the previous point. The existence of such a path would also be inaccurate from economic perspective because it would assume that the production can be rationalised if some edges or nodes (countries) are left out. That would assume that the structure of global trade is not optimal, which would contradict to the theory of perfect competition.

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<sup>1</sup> The term ‘size of the economy’ represents the total value added created by the country or industry.

<sup>2</sup> In some international IO tables – like the WIOD – there are elements with 0 value, which indicates no transaction between the units. However, this only means that the value is below a predefined threshold. The Eora database, which is utilised in this dissertation (see Chapter 4), does not apply such threshold values, and thus all elements in the IO matrix are above zero.

5. On account of the preceding point, it is worth examining dependency in the GVCs. At the same time, it is beyond the purview of network theory, because besides the dependency on raw materials, there are other (political, cultural, and historical) factors that play a crucial role in the development of trade (for example, in the form of a free trade agreement) (Pratono, 2019).
6. In contrast to the classical models in graph theory, the diagonal of the adjacency matrix ( $a_{ij}$ ) has a key role. In many cases in the traditional models, the values on the diagonal are zero. For IO matrices, however, it is not true – even more, they are the largest elements in the matrix (alternatively, if there were any larger elements in the matrix, it would mean that the industry exports more than it uses, which is highly unlikely). In dependency analysis, it means that all nodes depend mainly on themselves. It can be concluded that in competitiveness analysis, the effectiveness of domestic production is more important than foreign linkages.

Another reason is, that GVC data does not constitute time series, and must be handled as cross-sectional data. By mapping the value chains as a network, one could analyse the time dimension as well. This expands the statistical–econometric framework and could reveal certain aspects of value chains that were hidden. The analysis of production sequences can turn the static approach into a dynamic one, and by that, one can get more accurate position of a country or industry in any supply chain.

The aim of this dissertation was to connect the disciplines of graph theory and economics and to make the necessary adjustments in the toolbox of network theory. By that one can map the position of Hungarian industries and companies in GVCs.

The analysis of GVCs necessitates a complex and interdisciplinary approach, which develops the consistency between the estimations and their economic interpretation. This dissertation applies a holistic approach and analyses the GVCs through regional examples, while it puts the Hungarian economy and Hungarian firms into focus. This dissertation poses the following research questions:



- How do the globalised international trade, production in value chains, and new forms of trade affect official statistics?
- Considering the bilateral trade relations of Hungary, how could the companies and industries be positioned in the GVCs? How could the relations that have particular importance be visualised?
- How far can the Hungarian value-added get in the GVCs? Which routes are the most important? Where are the hubs?
- How do the sequential differences affect the value of the GVC indicators? Is it relevant when countries with similar production profiles enter the production chain?
- How do domestic firms participate in value-added flow in the GVCs?

## II. Methodology

The dissertation relies on the method of algebra, graph theory and statistics and it combines them to conclude inferences. The analyses are based on the IO transaction tables that record the bilateral flow of goods and services between the domestic industries and the consumers. The standard IO table is depicted in table 2:

**2. Table: Flow table for a two-sector economy**

		Processing sectors		Final demand (f)				Total output
		I	II					
Processing sectors	I	$z_{11}$	$z_{12}$	$c_1$	$i_1$	$g_1$	$e_1$	$x_1$
	II	$z_{21}$	$z_{22}$	$c_2$	$i_2$	$g_2$	$e_2$	$x_2$
Payment sectors	Value-added	$l_1$	$l_2$	$l_C$	$l_I$	$l_G$	$l_E$	$L$
		$n_1$	$n_2$	$n_C$	$n_I$	$n_G$	$n_E$	$N$
Import		$m_1$	$m_2$	$m_C$	$m_I$	$m_G$	$m_E$	$M$
Total outlays		$x_1$	$x_2$	$C$	$I$	$G$	$E$	$X$

Source: Miller and Blair (2009)

The total output (including  $i = j$ ) can be depicted with matrix notations:

$$\mathbf{x} = \mathbf{Z}\mathbf{i} + \mathbf{f} \quad (1)$$

The final demand of industry  $i$  is given by  $f_i = c_i + i_i + g_i + e_i$ , where  $c_i$  is the household consumption,  $i_i$  is the private gross capital formation,  $g_i$  is the public gross capital formation, and  $e_i$  is the net export. In the next section, the technological coefficient shall be introduced,

which is defined by  $a_{ij} = z_{ij}/x_j$ . Using  $(\hat{\mathbf{x}})(\hat{\mathbf{x}})^{-1} = \mathbf{I}$ , we get  $\hat{\mathbf{x}}^{-1} = \begin{bmatrix} 1/x_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/x_n \end{bmatrix}$ . If the  $\mathbf{Z}$

transaction matrix is multiplied by  $(\hat{\mathbf{x}})^{-1}$  from the right, we get the technological matrix denoted by  $\mathbf{A}$ :

$$\mathbf{A} = \mathbf{Z}(\hat{\mathbf{x}})^{-1} \quad (2)$$

The output function in matrix form is  $\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{f}$ . From this, one is able to derive  $(\mathbf{I} - \mathbf{A})\mathbf{x} = \mathbf{f}$ , which is the final demand. It must be noted that the aforementioned equations can be solved if and only if  $(\mathbf{I} - \mathbf{A})^{-1}$  exists, that is,  $|\mathbf{I} - \mathbf{A}| \neq 0$ . If the inverse matrix exists, the system of equations have a solution, which can be expressed as the following:

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} = \mathbf{B}\mathbf{f} \quad (3)$$

where  $(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{B} = [b_{ij}]$ , or in other words, the Leontief inverse. In the possession of world IO tables the global Leontief inverse can be estimated which can be utilised to disaggregate the value-added flow in the GVCs.

If  $\mathbf{B}$  is known, the flow of direct and indirect value-added between two random points can be disaggregated in the following way:

$$int.e_{i,x,m} = \underbrace{\langle \mathbf{VA}_i \rangle \mathbf{B}_{i,i} int.e_{i,x}}_{\text{domestic VA in } i} + \sum_{m \neq i}^n \underbrace{\langle \mathbf{VA}_m \rangle \mathbf{B}_{m,i} int.e_{i,x}}_{\text{import VA in } i} + \underbrace{\langle \mathbf{VA}_x \rangle \mathbf{B}_{x,i} int.e_{i,x}}_{\text{reimport VA in } x} \quad (4)$$

where

$int.e_{i,x}$ : the intermediate export from country  $i$  to country  $x$ ;

$\mathbf{VA}_i, \mathbf{VA}_m, \mathbf{VA}_x$ : the VA/output ratios in the exporter country  $i$ , the importer country  $m$  and in the export partner country  $x$ ;

$\langle \cdot \rangle$ : diagonal matrix;

$\mathbf{B}_{i,i}$ : the final direct demand country  $i$ ;

$\mathbf{B}_{m,i}$ : the Leontief inverse of the indirect import partner  $m$  in country  $i$ ;

$\mathbf{B}_{x,i}$ : the Leontief inverse of indirect import export partner  $x$  in country  $i$ .

Consequently, equation (4) can be interpreted as:

gross export = domestic value-added + re-imported domestic value-added from import partners + re-exported value-added from the export partners

The value-added from the import partners can be further disaggregated, as it contains all direct and indirect value-added from all other trade channels:

$$\begin{aligned}
 REII_{i,m_1,m_i,x} &= \underbrace{\langle VA_{m_1} \rangle B_{m_1,m_1} REII_{i,m_1,x}}_{\text{domestic VA in } m_1 \text{ directly to } i} + \underbrace{\sum_{i=2}^n \langle VA_{m_i} \rangle B_{m_i,m_i} REII_{i,m_1,x}}_{\text{domestic VA in } m_1 \text{ via } m_i \text{ to } i} \\
 &+ \sum_{i=2}^n \underbrace{\varepsilon_{i,m_1,m_i,x}}_{\text{reimported domestic VA in } m_1 \text{ via } m_i \text{ to } i} \\
 & \quad i = \{2,3 \dots n\}
 \end{aligned}
 \tag{5}$$

where

$REII_{i,m_1,x}$ : the value-added produced by country  $m_1$  and traded directly to country  $i$ , and then exported to country  $x$ ;

$REII_{i,m_1,x}$ : the value-added produced by country  $m_1$  and indirectly traded to country  $i$  via country  $m_i$ , and then exported to country  $x$ ;

$\varepsilon_{i,m_1,m_i,x}$ : value-added re-imported by country  $i$  from countries  $m_1, m_i$ , and then exported to country  $x$ .

Equation 5 depicts how far the value-added of a country or industry can get in the value chain directly and indirectly. With the help of that method the direct and indirect path of the Hungarian value-added produced by the automotive industry could be traced.

However, that did not reveal the position of Hungary in the global value chain. The question could have been answered by the segmentation algorithm often applied in network science, however adjustments must be made before that.

A method called *vertical and horizontal detection* will be proposed for which one must define some crucial characteristics of value chains:

**1. Definition:** Let  $G = (V, E(w))$  be a complete graph, where  $V$  denotes set of the vertices,  $E$  the set of edges, while  $w$  denotes the weights of the latter.

We are looking for  $\gamma_i(G/S_i)$ , which is the value of  $S_i$  partition of the complete graph  $G$ , which depends on the subgraphs of  $V(G/S_i)$  and  $E(G/S_i, w)$ .

**2. Definition:** The value of a random  $V$  node can be determined by  $\gamma(V) = \sum_{i=1}^k E(w_i)$ , that is the sum of all edge weights<sup>3</sup>. Consequently, the total value of graph  $G$  is  $\gamma(G) = \sum_{i=1}^v \gamma_i(V|G)$ , which is the sum of all edge weights in the network.

During segmentation, the  $S_k$  partition is compared to  $S_i$  ( $\#E(G/S_k) > \#E(G/S_i)$ ) in a way that we examine how much the value of  $S_k$  decreases if a random edge  $e_i(G/S_k, w_i)$  is cut from the graph. The outcome of the cut is the node set of  $V_e = \{v \in V: vIe_i\}$ , that is the set of those vertices that are still members of the subgraph after the cut of edge  $e_i$ .

**3. Definition:** Let  $C$  be the cost of cut and  $C_{i,k} = \frac{\gamma(S_k) - \gamma(S_i)}{\gamma(S_k)}$  be the normalised cost of the transition from  $S_k$  to  $S_i$  ( $0 \leq C_{i,k} \leq 1$ ). It shows how much value did the subgraph lose after pruning edge  $e_i$ . Owing to operationalisation requirements, the minimum value of  $C$  shall be fixed in the form of  $C \geq \varepsilon$  to decrease the computational demand of the algorithm.

In the first step, only the most important edges remain for the node in focus. In the second step, the algorithm maps the further links, which is the most important part of the process. In this round, the addition of those nodes happens that is not a member of the  $S_k$  subgraph, that is  $V \notin S_k$ . The value of the new vertex can be evaluated in two ways:

1. How much value does the new node add to the previous graph?
2. What is the relationship of the new vertex with the other nodes in the network?

The importance of the second point is that the network should be extended by those nodes that share strong links with the vertices that are already members, and the ‘old’ members are also important for the new one. In that way, it is ensured that the complexity (modularity) of the

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<sup>3</sup> Only the direct and indirect export of the value-added is analysed, thus only the outward edges are considered in the model.

subgraph increases, while the degree distribution is not skewed a lot towards the dominant nodes in the original complete graph. The result is not a star-structure network, but it more resembles a scale-free network<sup>4</sup>. This can be characterised by the degree distribution of the network.

4. Definition: The degree of  $V(G)$  is given by  $d_G(V) = |\{e \in E(G) : v \in e\}|$ , which is the number of nodes in  $G$ . Consequently, the complete degree of graph  $G$  is  $\sum_{i=1}^V d(v_i) = 2|E(G)|$ . The mass probability distribution is given by  $f(x) = P(\{d_G \in D : X(d_G) = x\})$ .

The extension of the network relies on the condition that the new node is valuable for the vertices that are already members of the subgraph, and the bias of the degree distribution is minimised.

5. Definition: Let  $S_1(G) - S_2(G) = \{V_p(G)\}$  true for  $S_1(G) S_2(G)$  subgraphs, that is, they differ in one ( $V_p(G)$ ) node only. Then, the bias caused in the degree distribution by the inclusion of node ( $V_p(G)$ ) is  $b = 1 + \sqrt{(Gini(S_2(G)) - Gini(S_1(G)))^2}$ , ( $1 \leq b \leq 2$ ).

To choose the optimal  $V_p(G)$  node, the algorithm should search for the optimal trade-off between the cut cost and the bias, which implies the following solution:

$$\operatorname{argmax} \left\{ 2 \frac{c_{i,k}}{b_{i,k}} : \frac{c}{b} \in \mathbb{R}^+, 0 \leq \frac{c}{b} \leq 2 \right\} \quad (6)$$

The value of the  $C/b$  fraction is 2 (theoretical maximum) if the new node adds the largest value to the network, while the degree distribution is not changed at all. This ensures that the largest and most crucial nodes in the GVC are involved in the subgraph only if it is important for all other members of the group. With the help of that method, the value chain of Hungary could be identified.

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<sup>4</sup> It is easy to see that in the GVC, the role of some countries (nodes) is crucial for the small economies. Still, it would be wrong to indicate these smaller economies as full members of value chain of the aforementioned dominant countries. For example, it is certain that Eastern Europe exports value-added to South America; however, it would be disproportionate to infer that the region is just as important for Argentina or Brazil as the other South American smaller economies.

One flaw of GVC mapping is that one needs to identify the network flows. The classical IO tables are not suitable for that, thus the product-based statistics should be utilised, because only they provide high frequency data.

The network representation of univariate time series was studied by Lacasa et al. (2008), who proposed a new approach called the visibility graphs (VG). In the  $X_t: \{x_1, x_2 \dots x_n\}$  time series, let two  $\{x_i, x_j\} \in X_t$  elements in any two  $\{t_i, t_j\} \in T$  time. Let two  $x_k \in X_t$  elements at the same time of  $t_k \in T$ , which satisfy the  $t_i < t_k < t_j$  inequality. Then,  $x_i$  and  $x_j$  are visible for each other if the following inequality is true:

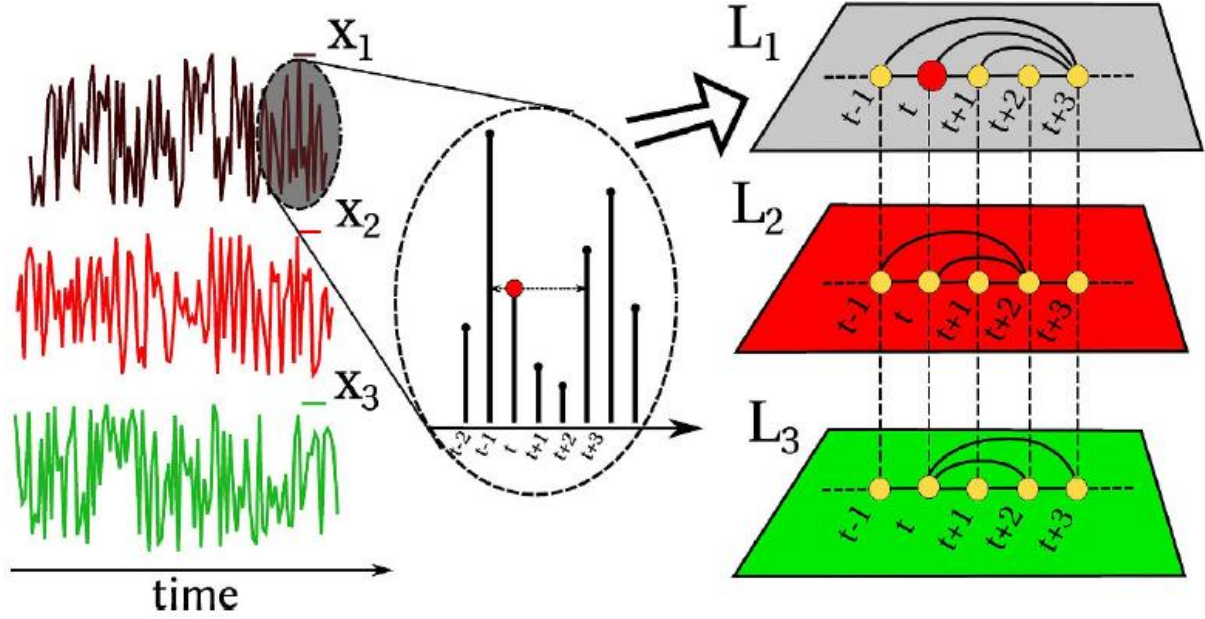
$$x_k < x_j + (x_i - x_j) \frac{t_j - t_k}{t_j - t_i} \quad (7)$$

If the inequality expressed in equation (39) is true, then the two elements at different times, represent two nodes that can be linked by an edge. The interpretation of the connection is that two time markers ‘see each other’, because all elements between the two markers are smaller. The visibility graph defined over the univariate time series can be readily extended to the multivariate case (Luque et al., 2009). Let  $\{x_i\}_{i=1 \dots N} x_i \in \mathbb{R}$  be  $N$  time series. However, as there are no constraints for the moments of the time series, it is worth standardising the values into a  $[0; \infty]$  scale. The standardisation does not change the distribution of the data. In the network representation of two time markers in two different  $x_i \in X_k$  and  $x_j \in X_l, k \neq l$  univariate time-series, two nodes can be linked only if the following geometrical inequality is satisfied:

$$x_i, x_j > x_n \quad \forall n: i < n < j \quad (8)$$

One can generate the visibility graphs for every  $M$  univariate time series, which can be horizontally connected if the inequality described by equation (40) is satisfied. Thus, a multidimensional, so-called horizontal visibility graph (HVG) can be created. Let  $A = \{A^1, A^2 \dots A^M\}$  the set of adjacency matrices for the  $1, 2, \dots M$  univariate time series, where  $a_{ij} = 1$  if and only if the nodes of  $i$  and  $j$  are linked with a unidirectional edge.

**1. Figure: A representative case of horizontal visibility graphs**



Source: Lacasa et al. (2015)

On the basis of the HVG diagrams, one can draw meaningful conclusions concerning the dynamics of the multivariate time series. Let  $x_t^1$  be a random value from a three-dimensional  $X: \{x_t^1, x_t^2, x_t^3\}$  time series. The algorithm examines the environment of  $[x_{t-\varepsilon}^2, x_{t+\varepsilon}^2]$  and  $[x_{t-\varepsilon}^3, x_{t+\varepsilon}^3]$ , where  $\varepsilon \in \mathbb{N}$  represents the time shift and creates the  $A^{|X|t \times |X|t}$  adjacency matrix, where  $|X|$  denotes the number of elements of set  $X$ . In this example, the  $A^{3t \times 3t}$  adjacency matrix is a symmetrical block matrix (for simplicity, let us denote it by  $A$ ):

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = A^T \quad (9)$$

In the lower triangular matrix<sup>5</sup>, let  $s_i = \sum_{j=1}^t a_{ij}, i \neq j$  be the sum of the  $i$ th row in the block of  $A_{ij}^{t \times t}$ , that is, the number of those connections<sup>6</sup> through which the particular node is linked to nodes in other time series. Let us define variable  $d_{ij}$ , which depicts the time relations of linked nodes<sup>7</sup>:

<sup>5</sup> Owing to the symmetrical adjacency matrix.

<sup>6</sup> Because the network is undirected and  $a_{ij} = [0,1]$  the sum of rows and columns is equal.

<sup>7</sup> As the network is undirected, it does not matter if one utilises the sum of the columns or the rows. However, in compliance with the algebraic conventions, the rows are used as the base of the analysis.



$$d_{ij} = \begin{cases} i - j & \text{ha } a_{ij} = 1 \\ 0 & \text{ha } a_{ij} = 0 \end{cases} \quad (10)$$

If the value of  $d_{ij}$  is negative, the node in row of the adjacency matrix is earlier in time compared to the node in the column. The value is positive if the row element happened later than the column element, and it is zero if the row node has the same time marker as the column or there is no edge between the two. The following equation defines the average time shift between the time series:

$$d^{ij} = (\sum_{i=1}^t s_i)^{-1} \sum_{j=1}^t d_{ij}, \forall d: i \neq j, s_i > 0 \quad (11)$$

As the length of the average shifts bears less importance, one can simplify the results by utilising indicator functions:

$$I(d^{ij} > 0) = 1 \quad (12)$$

$$I(d^{ij} < 0) = -1 \quad (13)$$

$$I(d^{ij} = 0) = 0 \quad (14)$$

On the basis of the indicator function, one can map the sequential shifts between the time series. With the help of HVG method, the order of supply and production process of automotive industry in the Central and Easter European region was estimated.

Owing to the aggregated IO data, no analysis at IO level can be made to map local GVCs. Therefore a novel approach was utilised to create a link between the nation IO tables and the balance sheet of the company register. By that one can disaggregate the nation IO according to company size. The intra-industrial trade of intermediate goods was estimated by the so-called RAS<sup>8</sup> method, which is a balancing algorithm (Miller & Blair, 2009). RAS is an iterative algorithm with an objective function that aims to get an estimated matrix in which the edges are equal to the predefined values. The estimation is based on a previous full matrix, and thus, the inner structure is preserved. In particular, the base matrix was the IO table containing the national intermediate-use data, and thus, one could take advantage that the objective function

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<sup>8</sup> RAS is not an abbreviation but the actual name of the process. Its name comes from the original article in which the authors calculated with three matrices that were denoted by R and S, while A refers to the transaction matrix.

was such that the sum data by firm size must be equal to the sum of data at national level. The used is summarised in the next table:

**3. Table: Introduction to data sources**

<b>Data type</b>	<b>Data description</b>	<b>Data source</b>
Industrial output, import use, value-added, export, final use	The production components that can be found on the edge of the transaction matrix of the IO table. Values are at 2008 prices.	Central Statistical Office, table PP1109
Merchandise trade according to firm size	Foreign trade by corporate self-declaration according to firm size. In case of export the threshold is 100 million HUF, for the import it is 170 million HUF in case of trade within the European Union. Data collection in case of trade with third countries is fully covered.	Central Statistical Office, table 3.5.27.
Corporate export and financial data	The export data serve as a control variable to check the data of the CSO. Financial data are for creating distributions of output, value-added, and export according to firm size.	NAV database

Source: own collection

With the help of the IO methods introduced earlier, one can evaluate the participation of Hungarian firms in the GVCs.

### III. Scientific achievements of the dissertation

With the help of the summarised methods, all research questions could be answered, as well as all hypothesis could be confirmed. For the sake of clarity this is summarised in the following table:

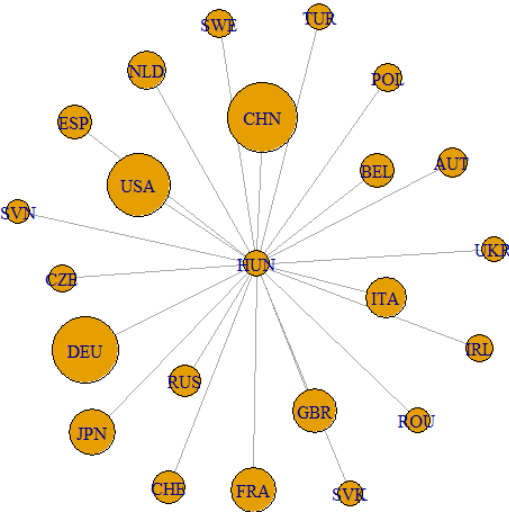
**4. Table: Summary of the research questions, hypothesis and results**

Research question	Hypothesis	Result
How do the globalised international trade, production in value chains, and new forms of trade affect official statistics?	The current accounting practices of official statistics concerning the transactions between companies in the same value chain can significantly bias macroeconomic statistical indicators.	Confirmed
Considering the bilateral trade relations of Hungary, how could the companies and industries be positioned in the GVCs? How could the relations that have particular importance be visualised?	Hungary has strong relationship with other regional countries, while the connection with economies outside the European Union is rather weak.	Confirmed
How far can the Hungarian value-added get in the GVCs? Which routes are the most important? Where are the hubs?	The Hungarian value-added is circulating mainly in Europe, and typically one cannot measure its presence outside Europe.	Confirmed
How do the sequential differences affect the value of the GVC indicators? Is it relevant when countries with similar production profiles enter the production chain?	Compared to economies of similar profiles, Hungary has joined the value chains later and this has biased the value of GVC indicators downwards.	Confirmed
How do domestic firms participate in value-added flow in the GVCs?	The volume of indirect value-added of the participating Hungarian companies in the value chains is larger than the direct flow, and that amount is mainly produced by the small- and medium-size enterprises.	Confirmed

Source: own edition

This dissertation presented two self-developed models. In their current form, these models have not been discussed yet in the literature. The algorithm introduced above was the first one that proposed an accurate position of Hungary in GVCs by narrowing the complete global graph, while it keeps the network of Hungary in focus.

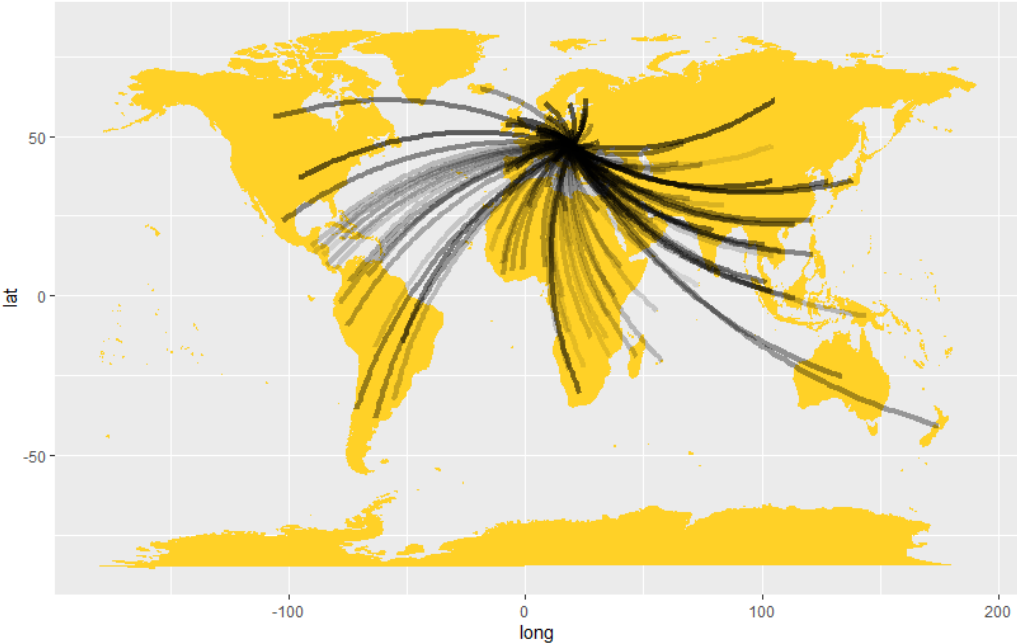
**2. Figure: Value-added flowing directly and indirectly to Hungary based on the vertical and horizontal segmentation algorithm**



Source: own estimation based on Eora data

The path of direct and indirect value-added became traceable even via multiple mediators. Among many industries the one that is the most important for Hungary, the automotive sector, was analysed. The following maps show the paths of value-added (direct and indirect) of the Hungarian carmakers to Germany.

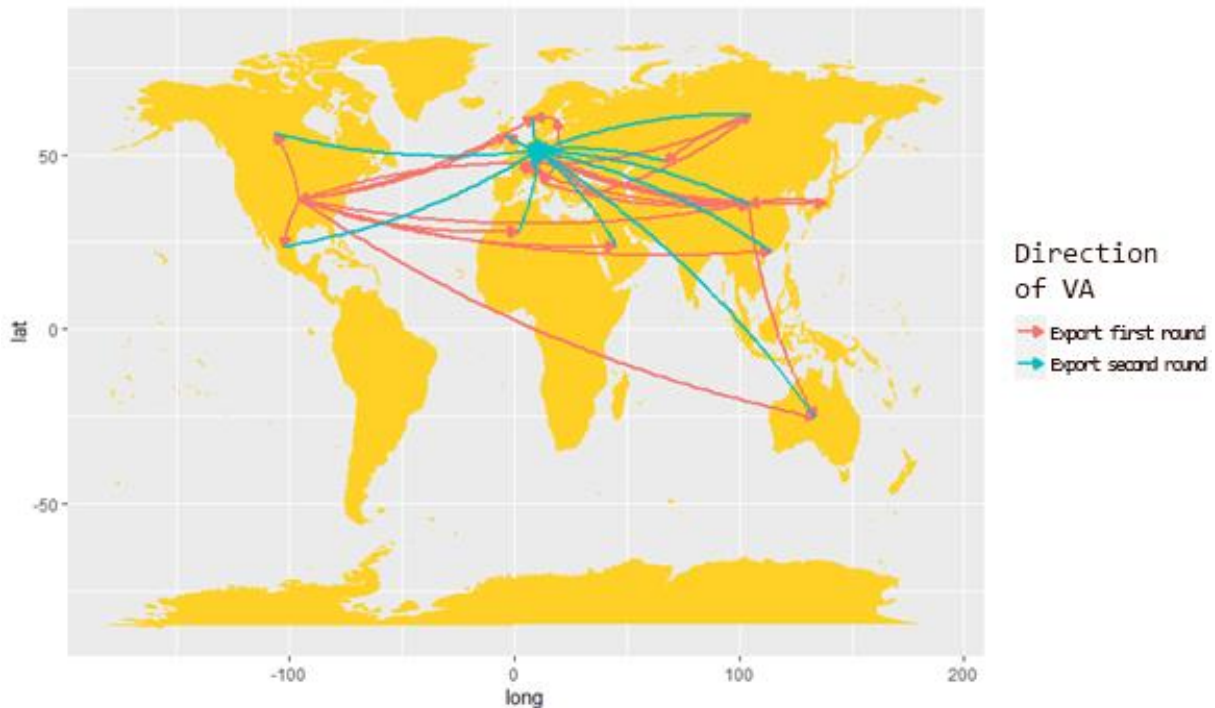
**3. Figure: The paths of value-added produced by the Hungarian automotive sector to Germany (word map)<sup>9</sup>**



Source: own estimation based on Eora data

<sup>9</sup> For the sake of transparency, the edges linking the partner country to Germany are not displayed. The width of the curves is proportional to the volume of value-added.

**4. Figure: The paths of value-added produced by the Hungarian automotive sector to Germany (top30 partners)**



Source: own estimation based on Eora data

This dissertation contributes to the literature with the following theses:

**1. Thesis:** The current statistical accounting procedures may cause biases in the macroeconomic statistical indicators. Owing to the inadequate and ambiguous definition of resident companies and to the new forms of trade emerged in the past decades, even the GDP can be biased. That could be eased by applying modern technologies (such as blockchain) that can connect the production, trade and domestic demand statistics.

**2. Thesis:** The analysis of global value chains in their current aggregated form is burdensome. For a more accurate investigation one needs to extract the regional and industrial chains. The application of classical network segmentation procedures is very limited in that case, thus a modified algorithm was proposed. The basis of the process is similar to the one applied in random graph growing.

**3. Thesis:** Mapping the value-added flow in a disaggregated value chain is hindered by the lack of source point in the network. A special method was utilised in order to reveal the path of value-added of any country or industry.

**4. Thesis:** Another aspect of value chain analysis is proposed by sequence mapping approach. This method can order suppliers of different countries on the same value chain, which is crucial to study GVC dynamics.

**5. Thesis:** Hungarian firms participate in global value chains both directly and indirectly. The latter one is generally valid for companies that cannot have their own export market. The direct investigation of this problem is not possible through the input-output data, thus a new method was developed, which forms the connection between the national input-output tables and the company balance sheet dataset.

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