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Ph.D. Dissertation entitled

ESSAYS ON THE THEORY OF TWO-SIDED MARKETS

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Introduction

Several markets have emerged in which consumers and sellers interact and make transactions through an intermediary or platform. Examples include smart phones, bank cards, game consoles, shopping centers but even airports. The peculiarity of these markets is that the actors join them because the platforms have a large number of members of either their own or the other segment. Such markets are referred to in the literature as two-sided markets, which, through the development of technology, have now appeared in many forms in our daily lives. Although the analysis of two-sided markets is already extensive, it still has unexplored areas. In addition, due to their different functioning from traditional markets, they are also of great interest from a regulatory point of view, whereas the impact of the regulatory tools used may be different and may be particularly harmful in some cases. The chapters in this thesis contribute to a better understanding of two-sided markets.

The results presented in the different chapters are not closely related, so they can be interpreted independently. Therefore, at the beginning of each chapter, the analyzed problem, its justification, as well as a summary of the relevant literature are covered. In each chapter, duopolistic models are presented to determine the impact of platforms’ product differentiation decisions, the use of public and private contracts, and the entry of the state in the market on optimal pricing. Because consumers typically do

\[1\]

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not consider when purchasing a platform’s product or service that they will later purchase compatible products, they do not internalize the benefits of their future consumption, so the models presented are sequential in nature. To analyze these and determine their equilibrium, the tools of game theory are used. Depending on the type of model, the aim is to determine the subgame-perfect Nash equilibrium or the symmetric perfect Bayesian equilibrium.

When it comes to products, one of the most important issues companies have to decide is how to differentiate their products. The more differentiated a product is, the better it meets consumer tastes, and the higher the demand for it. In the literature, product differentiation is typically determined by the location decision of the companies, we can think of models of Hotelling (1929) or Salop (1979). However, in practice, product differentiation is not achieved through a well-chosen location. Rather, by designing the product or service in such a way that it can satisfy as many needs as possible, it can also meet a more specific goal. Thus, if products are differentiated, the consumer suffers a much smaller loss of utility than if the products serve a more general purpose. In light of the above, in our analysis, the platforms decide on product differentiation by determining the transportation cost. Furthermore, instead of the usual symmetric network effect in the literature of two-sided markets, asymmetric markets are examined. It is concluded that profit-maximizing platforms offer either full customizability to their consumers, but therefore charge a positive price, or limited customizability but at a zero price. Thus, it is argued that in two-sided markets, it is not necessarily a problem if the platform has a large market share in one segment, provided that it has to compete for subscribers on the other side. Based on our results, the competition on one side will automatically spill over to the other side, and if not in the prices charged, it will feel its effect in a different dimension.

In Chapter 2, the impact of observable and unobservable contracts applied in two-sided markets are examined. The literature typically assumes public contracts in the analysis, the examination of private contracts rather neglected. These appear only as an alternative strategy, but their
effects have not been incorporated into the models until the paper of Llanes and Ruiz-Aliseda (2015). Under the assumption of a monopolistic platform, the authors analyzed the effect of the above two contract types on equilibrium. However, to the best of our knowledge, a similar attempt has not yet been made for competing platforms. Thus, in the framework of this chapter, the use of public and private contracts are analyzed in a duopolistic environment, their impact on pricing, and sellers’ decision to join. On the one hand, it is confirmed that platforms change their optimal strategy even in the event of competition, as long as they have the opportunity to offer private contracts to sellers and set a high price for the previously supported segment. As a result of private contracts, sellers set a collusive price for their products. This is consistent with the findings of Armstrong and Wright (2007) and Llanes and Ruiz-Aliseda (2015). On the consumer side, on the other hand, in contrast to the results of the above two papers, the platforms do not offer discounts but continue to set a positive subscription fee. However, this is lower for private contracts than for public contracts, and the increase in network effect reduces the subscription fee charged to consumers, while it increases it for public contracts. The results also highlight that the use of public contracts is preferred for platforms, while for consumers, a different outcome may occur depending on the travel cost. It is also shown that platforms offer sellers contracts that they can attract as many of them as possible.

In many markets, it is common for the state to intervene by setting up a company. Two-sided markets are no exception: the most typical example is the media market. In Chapter 3, the answer to the question of how the product differentiation and pricing will be decided in the case of competition between a public firm and a privately-owned platform is looked for. This is justified by the fact that there have already been several attempts in the European Union to ban advertising on state TV broadcasts. However, this makes the state platform inaccessible to one side, and thus essentially ceases to be a platform and continues to operate as a normal company. The results show that if the state appears with a company in a two-sided market, it does not influence its competitor’s
decision to differentiate products. Conversely, on the side of the segment in which the public firm and the private platform compete, the platform approximates the price charged to that of its competitor, while it imposes a monopolistic price on the other segment. Based on the results, the regulation of such markets can be achieved either by applying regulatory tools (tax, price regulation, competition law) or by establishing a state-owned platform.
Chapter 1

Product differentiation in two-sided markets

In this chapter, the decision to differentiate products and the impact of this decision on pricing in the case of two-sided markets are examined. Instead of assuming symmetric network effect, asymmetric markets are analyzed where platforms directly differentiate their products by determining transportation costs. The idea is based on an article by von Ungern-Sternberg (1988), but instead of the Salop’s (1979) circular city model, we examine it in the framework of Hotelling’s (1929) linear city model. The possibility of customization can be important because it also makes the service of the platform more attractive to users who would not otherwise enter the platform.

The analysis is supported by the practice of making products available in two forms, typically in the case of online products: in a form that provides a limited user experience but is available free of charge, or at a positive price in the form of a product or service that provides a number of customizability options, even additional services. It is also pointed out that in two-sided markets, it is not necessarily a problem for a platform to have a high market share in one segment, provided it has to compete for

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1 Based on Bakó and Horváth (2020).
the other side.

1.1 The equilibrium

1.1.1 Competition

In the case of competing platforms, in equilibrium it is found that

\[ t^j_k = \frac{b_l^2}{4} + 4b_k \quad \forall j = 1, 2. \] (1.1.1)

In addition to these transportation costs, the platforms choose the prices \( p^j_k = 0 \) and \( p^j_l = \frac{b_l}{4} \) and realize profits of \( \pi_j = \frac{b_l^2}{16} \). In equilibrium, on the user side, the marginal consumer is at the midpoint, while on the advertiser side, advertisers with \( x \in \left( \frac{b_l}{4}, \frac{4-b_l}{4} \right) \) locations will advertise on both platforms, and other advertisers will only advertise on the platform closer to them.

However, the above result will only be the equilibrium outcome of the game if the basic assumption is true that both sides of the market are fully covered, i.e. that all consumers are willing to enter a platform at given prices. However, this is only the case if the realized utility at the given prices to the consumer realizing the lowest utility after the entry is at least non-negative. That is if

\[ v_k - \frac{1}{2} \frac{b_l^2}{4} + 4b_k + \frac{1}{2} b_k \geq 0, \] (1.1.2)

and

\[ \frac{b_l}{2} - \frac{b_l}{4} - \frac{b_l}{4} \geq 0 \] (1.1.3)

hold. These conditions are met for all \( v_k \geq \frac{b_l^2}{8} \).

1.1.2 Local monopolies

With low \( v_k \) values, some consumers decide that they do not want to connect to any of the platforms at the given prices, i.e. the markets of the
platforms on the user side do not meet. The platforms then act as a local monopoly.

In this case, in the absence of direct competition for users, the platforms will charge prices so that the net benefits realized by the most distant consumers connected to the platforms will be zero, i.e. $v_k + b_{k,j} n_{k,j} - p_{k,j} - t_{k,j} \tilde{x}_{k,j} = 0$ will be satisfied for all $j = 1, 2$. Using the methodology used previously, we get

$$p_k^j = \frac{v_k}{2}, \quad p_l^j = \frac{b_l}{4},$$

$$t_k^j = 0, \quad \pi_j = \frac{v_k}{2} + \frac{b_k}{4} + \frac{b_l}{4} \left(1 - \frac{b_l}{4}\right)$$

(1.1.4)

In equilibrium, therefore, platforms can attract the same number of users as in the previous case. As a result, indierent consumers on the advertiser side will be located at $\frac{b_l}{4}$ and $\frac{4 - b_l}{4}$, and will continue to be advertisers who will advertise on both platforms.

1.2 Results

**Proposition 1.** Profit-maximizing platforms provide users with either maximum customizability at a positive price or somewhat limited customizability at a zero price, depending on the utility from consumption.

**Proposition 2.** Leveraging the dominance of a platform from market power is somewhat hampered by the possibility of customization. In two-sided markets, competition on one side of the market will automatically spread to the other side of the market and, if not in terms of prices, will have an impact on some other dimension.
Chapter 2

Public and private contracts in two-sided markets

The literature analyzing two-sided markets usually assumes public contracts in the market, i.e. all actors have all the information available in the market. One of the earliest articles, which also deals with the application of private contracts, can be linked to Armstrong and Wright (2007), but here their application by platforms only emerges as an alternative strategy. The use of public and private contracts in two-sided markets was first examined by Llanes and Ruiz-Aliseda (2015) in a model framework. According to the literature, private contracts change the optimal behavior of platforms and have a significant impact on the market price structure, and the price charged to the previously unsupported segment decreases, while the previously subsidized segment experiences a substantial increase in fees.

In this chapter, a Hotelling model is build where the impact of the above two types of contracts is examined, namely public and private contracts, within the framework of duopolistic two-sided markets. It is examined whether, in the case of competing platforms, the claim of Armstrong and Wright (2007) that the platforms change their optimal strategy in addition to private contracts also exists, and the finding of Llanes and Ruiz-Aliseda (2015) that the franchise and fixed access fees offered under
the contracts result in collusive prices for platform-specific products. It is also addressed whether, in the context of duopolistic two-sided markets, in addition to public and private contracts, single-homing or multi-homing behavior is preferred by sellers.

In the case of public contracts, every actors know the content of the contracts, while in the case of private contracts, only the platform which made the contract offer and the seller who received it know the content of the contract. The other actors form beliefs about it. Consumers form passive beliefs meaning that the difference in the subscription price causes a change in the access fee of the contract offered to the sellers, leaving the franchise fee unchanged, while sellers form cautious belief meaning that the seller receiving the contract offer believes that the platform has made a contract offer to the other seller as well, and thus, seeks to maximize its realizable profit in the market. These beliefs are determined by the franchise fee and fixed access fee included in the contract. And the seller who receives the contract offer is perfectly able to predict what offer the platform has made to the other seller, i.e. how the platforms have deviated from their optimal behavior. It is also assumed that, according to the seller who forms cautious belief, other sellers may have similar beliefs, and that the platforms are not intended to exclude sellers from the market by offering private contracts.

2.1 The equilibrium

2.1.1 Public contracts

The model of single-homing sellers

The platform optimization results in the following equilibrium when using public contracts and when sellers are connected to only one platform:

\[ w = \frac{1}{3} \left( 1 + b - 4\sqrt{1 + b^2} \right) \]  

(2.1.1)
\[ p_{0,j} = \frac{1}{4} \left( (1 + b)^2 + 2v_k \right) \quad (2.1.2) \]

\[ p_i = -\frac{2}{3} \left( -1 - b + \sqrt{(1 + b)^2} \right) \quad (2.1.3) \]

\[ q_{i,j} = \frac{1}{3} \left( 1 + b + 2\sqrt{(1 + b)^2} \right) \quad (2.1.4) \]

\[ x_{0,1} = \frac{(1 + b)^2 + 8\sqrt{(1 + b)^2} + 8b\sqrt{(1 + b)^2} + 18v_k}{36t} \quad (2.1.5) \]

\[ x_{0,2} = 1 - x_{0,1} \quad (2.1.6) \]

\[ \pi^j = \frac{\left( (1 + b)^2 + 8\sqrt{(1 + b)^2} + 8b\sqrt{(1 + b)^2} + 18v_k \right)^2}{1296t} \quad (2.1.7) \]

\[ CS = -\frac{1}{1296t} \left( 65 + 65b^4 + 16\sqrt{(1 + b)^2} + 4b^3 \left( 65 + 4\sqrt{(1 + b)^2} \right) + 
648t^2 + 6b^2 \left( 65 + 8\sqrt{(1 + b)^2} - 12t + 6v_k \right) + 
36v_k \left( 1 + 8\sqrt{(1 + b)^2} + 9v_k \right) - 
72t \left( 1 + 8\sqrt{(1 + b)^2} + 18v_k \right) - 4b \left( -65 - 12\sqrt{(1 + b)^2} + 
36 \left( 1 + 4\sqrt{(1 + b)^2} \right) t - 18 \left( v_k + 4\sqrt{(1 + b)^2} v_k \right) \right) \right) \quad (2.1.8) \]
The model of multi-homing sellers

In this model, the equilibrium is as follows:

\[ w = \frac{1 + b - 4\sqrt{(1 + b)^2}}{3} \quad (2.1.9) \]

\[ p_{0,j} = \frac{(1 + b)^2}{2 + \phi} + \frac{v_k}{2} \quad (2.1.10) \]

\[ p_i = -\frac{2}{3} \left( -1 - b + \sqrt{(1 + b)^2} \right) \quad (2.1.11) \]

\[ q_{i,j} = \frac{2 \left( 1 + b + 2\sqrt{(1 + b)^2} \right)}{3 (2 + \phi)} \quad (2.1.12) \]

\[ x_{0,1} = \frac{2 + 16\sqrt{(1 + b)^2} + 9 (2 + \phi) v_k}{18t (2 + \phi)} + \frac{2b \left( 2 + b + 8\sqrt{(1 + b)^2} \right)}{18t (2 + \phi)} \quad (2.1.13) \]

\[ x_{0,2} = 1 - x_{0,1} \quad (2.1.14) \]

\[ \pi^j = \frac{\left( 2 + 16\sqrt{(1 + b)^2} + 2b \left( 2 + b + 8\sqrt{(1 + b)^2} \right) + 9 (2 + \phi) v_k \right)^2}{324t (2 + \phi)^2} \quad (2.1.15) \]

\[ CS = -\frac{1}{324t (2 + \phi)^2} \left( 260 + 260b^4 + 64\sqrt{(1 + b)^2} + \right. \]
\[
16b^3 \left( 65 + 4\sqrt{(1+b)^2} \right) + 162(2+\phi)^2 + 12b^2. \\
\left( 130 + 16\sqrt{(1+b)^2} - 6t(2+\phi) + 3v_k(2+\phi) \right) - 36t(2+\phi) \cdot \left( 2 + 16\sqrt{(1+b)^2} + 9v_k(2+\phi) \right) + 9v_k(2+\phi) \left( 4 + 32\sqrt{(1+b)^2} + 9v_k(2+\phi) \right) - 8b \left( -130 - 24\sqrt{(1+b)^2} + 18t(2+\phi) \right). \\
\left( 1 + 4\sqrt{(1+b)^2} \right) - 9v_k(2+\phi) \left( 1 + 4\sqrt{(1+b)^2} \right) \right) \tag{2.1.16}
\]

### 2.1.2 Private contracts

The model of multi-homing sellers

Optimally, the following four conditions must be met:

\[
1 + 4\Phi \Sigma + \Theta - 4\Phi \Theta + b(1+\Theta) + \Gamma(4\Theta - 1 - 4\Sigma) = 0 \tag{2.1.17}
\]

\[
2 \left( 2\Sigma^2 - 1 - \Theta - 4\Sigma\Theta + 2\Theta^2 \right) = 0 \tag{2.1.18}
\]

\[
1 + b - 2\Phi = 0 \tag{2.1.19}
\]

\[
1 - 2\Sigma = 0 \tag{2.1.20}
\]

based on which \( \Sigma = \frac{1}{2}, \Phi = \frac{1+b}{2}, \)

\[
\Theta = \frac{1}{4} \left( 3 + \sqrt{13} \right)
\]

and

\[
\Gamma = \frac{-5 + \sqrt{13} - 5b + \sqrt{13}b}{4\sqrt{13}}.
\]
In equilibrium, therefore, the following occurs:

\[ w^* = \frac{\Gamma}{1 - \Theta} \]  \hspace{1cm} (2.1.21)

\[ p_0^* = \frac{1}{4 (2 + \phi) (\Theta - 1)^2} \left[ \Gamma^2 (1 - 8 (\Sigma - 1) \Sigma) + 2\Gamma (1 + b + 4\Phi (2\Sigma - 1)) \right] \cdot (\Theta - 1) + \left[ (1 + b)^2 + 2 (2 + \phi) v_k - 8\Phi^2 \right] (\Theta - 1)^2 \]  \hspace{1cm} (2.1.22)

\[ p^* = \Phi - \frac{\Gamma \Sigma}{\Theta - 1} \]  \hspace{1cm} (2.1.23)

\[ q_{i,j}^* = \frac{2 (\Phi (\Theta - 1) - \Gamma (\Sigma - 1))}{2 (2 + \phi) (\Theta - 1)} \]  \hspace{1cm} (2.1.24)

\[ x_{0,1}^* = \frac{1}{4t (2 + \phi) (\Theta - 1)^2} \left[ \Gamma^2 (1 + 8 (\Sigma - 1) \Sigma) + 2\Gamma (1 + b + 4\Phi (1 - \Sigma)) \right] \cdot (\Theta - 1) + \left[ (1 + b)^2 + 2 (2 + \phi) v_k + 8\Phi^2 \right] (\Theta - 1)^2 \]  \hspace{1cm} (2.1.25)

\[ x_{0,2}^* = 1 - x_{0,1}^* \]  \hspace{1cm} (2.1.26)

\[ \pi^* = \frac{1}{16t (2 + \phi)^2 (\Theta - 1)^4} \left[ \Gamma^2 (1 + 8 (\Sigma - 1) \Sigma) + 2\Gamma (1 + b + 4\Phi (1 - 2\Sigma)) \right] \cdot (\Theta - 1) + \left[ (1 + b)^2 + 2 (2 + \phi) v_k + 8\Phi^2 \right] (\Theta - 1)^2 \]  \hspace{1cm} (2.1.27)

\[ CS^* = \frac{1}{32t (2 + \phi)^2 (\Theta - 1)^4} \left[ \Gamma^2 (1 + 8 (\Sigma - 1) \Sigma) + 2\Gamma (1 + b + \right] \]
\[
4\Phi (1 - 2\Sigma)) (\Theta - 1) + \left( (1 + b)^2 + 2 (2 + \phi) v_k + 8\Phi^2 \right) (\Theta - 1)^2 \cdot \\
\left(-2\Gamma^2 (19 + 8\Sigma (-7 + 5\Sigma)) - 4\Gamma (\Phi (28 - 40\Sigma) + \\
(1 + b) (-13 + 16\Sigma)) (\Theta - 1) - \\
2 \left(3 (1 + b)^2 - 2 (2 + \phi) v_k + 8\Phi (-4 (1 + b) + 5\Phi) \right) (\Theta - 1)^2 \right) \tag{2.1.28}
\]

2.2 The results

**Proposition 3.** The platforms offer sellers contracts that encourage them to multi-homing to take advantage of the network effect of the market. For consumers, the preferred outcome is for both sellers to be present on the platform and for platform-specific products to complement each other.

**Proposition 4.** Beliefs are not affected by the relationship between platform-specific products for competing platforms.

**Proposition 5.** In the case of competing platforms, the use of private contracts changes the pricing strategy of the platforms on the seller side: a positive price is charged for the previously supported segment. However, on the consumer side, a positive price is still set, in contrast to the result obtained in the literature, where consumers were supported by the platforms. In contrast, the subscription fee for private contracts is lower than for public contracts.

**Proposition 6.** As a result of the franchise fee included in the private contract, the sellers set collusive prices.

**Proposition 7.** Private platforms are encouraged to use public contracts, while for consumers, the preferred output is a function of the network effect, the transportation cost, and the relationship between products.
Chapter 3

Mixed duopoly in two-sided markets

In this chapter, the case of "zero" duopolist by González-Maestre and Martínez-Sánchez (2015) was analyzed in a different approach. This is because the idea behind the model is that the state broadcaster chooses the level of advertising to be zero, i.e. it excludes the advertiser side from the given platform. And if the advertisers do not have access to the product or service of the public company, the public company will cease to be a platform and a public company will continue to compete with a private platform in that market. The effect of this on pricing and product differentiation is the subject of the study. This issue is justified by the efforts made in the European Union to ensure that public service broadcasters do not cover their operations from advertising revenues. It is shown in a Hotelling framework that the decision to differentiate a product is not influenced by the relevant decision of the competitor, whereas the prices on the consumer side are converging. It is also pointed out that if the state wants to take action against a private platform to represent the interests of consumers, it either uses other tools of regulation (taxation, price regulation, competition law) or creates a public platform in the given market.
3.1 The equilibrium

The state-owned company will make the location decision in equilibrium, while the private platform would choose the location

\[ l^*_s = \frac{v_v}{t_{v,s}} \]  

(3.1.1)

However, since for the location, \( l_p \leq 1 \) must hold, the private platform will be located on the right endpoint of the section. That is, \( l^*_p = 1 \) will be in equilibrium.

Viewers will pay a price of

\[ p^*_{v,p} = \frac{(b_v + b_a - 2t_{v,p}) v_v}{(b_v + b_a)^2 - 4t_{v,p}} \]  

(3.1.3)

for the product of the private platform, while the price charged to advertisers will be

\[ p^*_{a,p} = \frac{(b_v - b_a) v_v}{(b_v + b_a)^2 - 4t_{v,p}}. \]  

(3.1.4)

At these prices, the private platform makes a profit of

\[ \pi^*_p = -\frac{v_v^2}{(b_v + b_a)^2 - 4t_{v,p}}. \]  

(3.1.5)

The surplus for viewers and advertisers will be

\[ W^* = \frac{1}{2} \left( \frac{2}{t_{v,s}} + \frac{(b_v + b_a)^2 + 4t_{v,p}}{(b_v + b_a)^2 - 4t_{v,p}} \right) v_v^2. \]  

(3.1.6)

In equilibrium, viewers in location

\[ x \in \left[ 1 + \frac{2v_v}{(b_v + b_a)^2 - 4t_{v,p}}, \frac{2v_v}{t_{v,s}} \right] \]  

(3.1.7)
will buy the products of both companies, while on the advertiser side, the indifferent player will be located at point

\[ x_a^* = 1 + \frac{(b_v + b_a) v_v}{(b_v + b_a)^2 - 4t_{v,p}} \]  
(3.1.8)

3.2 The results

**Proposition 8.** *Neither service provider’s decision to differentiate a product is influenced by the competitor’s decision, while on the consumer side, as a result of competition, subscription fees converge.*

**Proposition 9.** *With the abolition of the state platform, the strategy of regulation of entry will lose its strength in the case of two-sided markets.*
Bibliography


Publications