Doctoral School of Economics, Business and Informatics

THESIS
on

Financial intermediation under liquidity and regulatory constraints

Judit Hevér

Ph.D. dissertation

Supervisor:

Dr. Péter Csóka Ph.D.
professor

Budapest, 2021

Department of Finance

THESIS
on

Financial intermediation under liquidity and regulatory constraints

Judit Hevér

Ph.D. dissertation

Supervisor:

Dr. Péter Csóka Ph.D.
professor

Contents

List of Figures

List of Tables

Chapter 1

Introduction

The liquidity crises of the past decades such as Black Monday in 1987, the one related to the Iraq War in 1990, the collapse of LTCM in 1998 and the subprime mortgage crisis in 2007 (Brunnermeier and Pedersen, 2008) evidence the paramount importance of liquidity in financial markets. Liquidity crises and studies covering them have drawn regulatory attention to the importance of understanding and analyzing liquidity. Therefore, liquidity and the management of liquidity risk are topics well represented in regulatory practice as well as economic theory.

In January 2013, the Basel Committee (*Basel Committee on Banking Supervision, BCBS*) introduced two new measures, Liquidity Coverage Ratio and Net Stable Funding Ratio, as part of the Basel III international regulatory framework for banks (BCBS, 2013). Beside regulations for the banking system, the recommendations and good practices of IOSCO (*International Organisation of Securities Commissions*) on the management of investment funds are also being updated. The objective of the recommendation IOSCO (2018) is to improve the management of liquidity risk of open-end investment funds with a view to protecting investors, increasing the efficiency of financial markets and reducing systemic risk. In 2016, the SEC (*Securities and Exchange Commission*) adopted New Rule 22e-4 to regulate the liquidity risk of registered open-end funds [1]. The mission statement of the SEC [2] includes the aim of protecting households who borrow funds or invest in financial markets. Beside the regulation of financial institutions, the development of financial literacy of households and avoidance of excessive risk-taking and over-indebtedness are certainly also key to achieving this objective.

[1] Securities and Exchange Commission's Investment Company Liquidity Risk Management Programs, 17 CFR Parts 210, 270, 274, pp. 90 and 195. https://www.sec.gov/rules/final/2016/33-10233.pdf

[2] *"The SEC enforces the securities laws to protect the more than 66 million American households that have turned to the securities markets to invest in their futures – whether it's starting a family, sending kids to college, saving for retirement or attaining other financial goals."* https://www.sec.gov/

In addition to regulatory practice, the topic is also amply covered by theoretical models. The liquidity of assets and markets may fluctuate over time due to the varying level of transparency of information on asset values, the number and capital of intermediaries providing liquidity, and uncertainty. Therefore, it is important to capture liquidity risk in models (Amihud, Mendelson and Pedersen, 2013). Based on Acerbi and Scandolo (2008), the modelling of liquidity risk (*liquidity risk*) covers the cash-flow risk of portfolios (generally speaking, that of companies) (Acerbi and Scandolo, 2008), the risk of trading in illiquid markets, i.e. the risk of price impact (Almgren and Chriss, 2001; Amihud, 2002; Acharya and Pedersen, 2005), and the risk of drying up of the liquidity circulating in the financial system (Amihud, Mendelson and Wood (1990), Brunnermeier and Pedersen (2008), Mitchell, Pedersen and Pulvino (2007) were the first to cover this topic). Accordingly, the theoretical literature of the topic is diverse.

Acerbi and Scandolo (2008) define portfolio value under liquidity policy, which, instead of valuing portfolios at the best bid or ask offer, takes into account the fact that the execution of strategies requires the liquidation of part of the assets. In Bigio (2015)'s model, companies face the problems of instant liquidation and collateralization of part of their assets due to the uncertainty arising from the limited enforceability of agreements. Kiyotaki and Moore (2019) limit the leverage of entrepreneurs. Therefore, they need to liquidate part of their illiquid assets in order to be able to invest. Csóka (2017) models the possibility of risk sharing among the divisions of an indebted company under financial constraints. Gromb and Vayanos (2010) model the relationship between intermediary capital and market liquidity.

Several studies assess the effectiveness and potential costs of regulatory requirements. De Nicolò, Gamba and Lucchetta (2014) demonstrate in a partial market equilibrium model that the application of the Basel III Liquidity Coverage Ratio restricts lending and reduces the levels of efficiency and welfare. Begenau (2019) uses a dynamic general equilibrium model to determine the optimal level of capital requirement. Increasing capital requirement reduces the leverage, and thus the amount of coveted deposit funding, of banks, which, through a reduction of deposit rates, reduces the cost of capital, increases profitability and, ultimately, lending. On the other hand, IOSCO (2019a) stresses that the regulation of the secondary market of corporate bonds has limited financial intermediaries in the provision of liquidity since the crisis. Stress test results show that market pressure may lead to more severe shifts in yields than before. According to Sommer and Sullivan (2018), the abolition of tax credits for mortgage loans would result in a drop of real estate prices and the stock of mortgage loans, and an increase in welfare.

Each chapter of the present dissertation analyzes the change in decisions of optimizing market agents upon the introduction of liquidity and regulatory constraints.

Chapter 2 presents a portfolio optimization problem, while Chapters 3, 4 and 5 cover the consumption and portfolio allocation decisions of market agents and the profit maximization of the market maker in a general equilibrium model. Liquidity and regulation play an important role in these optimization problems. In order to take transaction cost into account, we use the marginal supply-demand curve (MSDC) to determine prices and portfolio value. In a given period, the MSDC of a risky asset expresses the marginal bid prices (for positive quantities) and marginal asks (for negative quantities) at which the particular asset can be traded. Using the approach of Acerbi and Scandolo (2008), we determine portfolio value under a liquidity policy defining the set of acceptable future portfolios. Portfolio value is the maximum of the market values of the portfolios attainable from the initial one (through liquidation of part of it) that comply with the liquidity policy. A liquidity policy specifies the set of acceptable portfolios, for instance, the minimum level of cash to be reached. It captures investor objectives, aspects of pre-defined fund investment policies of institutional investors or regulatory requirements. Consumption and portfolio allocation decisions are made under regulatory requirements introduced as a function of *expected shortfall*.

For the valuation of illiquid portfolios, Acerbi and Scandolo (2008) assume that the liquidation of a part of the portfolio has no permanent price impact. Therefore, the final optimal portfolio that complies with the liquidity policy can be valued using the initial MSDC. This assumption is reasonable for small transaction volumes, long periods and low-risk assets (bonds). However, the permanent price impact of the trading of institutional investors can be significant. In order to better address the specific case of institutional investors, we modify portfolio value under liquidity policy by introducing permanent price impact in Chapter 2. In an example with linear permanent price impact, liquidity policy requiring a minimum level of cash and MSDC approximated by an exponential function, even a moderate permanent price impact can completely change the range of attainable portfolios, while slightly changing the value of the portfolio. The use of permanent price impact implies a breach of the price acceptance assumption due to the endowment effect, as agents can shift or even manipulate market prices. Such alternative portfolio value can be used for the calculation of risk metrics, definition of capital requirements, and performance evaluation of portfolio managers.

Chapters 3, 4 and 5 of the dissertation are linked through the applied general equilibrium model framework and summarized in a single section hereunder. For the construction of the model, I rely on the book of Le Roy and Werner (2001). New features introduced herewith include regulatory requirement as a function of expected loss, the use of endogenic MSDC and bid-ask spread of the market maker, and the distinction of cash from other assets, which makes saving in risk-free assets possible for all agents simultaneously. Chapter 3 of the present abstract examines the relationship between market liquidity and the introduction of regulatory requirements. The model uses an MSDC based on Chapter 3 of the dissertation to derive the solution to, and findings of, a two-agent scenario. In Chapter 4 of the dissertation, the change in market liquidity is measured through the change in bid-ask spread.

The introduction of regulatory requirements represents an additional constraint to the optimization problem of market agents; thus, their previous optimal portfolio may not be attainable any longer. Therefore, their utility for a given MSDC and bid-ask spread may not increase. If the market agent is constrained in its optimal decision by regulatory requirements, it makes sense for the market maker to increase transaction costs as long as the optimal portfolio of the market agent under the given regulatory requirements does not change. Based on the model output, the introduction of regulatory requirements results in a reduction of market liquidity, which restrains trading and risk sharing among agents. In the new equilibrium, the profile of market agents remains riskier, and they achieve a lower level of utility. In real life, regulation is much more complex, and intervention is justified by market imperfections. Nevertheless, the dissertation confirms that intervention has its costs and market liquidity is impacted by regulatory requirements, which should be considered during the impact assessment of regulatory proposals.

The structure of the present thesis is as follows: Section 2.2 of Chapter 2 provides notation, summarizes portfolio valuation theory under liquidity constraints without permanent price impact, and illustrates theory through numeric examples. In Section 2.3, we propose new methodology to value illiquid portfolios with linear permanent price impact functions.

In Chapter 3, the general equilibrium framework is introduced. In Section 3.2, we give the notation, present the consumption-portfolio choice problem with and without capital requirement. Then we introduce the market-clearing conditions and the problem of the market maker. Finally, we compare equilibrium with and without regulatory requirement. In Section 3.3, a specific, two agent model is examined through analytical derivation and examples. Due to limitations on length, the present abstract solely covers the general model of Chapter 3 of the dissertation in detail; Chapters 4 (The relationship between bid-ask spread and regulation in a general equilibrium model) and 5 (Institutions and economic growth: the effect of transaction costs on risk sharing) of the dissertation are summarized only in a subsection each (Subsections 3.3.3 and 3.4).

Chapter 4 concludes and outlines avenues for future research.

Chapter 2

Portfolio valuation under liquidity constraints with permanent price impact

This chapter is virtually identical to the paper of Csóka and Hevér (2018).

2.1 Introduction

In 2016, the SEC adopted New Rule 22e-4 to regulate the liquidity risk of registered open-end funds.[3] In the liquidity risk management program, there are four investment classification categories (highly, moderately, less liquid and illiquid) based on the level of convertibility to cash. To determine the value of an illiquid portfolio, Acerbi and Scandolo (2008) use the MSDCs of the assets within the portfolio, and a liquidity policy.

When institutional investors convert part of their portfolio to cash, they should also take into account the permanent price impact on the remaining portfolio. Almgren and Chriss (2001) and Almgren (2003) defined temporary and permanent price impact functions to determine optimal execution strategies. During trading, supply-demand imbalances cause temporary price impact. There are temporary price fluctuations from the equilibrium price, but by the end of the period, the order book recovers to eliminate the temporary price impact. On the other hand, permanent impact changes the equilibrium price for the whole liquidation time horizon. In general, some impact is temporary, and the rest is permanent. For valuing illiquid portfolios, Acerbi and Scandolo (2008) assumed that there is no permanent price impact, which is a reasonable assumption for smaller trades and longer time horizons, or for assets

[3]Securities and Exchange Commission's Investment Company Liquidity Risk Management Programs, 17 CFR Parts 210, 270, 274, pp. 90 and 195. https://www.sec.gov/rules/final/2016/33-10233.pdf.

having a relatively certain cash flow, such as bonds.

In this paper, we provide a new method for valuing illiquid portfolios with permanent price impact. We also assume that temporary effects dissipate by the time the liquidity policy should be satisfied, but we incorporate permanent price impact. Intuitively, there is an endowment effect on top of the transaction cost effect, which should be taken into account in case of permanent price impact. The trade-off is between trading more from a rather liquid asset to have lower transaction costs, and trading less to cause lower permanent price impact on the remaining endowment of the particular asset. As a specific case, we consider a cash liquidity policy with no short positions (Csóka, 2017), where the acceptable portfolio must have c units of cash, and short positions should be closed.

In a sense, we capture that an institutional investor has the power to influence or even manipulate the value of certain assets, which is also documented in recent studies. In the Saudi Stock Market, Alzahrani, Gregoriou and Hudson (2012) found an asymmetry in the price impacts of block purchases and sales. Han et al. (2016) and Kitamura (2016) analyze data from order-driven markets to examine the price impact of informed trading. Han et al. (2016) confirm the presence of informed trading by finding that a substantial portion of the price impact is persistent. Comparing the market impact of small and large trades, Han et al. (2016) find that the price impact of institutional investors is larger than that of individuals, and concludes that stealth trading is usual. Kitamura (2016) also shows that there is a significant price impact of informed trading.

Measuring the permanent price impact is challenging. Almgren et al. (2005) use a unique data sample of US institutional orders executed by the Citigroup equity trading desk to calibrate the models of Almgren (2003) and Almgren and Chriss (2001). Unfortunately, based on publicly available data sets, the estimation cannot be reproduced. We would need a reliable classification of individual trades as buyer- or seller-initiated, and information on sequences of trades that form part of a large transaction. Huberman and Stanzl (2004) show that a linear permanent price impact function is needed for arbitrage-free pricing. Moreover, Almgren et al. (2005) test empirically and cannot reject the hypothesis of a linear permanent price impact. We also assume a linear permanent price impact and analyze the proposed optimization problem. We show that solving the problem requires numerical methods or further assumptions. To get analytical results, one can approximate the MSDCs by exponential functions (Tian, Rood and Oosterlee, 2013).

2.2 Portfolio valuation without permanent price impact

In this section, we combine the notation of Acerbi and Scandolo (2008) and Csóka and Herings (2014) to summarize how the value of an illiquid portfolio can be defined without permanent price impact.

An investor can hold cash as well as risky assets from the set J. Let $\boldsymbol{\theta} = (\theta_0, \theta) \in \mathbb{R} \times \mathbb{R}^J$ denote a portfolio, where θ_0 is the amount of cash in the portfolio ($\theta_0 < 0$ means an immediate payment requirement) and θ_j is the quantity of assets held from asset $j \in J$. Let $\boldsymbol{\Theta} \in \mathbb{R} \times \mathbb{R}^J$ denote the space of portfolios. Moreover, let $\boldsymbol{\theta} \oplus a$ denote adding $a \in \mathbb{R}$ amount of cash to portfolio $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, which results in portfolio $\boldsymbol{\nu} \in \boldsymbol{\Theta}$ satisfying $\nu_0 = \theta_0 + a$ and $\nu_j = \theta_j$ for all $j \in J$.

The value of a portfolio depends on the order books for the risky assets to be specified as follows. We follow Cetin, Jarrow, and Protter (2004), Jarrow and Protter (2005) and Acerbi and Scandolo (2008) in modeling the order books for every asset $j \in J$ by a marginal supply-demand curve m_j.

Definition 2.1. The *marginal supply-demand curve* (MSDC) for asset $j \in J$ is given by the map $m_j : \mathbb{R} \setminus \{0\} \mapsto \mathbb{R}$ satisfying

(i) $m_j(h) \geq m_j(\overline{h})$ if $h < \overline{h}$;

(ii) $m_j(h)$ is right-continuous with left limits for $h < 0$ and left-continuous with right limits for $h > 0$.

The amount $m_j(h)$ for $h > 0$ expresses the marginal bids at which asset $j \in J$ can be sold. Similarly, $m_j(h)$ for $h < 0$ represents the marginal asks at which asset j can be bought. Let $m_j(0^+)$ denote the best bid and $m_j(0^-)$ the best ask price of asset $j \in J$. For negative (positive) values of x, MSDC $m_j(h)$ corresponds to the supply (demand) of asset $j \in J$ by others. Note that the MSDCs are not defined at zero. Since we are working with assets, it is natural to assume that $m_j(h) \geq 0$ for all $j \in J$. However, the MSDC of a contract (a swap agreement, for instance) could admit positive and negative values as well. MSDCs can be used to calculate the liquidation value of a portfolio.

Definition 2.2. The *liquidation mark-to-market value* of a portfolio $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ is defined by

$$L(\boldsymbol{\theta}) = \theta_0 + \sum_{j \in J} \int_0^{\theta_j} m_j(h)dh.$$

The liquidation mark-to-market value of a portfolio $L(\boldsymbol{\theta})$ is its initial cash, plus the proceeds one recieves by liquidating long positions, and the amount of money to be paid for closing short positions.

The other extreme is to use the best bid prices for long positions and the best ask prices for short positions.

Definition 2.3. The *uppermost mark-to-market value* of a portfolio $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ is defined by

$$U(\boldsymbol{\theta}) = \theta_0 + \sum_{j\in J} \left[m_j(0^+) \max(\theta_j, 0) + m_j(0^-) \min(\theta_j, 0) \right].$$

In case of perfect liquidity (when all MSDCs are constant), we could liquidate and value all the assets at the best prices. However, depending on the MSDCs, the cost of prompt liquidation, which is the difference between the liquidation value and the uppermost mark-to-market value of a portfolio, can be significant. The extent to which liquidation is required is related to the so-called liquidity policy defined as follows.

Definition 2.4. A *liquidity policy* $\mathcal{L} \subseteq \boldsymbol{\Theta}$ is a closed and convex subset of the portfolio-space satisfying

1. if $\boldsymbol{\theta} \in \mathcal{L}$ and $a \geq 0$, then $\boldsymbol{\theta} \oplus a \in \mathcal{L}$,
2. if $\boldsymbol{\theta} \in \mathcal{L}$, then $(\theta_0, 0^J) \in \mathcal{L}$.

As a specific case, we will consider a cash liquidity policy with no short positions (Csóka, 2017), where the acceptable portfolio should have c units of cash and short positions must be closed.

Definition 2.5. Given $c \in \mathbb{R}$, the *c-cash liquidity policy with no short positions* $\mathcal{L}^+(c)$ is given by

$$\mathcal{L}^+(c) = \left\{ (\theta_0, \theta) \in \boldsymbol{\Theta} | \theta_0 \geq c \text{ and } \theta \geq 0^J \right\}.$$

Note that the required cash c could be negative as well.

As we will see, the value of a portfolio in this framework depends on how we can reach the liquidity policy from it. A portfolio is attainable from another portfolio if we can reach both its level of cash and amount of risky assets by trading.

Definition 2.6. Given a portfolio $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, the portfolio $\boldsymbol{\nu} \in \boldsymbol{\Theta}$ is *attainable from p*, $\boldsymbol{\nu} \in Att(\boldsymbol{\theta})$ if there is $\boldsymbol{\rho} \in \boldsymbol{\Theta}$ such that

$$\boldsymbol{\nu} = \boldsymbol{\theta} - \boldsymbol{\rho} \oplus L(\boldsymbol{\rho}).$$

Note that the liquidated part $\boldsymbol{\rho}$ could also have short positions, meaning that there will be more of those assets in the new portfolio, which requires cash. Now we can define the (mark-to-market) value of a portfolio.

Definition 2.7 (Acerbi and Scandolo (2008))**.** The *mark-to-market value* of portfolio $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ under the liquidity policy $\mathcal{L}$ is a function $V^{\mathcal{L}} : \boldsymbol{\Theta} \to \mathbb{R}$ defined by

$$V^{\mathcal{L}}(\boldsymbol{\theta}) = \sup \{U(\boldsymbol{\nu}) | \boldsymbol{\nu} \in Att(\boldsymbol{\theta}) \cap \mathcal{L}\} . \tag{2.1}$$

In (2.1), we are looking for the most valuable portfolio that is attainable from the initial one and satisfies the liquidity policy. According to the next proposition, this portfolio can be found as a solution to a convex optimization problem, which is crucial for industry implementation.

Proposition 2.1 (Acerbi and Scandolo (2008))**.** *Optimization problem* (2.1) *in* $\boldsymbol{\nu}$ *is equivalent to a convex optimization problem in* $\boldsymbol{\rho}$*, given by*

$$V^{\mathcal{L}}(\boldsymbol{\theta}) = sup \{ U(\boldsymbol{\theta} - \boldsymbol{\rho}) + L(\boldsymbol{\rho}) | \boldsymbol{\rho} \in C_{\mathcal{L}}(\boldsymbol{\theta}) \} , \tag{2.2}$$

where $C_{\mathcal{L}}(\boldsymbol{\theta})$ *is a convex set given by*

$$C_{\mathcal{L}}(\boldsymbol{\theta}) = \{\boldsymbol{\rho} | \boldsymbol{\theta} - \boldsymbol{\rho} \oplus L(\boldsymbol{\rho}) \in \mathcal{L}\} .$$

If $C_{\mathcal{L}}(\boldsymbol{\theta})$ *is empty, then* $V^{\mathcal{L}}(\boldsymbol{\theta}) = -\infty$*, else supremum* $V^{\mathcal{L}}(\boldsymbol{\theta}) \in \mathbb{R}$.

To illustrate the definitions, we provide the following example.

Example 2.1. Consider a market with cash and a single risky asset, where the space of portfolios is given by $\boldsymbol{\Theta} = \mathbb{R}^2$. We are interested in the value of the initial portfolio $\boldsymbol{\theta} = (\theta_0, p_1) = (4, 4)$, that is, we have 4 units of cash and 4 units of an illiquid risky asset. Assume that the MSDC of the risky asset is given by

$$m_1(h) = \begin{cases} 5 & \text{if } h < 0, \\ 4 & \text{if } 0 < h \leq 1, \\ 2 & \text{if } 1 < h \leq 3, \\ 1 & \text{if } 3 < h. \end{cases}$$

We can calculate the liquidation and uppermost mark-to-market value of $\boldsymbol{\theta}$ as

$$L(\boldsymbol{\theta}) = 4 + 4 \times 1 + 2 \times 2 + 1 \times 1 = 13,$$
$$U(\boldsymbol{\theta}) = 4 + 4 \times 4 = 20.$$

Suppose that the liquidity policy is a 10-cash liquidity policy with no short positions $\mathcal{L}^+(10)$, where the acceptable portfolio must have 10 units of cash and short positions must be closed.

In $\boldsymbol{\theta}$, there are 4 units of cash and no short positions, so we need 6 more units of cash. To meet the liquidity policy, we can do nothing else but sell 2 units of the risky asset, $\boldsymbol{\rho} = (0, 2)$. The first unit is sold for 4, and the second unit for 2, $L(\boldsymbol{\rho}) = 6$. Indeed, we get that $\boldsymbol{\nu} = \boldsymbol{\theta} - \boldsymbol{\rho} + L(\boldsymbol{\rho}) = (10, 2) \in Att(\boldsymbol{\theta}) \cap \mathcal{L}^+(10)$. The mark-to-market value of $\boldsymbol{\theta}$ is

$$V^{\mathcal{L}^+(10)}(\boldsymbol{\theta}) = U(\boldsymbol{\nu}) = 10 + 4 \times 2 = 18.$$

In the spirit of Proposition 2.1, we get the same number from $U(\boldsymbol{\theta} - \boldsymbol{\rho}) + L(\boldsymbol{\rho}) = 4 + 4 \times 2 + 6 = 18$.

Notice that even though we have sold 2 units, the resulting portfolio was valued using the original MSDC. The implicit assumption is that liquidity recovers by means of new limit orders, i.e. the trading has no permanent price impact.

If there were full permanent price impact and, hence, no new limit orders, only one more unit could be sold for 2 and the rest only for 1. In this case, the MSDC of the risky asset $\hat{m}_1$ is

$$\hat{m}_1(h) = \begin{cases} 5 & \text{if } h < 0, \\ 2 & \text{if } 0 < h \le 1, \\ 1 & \text{if } 1 < h. \end{cases}$$

Using $\hat{m}_1$, the uppermost mark-to-market value of $\boldsymbol{\nu}$ is $\hat{U}(\boldsymbol{\nu}) = 10 + 2 \times 2 = 14$, which is, of course, lower then the one calculated with the original MSDC.

Motivated by Example 2.1, we incorporate the effects of permanent price impact on the valuation of illiquid portfolios in the next section.

2.3 Portfolio valuation with linear permanent price impact

We use a linear permanent price impact function in this paper. Huberman and Stanzl (2004) show that a linear permanent price impact function (called price update function in their paper) is needed for arbitrage-free pricing. Moreover, Almgren et al. (2005) test empirically, and cannot reject, the hypothesis of a linear permanent price impact. Translated to MSDCs, a linear permanent price impact means that in function of the size of the trade, selling decreases and buying increases the level of the MSDC linearly, formally defined as follows.

Definition 2.8. Let a risky asset $j \in J$, its MSDC m_j and a parameter $\beta_j \in \mathbb{R}^+$ be given. After liquidating $\rho_j \in \mathbb{R}$ amount of asset i, the *MSDC of asset i, modified* *with linear permanent price impact* $\overline{m}_j^{\beta_j}(h)$ is

$$\overline{m}_j^{\beta_j}(h) = m_j(h) - \beta_j \rho_j.$$

Again, $\rho_j > 0$ means selling, $\rho_j < 0$ means buying from asset i.

Let us reconsider Example 2.1 with linear permanent price impact.

Example 2.2. [Example 2.1 continued.] In a more realistic case, the permanent impact is smaller than the market impact. Suppose a linear permanent price impact function with $\beta_1 = 0.2$. Since we still can do nothing else but liquidate 2 units of asset 1, the initial MSDC is lowered by 2×0.2 and the MSDC of asset 1, modified with linear permanent price impact, $\overline{m}_1(h)$ becomes

$$\overline{m}_1^{0.2}(h) = \begin{cases} 4.6 & \text{if } h < 0, \\ 3.6 & \text{if } 0 < h \leq 1, \\ 1.6 & \text{if } 1 < h \leq 3, \\ 0.6 & \text{if } 3 < h. \end{cases}$$

Using $\overline{m}_1^{0.2}$, the uppermost mark-to-market value of $\boldsymbol{\nu}$ is

$$\overline{U}(\boldsymbol{\nu}) = 10 + 3.6 \times 2 = 17.2,$$

which is somewhere between the case with no permanent price impact and that with full permanent price impact.

Given linear price impact parameters $\beta_j \in \mathbb{R}^+$ for each $j \in J$, analogously to Definition 2.3, let $\overline{U}$ denote the uppermost mark-to-market value function with linear price impact, calculated from the MSDCs modified with linear permanent price impact, that is, from $\overline{m}_j^{\beta_j}$ for each $j \in J$.

Now we can define the mark-to-market value of a portfolio with linear permanent price impact.

Definition 2.9. Given $\beta_j \in \mathbb{R}^+$ for each $j \in J$, the *mark-to-market value of portfolio* $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ *with linear permanent price impact* under liquidity policy $\mathcal{L}$ is a function $\overline{V}^{\mathcal{L}} : \boldsymbol{\Theta} \to \mathbb{R}$ defined by

$$\overline{V}^{\mathcal{L}}(\boldsymbol{\theta}) = \sup \left\{ \overline{U}(\boldsymbol{\nu}) | \boldsymbol{\nu} \in Att(\boldsymbol{\theta}) \cap \mathcal{L} \right\}. \tag{2.3}$$

Next, we provide the following proposition, the proof of which is omitted since it is straightforward along the lines of the proof of Proposition 2.1.

Proposition 2.2. *Given $\beta_j \in \mathbb{R}^+$ for each $j \in J$, optimization problem* (2.3) *in* $\boldsymbol{\nu}$ *is equivalent to an optimization problem in* $\boldsymbol{\rho}$*, given by*

$$\overline{V}^{\mathcal{L}}(\boldsymbol{\theta}) = \sup\left\{\overline{U}(\boldsymbol{\theta} - \boldsymbol{\rho}) + L(\boldsymbol{\rho}) | \boldsymbol{\rho} \in C_{\mathcal{L}}(\boldsymbol{\theta})\right\}, \tag{2.4}$$

where $C_{\mathcal{L}}(\boldsymbol{\theta})$ *is again a convex set given by*

$$C_{\mathcal{L}}(\boldsymbol{\theta}) = \{\boldsymbol{\rho} | \boldsymbol{\theta} - \boldsymbol{\rho} \oplus L(\boldsymbol{\rho}) \in \mathcal{L}\}.$$

If $C_{\mathcal{L}}(\boldsymbol{\theta})$ *is empty, then* $V^{\mathcal{L}}(\boldsymbol{\theta}) = -\infty$*, else supremum* $V^{\mathcal{L}}(\boldsymbol{\theta}) \in \mathbb{R}$.

Note that in Proposition 2.2, we only defined a general optimization problem due to the appearance of $\overline{U}$. We leave it as further research to examine the convexity property of (2.4). Consider the following example.

Example 2.3 (Examples 2.1 and 2.2 continued.)**.** Let us extend the market to two risky assets, so the space of portfolios is now given by $\boldsymbol{\Theta} = \mathbb{R}^3$. Suppose that in the initial portfolio, there are also 4 units of a second risky asset, that is $\boldsymbol{\theta} = (\theta_0, p_1, p_2) = (4, 4, 4)$.

Let the MSDC of asset 2 be given by

$$m_2(h) = \begin{cases} 2 & \text{if } h < 0, \\ 1 & \text{if } 0 < h \leq 1, \\ 0.6 & \text{if } 1 < h. \end{cases}$$

Let asset 2 also have a linear permanent price impact parameter of 0.2, that is let $\beta_1 = \beta_2 = 0.2$. Finally, let us keep the 10-cash liquidity policy with no short positions $\mathcal{L}^+(10)$. If there is no permanent price impact, and liquidity recovers, then using Proposition 2.1, we get that it is optimal to liquidate $\boldsymbol{\rho}^* = (0, 1, \frac{8}{3})$ to get the additional liquidity of $L(\boldsymbol{\rho}^*) = 6$ and we obtain $\boldsymbol{\nu}^* = \boldsymbol{\theta} - \boldsymbol{\rho}^* = (10, 3, \frac{4}{3})$. The mark-to-market value of $\boldsymbol{\theta}$ is then

$$V^{\mathcal{L}^+(10)}(\boldsymbol{\theta}) = U(\boldsymbol{\nu}^*) = 10 + 4 \times 3 + 1 \times \frac{4}{3} = \frac{70}{3} \approx 23.33.$$

If there is permanent price impact, then, after solving (2.4) the new optimum is to liquidate $\boldsymbol{\rho}^{**} = (0, 0.8, 4)$ to get the additional liquidity of $L(\boldsymbol{\rho}^{**}) = 6$, and we obtain $\boldsymbol{\nu}^{**} = (10, 3.2, 0)$. The new best bid of asset 1 becomes $4 - 0.8 \times 0.2 = 3.84$ and we get that the mark-to-market value of $\boldsymbol{\theta}$ with linear permanent price impact is

$$\overline{V}^{\mathcal{L}^+(10)}(\boldsymbol{\theta}) = \overline{U}(\boldsymbol{\nu}^{**}) = 10 + 3.84 \times 3.2 = 22.288.$$

Note that using the optimal trade with no permanent price impact $\boldsymbol{\rho}^* = (0, 1, \frac{8}{3})$

would result in $\boldsymbol{\nu}^* = (10, 3, \frac{4}{3})$, and the new best bids of asset 1 and asset 2 would be $4 - 1 \times 0.2 = 3.8$ and $1 - \frac{8}{3} \times 0.2 = \frac{7}{15}$, respectively. Then $\overline{U}(\boldsymbol{\nu}^*) = 10 + 3.8 \times 3 + \frac{7}{15} \times \frac{4}{3} = \frac{991}{45} \approx 22.02 < 22.228 = \overline{U}(\boldsymbol{\nu}^{**})$, which is, of course, not optimal.

Intuitively, there is an *endowment effect* on top of the *transaction cost effect*, which should be taken into account with permanent price impact. The trade-off is between trading more from a relatively liquid asset to have lower transaction costs and trading less to cause lower permanent price impact on the remaining endowment of the particular asset. In a sense, an institutional investor has the power to influence or even manipulate the market.

2.3.1 Optimization problem with continuous MSDCs and $\mathcal{L}^+(c)$ liquidity policy

To give more insights about the problem in (2.4), in the rest of the paper we assume continuous MSDCs such that they are defined at zero, implying that $m_j(0) = m_j(0^+) = m_j(0^-)$ for all $j \in J$. Moreover, we work with a c-cash liquidity policy with no short positions $\mathcal{L}^+(c)$. The following formulation of the problem is straightforward.

Proposition 2.3. *Given $\beta_j \in \mathbb{R}^+$ and continuous MSDCs m_j for each $j \in J$, and $\mathcal{L}^+(c)$, optimization problem* (2.4) *is equivalent to an optimization problem in $\boldsymbol{\rho}$, given by*

$$\overline{V}^{\mathcal{L}^+(c)}(\boldsymbol{\theta}) = \max \overline{U}(\boldsymbol{\theta} - \boldsymbol{\rho}) + L(\boldsymbol{\rho}) \tag{2.5}$$

subject to

$$\begin{aligned} \theta_0 - \rho_0 + L(\boldsymbol{\rho}) &= c, \ \textit{and} \\ \theta_j - \rho_j &\geq 0 \textit{ for all } j \in J. \end{aligned}$$

Recall that using continuous MSDCs, the uppermost mark-to-market value function with linear price impact $\overline{U}$ is given by

$$\overline{U}(\boldsymbol{\theta} - \boldsymbol{\rho}) = \theta_0 - \rho_0 + \sum_{j \in J} [m_j(0) - \beta_j \rho_j](\theta_j - \rho_j). \tag{2.6}$$

Based on Boyd and Vandenberghe (2004), we can use the Karush-Kuhn-Tucker conditions to find the solution to (2.5). The Lagrangian function can be given by

$$G(\boldsymbol{\rho}, \lambda, \mu) = -\overline{U}(\boldsymbol{\theta} - \boldsymbol{\rho}) - L(\boldsymbol{\rho}) - \lambda [\theta_0 - \rho_0 + L(\boldsymbol{\rho}) - c] - \sum_{j \in J} \mu_j [\theta_j - \rho_j].$$

Using (2.6) and the definition of liquidation value,

$$G(\boldsymbol{\rho}, \lambda, \mu) = -\theta_0 - \sum_{j\in J} [m_j(0) - \beta_j \rho_j](\theta_j - \rho_j) - \sum_{j\in J} \int_0^{\rho_j} m_j(h)dh$$
$$-\lambda \left[\sum_{j\in J} \int_0^{\rho_j} m_j(h)dh + \theta_0 - c \right] - \sum_{j\in J} \mu_j [\theta_j - \rho_j].$$

Let ρ_j^{**} denote the optimal ρ_j for all $j \in J$. Without losing generality, we can assume that its cash component $\rho_0^{**} = 0$, since the optimization problem does not depend on ρ_0.

The Karush-Kuhn-Tucker conditions are as follows. First,

$$\frac{\partial G(\boldsymbol{\rho}, \lambda, \mu)}{\partial \rho_j} = m_j(0) - 2\beta_j \rho_j^{**} + \beta_j \theta_j - (1+\lambda) m_j(\rho_j^{**}) + \mu_j = 0 \text{ for all } j \in J.$$

Primal feasibility requires that

$$\frac{\partial G(\boldsymbol{\rho}, \lambda, \mu)}{\partial \lambda} = \sum_{j\in J} \int_0^{\rho_j^{**}} m_j(h)dh + \theta_0 - c = 0,$$
$$\frac{\partial G(\boldsymbol{\rho}, \lambda, \mu)}{\partial \mu_j} = \theta_j - \rho_j^{**} \geq 0$$

Due to complementary slackness,

$$\mu_j \left[\theta_j - \rho_j^{**}\right] = 0 \text{ for all } j \in J.$$

Finally, dual feasibility requires that

$$\mu_j \geq 0 \text{ for all } j \in J.$$

Now we can formally see the intuition of Example 2.3. Suppose that $\theta_j - \rho_j^{**} > 0$. Then $-2\beta_j \rho_j^{**} + \beta_j \theta_j$ appears in the partial derivative of the Lagrangian function with respect to ρ_j. So $\boldsymbol{\rho}$ depends on initial portfolio $\boldsymbol{\theta}$, the permanent price impact parameters, and the marginal revenue (cost) at $\rho_j^{**} > 0$ ($\rho_j^{**} < 0$) controlling both for the endowment effect and the transaction cost effect.

Solving the problem requires numerical methods or further assumptions. To get analytical results, one can approximate the MSDCs by exponential functions (Tian, Rood and Oosterlee, 2013).

2.3.2 Optimization problem with exponential MSDCs

The relevant question is how to model and estimate the MSDCs. The aim of Tian, Rood and Oosterlee (2013) is to complement the theoretical framework of Acerbi and Scandolo (2008) with practical implications. Tian, Rood and Oosterlee (2013) provide algorithms to portfolio valuation in case of different MSDCs (ladder MSDCs for actively traded products, exponential MSDCs for less liquid OTC markets). Their main finding that ladder MSDCs can be accurately approximated by exponential MSDCs is important for future applications.

Based on Tian, Rood and Oosterlee (2013), let m_j be given by $m_j(h) = A_j \exp^{-k_j h}$, so we approximate non-increasing step function MSDCs by exponential functions. The simplification is crucial because the existence of an analytical solution depends on the functional form of m_j. Exponential functions are defined at zero, thus without permanent price impact, the resulting $\boldsymbol{\theta} - \boldsymbol{\rho}$ portfolio is valued using the original MSDC at zero, i.e. $m_j(0) = A_j$. To determine the analytical solution to the problem, let us calculate the first order conditions with respect to ρ_j and λ

$$\begin{aligned}\frac{\partial G(\boldsymbol{\rho}, \lambda, \mu)}{\partial \rho_j} &= A_j - (1+\lambda) A_j \exp^{-k_j \rho_j^*} + \mu_j = 0 \ \forall j \in J, \\ \frac{\partial G(\boldsymbol{\rho}, \lambda, \mu)}{\partial \lambda} &= \sum_{j \in J} \frac{A_j}{k_j} \left(1 - e^{-k_j \rho_j^*}\right) + \theta_0 - c = 0.\end{aligned}$$

Now value the resulting portfolio using the uppermost portfolio value with permanent price impact. The modified MSDC at zero for any asset $j \in J$ is

$$\overline{m}_j(0) = A_j - \beta_j \rho_j,$$

so the analytical solution can be calculated by solving the equations

$$\begin{aligned}\frac{\partial G(\boldsymbol{\rho}, \lambda, \mu)}{\partial \rho_j} &= A_j - \beta_j \rho_j^{**} + \beta_j \theta_j - (1+\lambda) A_j e^{-k_j \rho_j^{**}} + \mu_j = 0 \ \forall j \in J \\ \frac{\partial G(\boldsymbol{\rho}, \lambda, \mu)}{\partial \lambda} &= \sum_{j \in J} \frac{A_j}{k_j} \left(1 - e^{-k_j \rho_j^{**}}\right) + \theta_0 - c = 0.\end{aligned}$$

Consider the following examples.

Example 2.4. The space of portfolios is given by $\boldsymbol{\Theta} = \mathbb{R}^3$, so consider a market of two illiquid securities. Suppose that in the initial portfolio, there are 600 units of risky asset 1 and 1000 units of risky asset 2, that is $\boldsymbol{\theta} = (\theta_0, \theta_1, \theta_2) = (0, 600, 1000)$. Let the exponential MSDCs of the two assets be given by $m_1(h) = 10e^{-0{,}0001h}$ and $m_2(h) = 10e^{-0{,}00005h}$. Let the assets have linear permanent price impact parameters of 0.00003 and 0.0004, respectively, i.e. $\beta_1 = 0.00003$ and $\beta_2 = 0.0004$. Finally, suppose $\mathcal{L}^+(6000)$ cash liquidity policy with a minimum of 6000 units and no short positions.

The analytically calculated optimal (ρ_1, ρ_2) without permanent price impact is $(202.03, 404.05)$, whereas in case of permanent price impact with numerical method we get $(310.32, 296.63)$. If we determine portfolio values under the previously defined assumptions, our results are

$$\begin{aligned} U(\boldsymbol{\theta}) &= 16000 \\ V^{\mathcal{L}^+}(\boldsymbol{\theta}) &= 15988 \\ \overline{V}^{\mathcal{L}^+}(\boldsymbol{\theta}) &= 15889 \\ L(\boldsymbol{\theta}) &= 15773. \end{aligned}$$

Example 2.5 (Examples 2.4 continued)**.** Let us use the parameters of Example 2.4 to perform a Monte Carlo simulation of the portfolio values with and without permanent price impact. Based on Acerbi and Scandolo (2008), we choose a liquidity

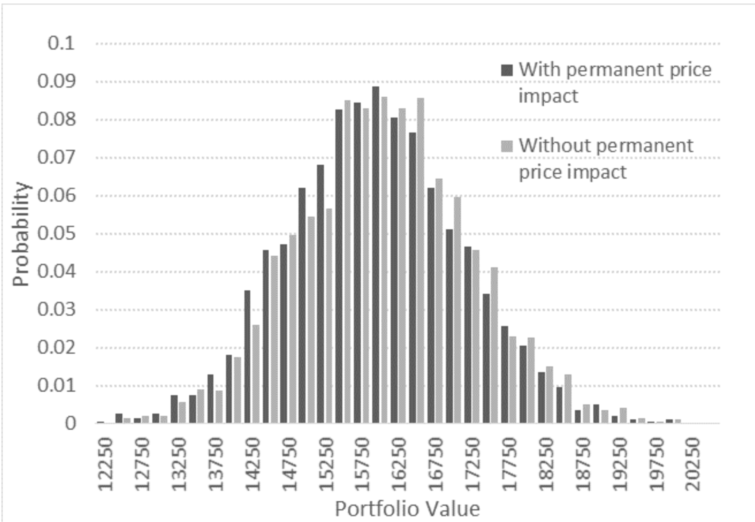

Figure 2.1:
Portfolio values under liquidity constraints with and without permanent price impact

risk model with one-factor non-trivial MSDCs. A_1 and A_2 are normally distributed with mean and volatility of $(10, 1)$, while k is fixed. For every realization, we calculate both portfolio values to correctly measure the difference. Our results are in line with our expectations: portfolio values diminish when we take the permanent price impact into consideration.

2.4 Conclusion

To take into account the impact of trading of institutional investors, we provided a method for portfolio valuation under liquidity constraints with linear permanent price impact. We have seen that incorporating permanent price impact can have significant impact on the optimal liquidation strategy. Given the cash and the number of assets in the portfolio, the building blocks of our model are the MSDCs, the linear price impact parameters of the assets, and the liquidity policy specifying the set of acceptable portfolios. All blocks could be state contingent for modeling liquidity risk and market risk together, running stress tests, and calculating capital requirements.

There are many further possible extensions for future research. One could model nonlinear price impact or a price impact which does not shift all the points of the MSDC in a parallel way. In fact, one of the reasons why we assume that short positions must be closed is that in case of short-selling of an extreme volume of an asset, the parallel shift of its MSDC results in negative prices at which short positions could be closed, and then the optimization problem has no solution. For industry implementation, the underlying optimization problem could be further investigated, even for more general liquidity policies. Finally, our method could also be adjusted to generate liquidity in high-frequency trading, since there is also permanent price impact on a smaller timescale.

Chapter 3

The effect of regulation on market liquidity: a general equilibrium approach

3.1 Introduction

Liquidity is a key consideration in financial markets, especially in times of liquidity crises. Brunnermeier and Pedersen (2008) list the Black Monday in 1987, U.S.-Iraq war in 1990, fall of LTCM in 1998, and the subprime crisis in 2007 as relevant examples. For this reason, regulatory attention to and measures in this field have been on the rise for the past years. Based on practical experience, regulation aiming at reducing liquidity risk appears to have the side-effect of reducing market liquidity itself (i.e. the higher regulatory requirements become, the lower market liquidity will be). The aim of this section is to define a framework in which these opposite dynamics between regulation and liquidity can be modeled.

We augment a general equilibrium model with transaction costs. The purpose of the trading of the agent is to smooth their future contingent payments (and thus their consumption) through risk sharing. As initial and future stochastic endowments determine the behavior and role of a given market agent in an economy, our framework is suitable for describing a number of practical problems. The agents can invest, borrow and hedge risky position by holding assets. In the model, investors have to meet regulatory requirement given as a function of the expected shortfall (ES) of their portfolio. The aim of regulation is to reduce risk-taking in investment decisions and the over-indebtedness of borrowing households, and ensure the payment for losses on risky future positions.

In the present chapter, we use MSDCs to describe market liquidity. In the model, market agents cannot trade directly with each other. Opposite orders are matched by a market maker who sets optimal MSDCs as a monopolist. Our finding is that the introduction of regulatory constraints, if hit by the optimizing agent, reduces market liquidity and the utility of agents.

In this section we summarize the models and results of Chapters 3 and 4 of the dissertation. We present the general version of the model introduced in the Hevér (2020) study. In Chapter 4 of the dissertation, the change in market liquidity is measured through the change in bid-ask spread. Chapter 5 uses the model in the context of the literature on institutions and economic growth, examining the relationship between the introduction of transaction costs and risk-sharing. The present chapter provides a brief summary of these Chapters of the dissertation in the context of the presentation of various model variants.

3.2 The general equilibrium model

As far as the structure of the general equilibrium model is concerned, the dissertation relies heavily on Le Roy and Werner (2001). New features introduced herewith include regulatory requirement as a function of expected loss, the use of endogenic MSDC and bid-ask spread of the market maker, and the distinction of cash from other assets, which makes saving in risk-free assets possible for all agents simultaneously.

3.2.1 Notation

In this section, we combine the notation of Csóka and Herings (2014) and Le Roy and Werner (2001). There are two periods in our model. An investor can hold cash, denoted by θ_0, as well as risky assets belonging to a set J. Securities are traded in period 0, while payoffs are occur in period 1. The payoff of an asset is subject to uncertainty. One out of S possible states of nature materializes in the future, where state of nature $s \in \{1, \ldots, S\}$ occurs with probability $\pi_s > 0$, such that $\sum_{s=1}^{S} \pi_s = 1$. The payoff of asset $j \in J$ in state of nature $s \in \{1, \ldots, S\}$ is denoted by $x_{js} \in \mathbb{R}$. Let us denote the payoff of asset $j \in J$ by the vector $x_j = [x_{j1}, \cdots, x_{jS}] \in \mathbb{R}^{\mathbb{S}}$ and the payoff-matrix by the matrix $X \in \mathbb{R}^J \times \mathbb{R}^S$[4]

A portfolio comprises J securities. Denote the space of risky portfolios by $\Theta = \mathbb{R}^J$ and a portfolio or position by $\theta \in \Theta$. The value of a portfolio depends on the order books for the various assets to be specified as follows. A function is càdlàg if it is right continuous with left limits and làdcàg if it is left continuous with right limits.

Definition 3.1. The *marginal supply-demand curve* (MSDC) for asset $j \in J$ is given by the map $m_j : \mathbb{R} \setminus \{0\} \mapsto \mathbb{R}$ satisfying

[4]Market is set to be complete if the rank of X is S. We do not assume complete markets.

1. $m_j(h) \geq m_j(h')$ if $h < h'$;

2. m_j is càdlàg at $h < 0$ and làdcàg at $h > 0$.

The MSDC can be used to calculate the liquidation value of a $\theta \in \Theta$ portfolio of risky assets.

Definition 3.2. The *liquidation mark-to-market value* of a risky portfolio $\theta \in \Theta$ is defined by

$$\ell(\theta) = \sum_{j \in J} \int_0^{\theta_j} m_j(h) dh. \tag{3.1}$$

We have agents/investors belonging to set I. The *portfolio* $\theta^i \in \mathbb{R}^J$ *of investor* $i \in I$ shows the amounts of assets held by investor i. Investor i consumes c_0^i in period 0 and $c_1^i = [c_{11}, \cdots, c_{1S}]$ in period 1, where c_{1s} represents consumption in state $s \in \{1, \cdots, S\}$. Investor i's endowments are given by ω_0^i capturing the cash in period 0 and $\omega_1^i = [\omega_{11}^i, \cdots, \omega_{1S}^i]$ representing the stochastic income and value of investments without capital requirements. Assume continuous utility function $u^i : \mathbb{R}^{S+1} \to \mathbb{R}$ to indicate investor i's preferences.

3.2.2 Consumption portfolio choice without capital requirements

Investor i's consumption-portfolio choice problem is

$$\max_{c_0^i, c_1^i, \theta^i, \theta_0^i} u^i(c_0^i, c_1^i) \tag{3.2}$$

subject to

$$\begin{aligned} c_0^i &\leq \omega_0^i + \ell(-\theta^i) - \theta_0^i & (3.3) \\ c_1^i &\leq \omega_1^i + \theta^i X + \theta_0^i 1^S. & (3.4) \end{aligned}$$

The agent determines optimal consumption level c_0^i and c_1^i, optimal portfolio θ^i and the amount of the risk free asset θ_0^i. Its utility maximization is subject to

1. its period 0 consumption being no more than initial endowments minus the amount of money needed to open position θ^i and keep risk-free asset (cash or bank deposit) θ_0^i

2. its period 1 stochastic consumption being no more than its stochastic endowment plus the payoff of position θ^i plus θ_0^i.

3.2.3 Regulatory requirement

Consider the case when investors have to meet a cash liquidity requirement. Define δ_j as the regulatory parameter for security j, so regulation determines different δs for different markets. The regulatory rule is to keep sufficient cash to cover a part of the negative payoffs realized in adverse states of nature[5].

Definition 3.3. Denote the *function of regulatory requirement* by $e : \mathbb{R}\times\mathbb{R}^S\times\mathbb{R}^S \to \mathbb{R}$. The required amount of the risk free asset is given by inequation

$$\theta_0^i \geq e\left[\delta, \theta^i X, \omega_1^i\right], \tag{3.5}$$

where *regulatory requirement function* $e\left[\delta_j, \theta^i X, \omega_1^i\right]$ is a function of payoff $\theta^i X$ of risky portfolio, regulatory parameters δ_j and stochastic endowment ω_1^i.

Modify investor i's consumption and portfolio choice problem with regulatory requirement. In this case, the maximization problem can be defined as

$$\max_{c_0^i, c_1^i, \theta^i, \theta_0^i} u^i(c_0^i, c_1^i) \tag{3.6}$$

subject to

$$\begin{aligned} c_0^i &\leq \omega_0^i + \ell(-\theta^i) - \theta_0^i & (3.7)\\ \theta_0^i &\geq e\left[\delta_j, \theta^i X, \omega_1^i\right] & (3.8)\\ c_1^i &\leq \omega_1^i + \theta^i X + \theta_0^i 1^S. & (3.9) \end{aligned}$$

Definition of Expected Shortfall

We will follow Csóka, Herings and Kóczy (2009) to define the expected shortfall (ES) of an asset or a portfolio. For security $j \in J$, denote the ordered values of outcomes $x_{j1}, \cdots, x_{js}$ by $x_{j,s:S}$, that is, $\{x_{j,1:S}, \cdots, x_{j,s:S}\} = \{x_{j1}, \cdots, x_{jS}\}$ and $x_{j,1:S} \leq x_{j,2:S} \cdots \leq x_{j,s:S}$. As a first step, we will give the definition with equiprobable outcomes, so suppose that $\pi_1 = \cdots = \pi_S = \frac{1}{S}$.

Definition 3.4. The outcomes are equiprobable and $k \in \{1, \dots, S\}$. The *k-expected shortfall* of a realization vector x_j for security $j \in J$ is defined by

$$ES_k(x_j) = -\sum_{s=1}^{k} \frac{1}{k} x_{j,s:S}. \tag{3.10}$$

[5]As an alternative approach, Le Roy and Werner (2001) suppose that collateral is needed in case of security holdings which involve strictly negative payoffs in some states. Investors can insure the fulfilment of their obligations by allocating their endowments to the securities as collateral

As a general case, we do not suppose that each state of nature occurs with the same probability. Denote $\pi_{j,s:S}$ the probability of the state where the expected outcome of security j is $x_{j,s:S}$.

Definition 3.5. The *k-expected shortfall* ($k \in \{1, \dots, S\}$) of a realization vector x_j for security $j \in J$ is defined by

$$ES_k(x_j) = -\sum_{s=1}^{k} \pi_{j,s:S} x_{j,s:S}. \tag{3.11}$$

The ES can be defined at portfolio level, too. For portfolio $\theta^i \in \Theta$, payoff $\sum_{j\in J} \theta^i_j x_{js}$ is realized in state of nature $s \in \{1, \cdots, S\}$. In this case, denote the ordered values of outcomes $\sum_{j\in J} \theta^i_j x_{j1}, \cdots, \sum_{j\in J} \theta^i_j x_{jS}$ by $(\sum_{j\in J} \theta^i_j x_{js})_{s:S}$, where

$$\{\sum_{j\in J} \theta^i_j x_{j1}, \cdots, \sum_{j\in J} \theta^i_j x_{jS}\} = \{(\sum_{j\in J} \theta^i_j x_{j1})_{1:S}, \cdots, (\sum_{j\in J} \theta^i_j x_{js})_{S:S}\}$$

and

$$(\sum_{j\in J} \theta^i_j x_{j1})_{1:S} \leq (\sum_{j\in J} \theta^i_j x_{j1})_{2:S} \cdots \leq (\sum_{j\in J} \theta^i_j x_{jS})_{S:S}.$$

Definition 3.6. The outcomes are equiprobable and $k \in \{1, \dots, S\}$. Given a portfolio $\theta^i \in \Theta$, the *k-expected shortfall* of realization vector $\theta^i X$ is defined by

$$ES_k(\theta^i X) = -\sum_{s=1}^{k} \frac{1}{k} (\sum_{j\in J} \theta^i_j x_{js})_{s:S}.$$

Let $\pi_{\theta^i X,s:S}$ denote the probability of the state where the expected outcome of portfolio θ^i is $(\sum_{j\in J} \theta^i_j x_{js})_{s:S}$.

Definition 3.7. Given a portfolio $\theta^i \in \Theta$, the *k-expected shortfall* ($k \in \{1, \dots, S\}$) of a realization vector $\theta^i X$ is defined by

$$ES_k(\theta^i X) = -\sum_{s=1}^{k} \frac{\pi_{\theta^i X,s:S}}{\sum_{l=1}^{k} \pi_{\theta^i X,l:S}} (\sum_{j\in J} \theta^i_j x_{js})_{s:S}.$$

If the expected average payoff in the worst k states of nature is negative, the k-expected shortfall is positive, whereas in case of expected gain in the worst k states (for instance when investing into riskless bonds), ES will be negative.

Regulatory requirement as a function of Expected Shortfall

Consider the case when $\forall i \in I$ investors have to meet a cash liquidity requirement given as a function of the expected shortfall of the portfolio. The regulator can determine the regulatory requirement as a function of the expected shortfall in the payoffs of assets or portfolios. Restrictions are introduced in all cases to discourage risk-taking, but different definitions result in significantly different equilibrium portfolios.

1. **Regulation at the level of assets**
   Suppose that the regulator specifies different parameters δ_j for each $j \in J$ asset. The regulatory requirement function is given by

$$e\left[\delta_j, \theta^i X\right] = \sum_{j \in J} \delta_j \max\left[0, ES_k(\theta^i_j x_j)\right]. \tag{3.12}$$

   According to the requirement, agents are required to hold risk-free assets corresponding to the amount of the capital requirement aggregated for $j \in J$ assets with positive ES

$$\theta^i_0 \geq \sum_{j \in J} \delta_j \max\left[0, ES_k(\theta^i_j x_j)\right]. \tag{3.13}$$

   In this case, the regulator aims to discourage holding risky assets with significant negative payoff in adverse states of nature without taking into account the risk-mitigating effects of portfolio diversification, and the stochastic endowment. The use of different regulatory parameters for each assets allows the promotion of ESG (environmental, social, governance) aspects, as the regulator can stimulate the demand for preferred assets at the expense of that for others. The introduction of the regulation excludes the possibility of borrowing in period 0.

2. **Regulation at the level of portfolios**
   If the regulator aims to contain the risk of the capital market positions of agents, it determines the regulatory requirement as a function of the ES quantified based on realization vector $\theta^i X$ of the portfolio

$$e\left[\delta, \theta^i X\right] = \delta ES_k(\theta^i X). \tag{3.14}$$

   The requirement can be given as

$$\theta^i_0 \geq \delta ES_k(\theta^i X), \tag{3.15}$$

   where regulatory parameter δ determines what proportion of the expected shortfall of the portfolio should be kept in the risk-free asset. If the average expected payoff of the portfolio calculated for the worst k states is positive, i.e. it provides a profit, the ES will be negative. Borrowing is possible, but its extent is limited by the expected profit. The regulator takes into account the diversification resulting from the holding of the portfolio, and formulates a lower capital requirement compared to the regulatory requirement formulated at the asset level. However, the holding of individual assets cannot be encouraged or discouraged.

3.2.4 The problem of the market maker

Le Roy and Werner (2001) suppose that there is a specialist/market maker who matches opposite orders for securities and consumes its profit in period 0.

In the model, market agents cannot trade directly with each other; the market maker matches orders from both sides and consumes its profit in period 1. If $|I| \geq 2$, i.e. there are at least 2 market agents, the market maker only acts as an intermediary in equilibrium. It does not hold any assets, thus it is not required to set aside capital to cover potential losses[6].

Erb and Havran (2015) present the microstructure of financial markets in detail, especially that of quote-driven ones. Their paper points out that reasons for market imperfections can include the costs of search for counterparties and contacting, network externalities, asymmetric information and the cost of holding inventory, which can lead to different market microstructures. In the present dissertation, I assume the simplest case based on Le Roy and Werner (2001): the market maker sets the marginal supply-demand curve as a transaction monopolist for each asset, thereby influencing the liquidity of the markets for these assets. Havran and Szűcs (2016) assume duopolistic behaviour for market makers, which could be further analyzed in the present modelling framework.

Definition 3.8. For security $j \in J$, the *transaction cost function* $T_j : \mathbb{R}^I \to \mathbb{R}$ is defined as

$$T_j(\theta_j^1, \cdots, \theta_j^I) = -\sum_{i \in I} \int_0^{-\theta_j^i} m_j(h) \mathrm{d}h. \tag{3.16}$$

The transaction cost collected by the market maker in respect of each transaction is given by function $T : \mathbb{R}^I \times \mathbb{R}^J \to \mathbb{R}$ as

$$T(\theta^1, \cdots, \theta^I) = \sum_{j \in J} T_j(\theta_j^1, \cdots, \theta_j^I) = -\sum_{j \in J} \sum_{i \in I} \int_0^{-\theta_j^i} m_j(h) \mathrm{d}h. \tag{3.17}$$

.

In the model, the market maker sets the MSDC by maximizing its profit, thus

[6]The model is significantly different in case of a single market agent who trades with the market maker: the latter takes some risk.

its optimization problem is

$$\max_{m_j()\ \forall j \in J} \sum_{j \in J} T_j(\theta_j^1, \cdots, \theta_j^I), \tag{3.18}$$

subject to each agent maximizes its utility when determining its portfolio θ^i.

By placing limit orders, the market maker determines MSDCs based on which market agents trade by submitting market orders. The market maker realizes revenue in the form of transaction fees when matching offers. The amount of the revenue depends on the functional form of the MSDC (the amount of bid-ask spread, and the distance between transaction price level and best price). The form of the transaction cost function is determined by the MSDCs. Based on the definition of MSDC, transaction cost function can be defined as follows:

Proposition 3.1. *The revenue of the market maker $T(\theta^1, \cdots, \theta^I)$ can be calculated as the sum of the liquidation mark-to-market values $\ell(-\theta^i)$ of the portfolios of agents $\theta^i \in \Theta$*

$$T(\theta^1, \cdots, \theta^I) = \sum_{j \in J} T_j(\theta_j^1, \cdots, \theta_j^I) = \sum_{i \in I} -\ell(-\theta^i). \tag{3.19}$$

Proof. By swapping the sums and using definition 3.2, the proposition becomes trivial. □

3.2.5 Market-clearing conditions

We determine the equilibrium for given MSDCs. An equilibrium consists of portfolio allocations and consumption plans $\{\theta^{*i}, \theta_0^{*i}, c_0^{*i}, c_1^{*i}\}$ which are solution to investor i's choice problem. The portfolio market clearing and consumption market clearing conditions are

$$\sum_{i \in I} \theta^i = 0 \tag{3.20}$$

$$\sum_{i \in I} c_0^i \leq \sum_{i \in I} \omega_0^i - T(\theta^1, \cdots, \theta^I) - \sum_{i \in I} \theta_0^i \tag{3.21}$$

$$\sum_{i \in I} c_1^i \leq \sum_{i \in I} \omega_1^i + \sum_{i \in I} \theta_0^i 1^S. \tag{3.22}$$

Proposition 3.2. *In equilibrium, when portfolio market clears*

$$\sum_{i \in I} \theta^i = 0,$$

consumption market-clearing conditions

$$\begin{aligned}\sum_{i\in I} c_0^i &\leq \sum_{i\in I}\omega_0^i - T(\theta^1,\cdots,\theta^I) - \sum_{i\in I}\theta_0^i \\ \sum_{i\in I} c_1^i &\leq \sum_{i\in I}\omega_1^i + \sum_{i\in I}\theta_0^i 1^S.\end{aligned}$$

hold as well because of budget constraints.

3.2.6 Equilibrium with and without regulatory requirement

It is key to understand what happens to market liquidity upon the introduction of regulatory requirement. We have to compare two equilibria: the one determined by the decision of agents in optimization problem (3.2) without requlatory requirement, and the one resulting from agent decisions in optimization problem (3.6) with regulatory requirement. With the introduction of a regulatory requirement, an additional constraint is added to the conditional maximization problem; therefore, the set of decision options may not expand. Let us distinguish two cases as follows:

Proposition 3.3. *Let* $\{\theta^{*i}, \theta_0^{*i}, c_0^{*i}, c_1^{*i}, m_j^*()\}$ *denote the equilibrium where* $\theta^{*i}\theta_0^{*i}, c_0^{*i}$, *and* c_1^{*i} *are solutions to optimization problem (3.2) of the market agents. If for* $\forall i \in I$

$$\theta_0^i \geq e\left[\delta, \theta^i X, \omega_1^i\right],$$

then $\{\theta^{*i}, \theta_0^{*i}, c_0^{*i}, c_1^{*i}, m_j^*()\}$ *remains the equilibrium, if market agents make decisions according to optimization problem (3.6).*

The trivial proposition formulates the case where the optimum determined without regulatory requirements meets the introduced constraint, i.e. the regulatory constraint is redundant. The initial equilibrium remains attainable, the decision of the market maker does not change, thus the MSDCs describing market liquidity are identical in both cases.

Proposition 3.4. *Let* $\{\theta^{*i}, \theta_0^{*i}, c_0^{*i}, c_1^{*i}, m_j^*()\}$ *denote the equilibrium where* $\theta^{*i}, \theta_0^{*i}, c_0^{*i}$, *and* c_1^{*i} *are solutions to optimization problem (3.2) of the market agents, and suppose that* $\exists \bar{i} \in I$*, for which*

$$\theta_0^{*\bar{i}} < e\left[\delta, \theta^{*\bar{i}} X, \omega_1^{\bar{i}}\right].$$

In this case, equilibrium $\{\theta^{**i}, \theta_0^{**i}, c_0^{**i}, c_1^{**i}, m_j^{**}()\}$ *where market partipants decide according to optimization problem (3.6), is not identical to equilibrium* $\{\theta^{*i}, \theta_0^{*i}, c_0^{*i}, c_1^{*i}, m_j^*()\}$.

If there is a market agent in the equilibrium determined without regulatory requirement who breaches the constraint introduced as a regulatory requirement, the portfolio chosen earlier will not be attainable to it after the introduction of the regulatory requirement. The equilibrium changes due to the introduction of the regulatory requirement; the question is what happens to it. I will explore this in the rest of this Chapter.

3.3 Specific cases of the model

Introducing portfolio value considering market liquidity into the optimization problem of market agents is a challenge for multiple reasons. The quantification of liquidation value as per definition 3.2 is performed using the marginal supply-demand curve. In practice, however, the order book changes upon each matching of orders, and the market maker also changes the quantities available at various price levels after each transaction. If $|I|$ number of agents decide simultaneously to buy some of asset $j \in J$, the market maker will sell the aggregate quantity of asset j at a different price from the one in the scenario in which a single market agent buys the same quantity of the asset. For this reason, an additional assumption needs to be made in the model. We can choose from the following ones:

- In case of $|I| = 2$ agents, if the market maker does not hold inventory, each asset is sold and purchased by a single agent, which eliminates the problem. I use this solution in the example at the end of this chapter.

- The model can only be used in case of $|I| > 2$ market agents using MSDCs if it can be ensured that in equilibrium, there is a single seller and a single buyer of each asset. This condition can be met by harmonising the exogenously defined stochastic endowment with the payoff of the risky assets. An exogenous constraint on the decisions of market agents pre-defining which agent can trade which asset, as in Faias and Luque (2018), is an alternative solution. How can we ensure that there is only one buyer and one seller for every asset? Following Faias and Luque (2018), we can constrain investors exogenously beforehand.

- In case of $|I|$ agents, a horizontal MSDC is an alternative way. In this case, we simply model market liquidity through the bid-ask spread and we do not examine the depth of the market. Chapters 4 and 5 introduce a model variant using the bid-ask spread.

- The assumption of a single representative agent leads to a substantially different model variant. A single agent can only trade if the market maker is assumed to provide liquidity on the opposite side. The market maker is a risk-taking agent, its decision problem changes, and the market-clearing condition is modified. The aim of the regulator may be to limit the risk-taking of the market maker.

3.3.1 Model with two investors and MSDCs

Model without regulatory requirement

Suppose $|I| = 2$, so two investors trade in the capital market. The consumption-portfolio choice problem of investor $i \in I$ is

$$\max_{c_0^i, c_1^i, \theta^i, \theta_0^i} u^i(c_0^i, c_1^i), \tag{3.23}$$

subject to

$$\begin{aligned} c_0^i &\leq \omega_0^i + \ell(-\theta^i) - \theta_0^i \\ \theta_0^i &\geq e\left[\delta, \theta^i X, \omega_1^i\right] \\ c_1^i &\leq \omega_1^i + \theta^i X + \theta_0^i 1^S. \end{aligned}$$

When the portfolio market clears, the sum of cash is not necessarily 0. The market-clearing condition can be given by the equation

$$\theta^1 + \theta^2 = 0,$$

which implies that $-\theta_j^1 = \theta_j^2$ for $\forall j \in J$ risky security. Generally, we have consumption market clearing conditions (one in period 0 and S in the S states of the period 1)

$$\begin{aligned} c_0^1 + c_0^2 &\leq \omega_0^1 + \omega_0^2 - T(\theta^1, \theta^2) - \theta_0^1 - \theta_0^2 \\ c_1^1 + c_1^2 &\leq \omega_1^1 + \omega_1^2 + \theta_0^1 1^S + \theta_0^2 1^S \end{aligned}$$

as well. By summing up budget constraints, they trivially hold in this specific case. The market maker maximizes the transaction cost function

$$\max_{m_1(), m_2()} T_1(\theta_1^1, \theta_1^2) + T_2(\theta_2^1, \theta_2^2) = -\ell(-\theta^1) - \ell(-\theta^2) \tag{3.24}$$

and consumes its profit in period 0.

Problem with regulatory requirement as a function of ES

When agents face regulatory requirements, the model can be modified with inequation constraints.[7] The introduction of regulatory requirements adds constraints defined through inequations to the optimization problem of market agents. If the regulator defines the requirement as a function of the expected shortfall of portfolios, the following conditions will limit Agent 1

$$\theta_0^1 \geq \delta ES_k(\theta^1 X), \tag{3.25}$$

and Agent 2

$$\theta_0^2 \geq \delta ES_k(\theta^2 X), \tag{3.26}$$

respectively. Assuming two equiprobable states of nature, $k = 1$, thus the regulatory requirement is δ times the portfolio value realised in the state of nature resulting in the lower payoff, i.e.

$$\begin{aligned} \delta ES_1(\theta^1 X) &= -\delta \min\{-\theta_1 x_{11} + \theta_2 x_{21}; -\theta_1 x_{12} + \theta_2 x_{22}\} \\ \delta ES_1(\theta^2 X) &= -\delta \min\{+\theta_1 x_{11} - \theta_2 x_{21}; +\theta_1 x_{12} - \theta_2 x_{22}\}. \end{aligned}$$

Suppose that Agent 1 has positive endowment in state of nature 1, while its endowment is 0 in state of nature 2. Conversely, Agent 2 has no endowment in state of nature 1 and positive endowment in state of nature 2. Our assumption models the very case where the natural exposures of agents are inverted, thus they can reciprocally reduce the uncertainty of future payoffs through trading. Risk sharing is performed by holding a capital market portfolio. We can assume that $x_{11} > 0$ and $x_{22} > 0$, while $x_{12} = 0$ and $x_{21} = 0$. Agent 1 sells asset 1 ($-\theta_1 < 0$) and buys asset 2 ($\theta_2 > 0$), whereas Agent 2 buys asset 1 ($\theta_1 > 0$) and sells asset 2 ($-\theta_2 < 0$). The regulatory requirement takes the form of

$$\begin{aligned} \theta_0^1 &\geq \delta \theta_1 x_{11} \\ \theta_0^2 &\geq \delta \theta_2 x_{22}. \end{aligned}$$

In case of this specific problem, the regulatory requirement defined at the level of assets would lead to the same constraining inequations if $\delta_1 = \delta_2$. With these simplifications, the optimization problem of market agents takes the functional form of

$$\max_{\theta_1, \theta_2, \theta_0^1} \ln(\omega_0^1 + \ell(-\theta^1) - \theta_0^1) + \frac{1}{2} \ln(\omega_{11}^1 - \theta_1 x_{11} + \theta_0^1) + \frac{1}{2} \ln(\theta_2 x_{22} + \theta_0^1)$$

[7] No capital requirement applies to the market maker because it matches opposite orders without taking risk on its own balance sheet once the market has cleared.

subject to

$$\theta_0^1 \;\geq\; \delta\theta_1 x_{11},$$

and

$$\max_{\theta_1,\theta_2,\theta_0^2} \ln(\omega_0^2 + \ell(-\theta^2) - \theta_0^2) + \frac{1}{2}\ln(\theta_1 x_{11} + \theta_0^2) + \frac{1}{2}\ln(\omega_{12}^2 - \theta_2 x_{22} + \theta_0^2)$$

subject to

$$\theta_0^2 \;\geq\; \delta\theta_2 x_{22}.$$

The Karush-Kuhn-Tucker conditions can be used to find the solution to the optimization problems. The Lagrangian functions of the agents are

$$\begin{aligned} G_1(\theta_1,\theta_2,\theta_0^1,\lambda_1) = \ln(\omega_0^1 + \ell(-\theta^1) - \theta_0^1) + \frac{1}{2}\ln(\omega_{11}^1 - \theta_1 x_{11} + \theta_0^1) + \\ +\frac{1}{2}\ln(\theta_2 x_{22} + \theta_0^1) - \lambda_1[\theta_0^1 - \delta\theta_1 x_{11}] \end{aligned}$$

and

$$\begin{aligned} G_2(\theta_1,\theta_2,\theta_0^2,\lambda_2) = \ln(\omega_0^2 + \ell(-\theta^2) - \theta_0^2) + \frac{1}{2}\ln(\theta_1 x_{11} + \theta_0^2) + \\ +\frac{1}{2}\ln(\omega_{12}^2 - \theta_2 x_{22} + \theta_0^2) - \lambda_2[\theta_0^2 - \delta\theta_2 x_{22}], \end{aligned}$$

where λ_1 and λ_2 are the Lagrange multipliers. The first order conditions are provided as follows:

$$\begin{aligned}
\frac{\partial G_1(\theta_1,\theta_2,\theta_0^1,\lambda_1)}{\partial\theta_1} &= \frac{A_1 e^{-k_1\theta_1}}{\omega_0^1 + \ell(-\theta^1) - \theta_0^1} + \frac{1}{2}\frac{-x_{11}}{\omega_{11}^1 - \theta_1 x_{11} + \theta_0^1} + \lambda_1\delta x_{11} = 0 \\
\frac{\partial G_1(\theta_1,\theta_2,\theta_0^1,\lambda_1)}{\partial\theta_2} &= \frac{-A_2 e^{k_2\theta_2}}{\omega_0^1 + \ell(-\theta^1) - \theta_0^1} + \frac{1}{2}\frac{x_{22}}{\theta_2 x_{22} + \theta_0^1} = 0 \\
\frac{\partial G_1(\theta_1,\theta_2,\theta_0^1,\lambda_1)}{\partial\theta_0^1} &= \frac{-1}{\omega_0^1 + \ell(-\theta^1) - \theta_0^1} + \frac{1}{2}\frac{1}{\omega_{11}^1 - \theta_1 x_{11} + \theta_0^1} + \frac{1}{2}\frac{1}{\theta_2 x_{22} + \theta_0^1} - \lambda_1 = 0 \\
\frac{\partial G_1(\theta_1,\theta_2,\theta_0^1,\lambda_1)}{\partial\lambda^1} &= \theta_0^1 - \delta\theta_1 x_{11} \geq 0 \\
\frac{\partial G_2(\theta_1,\theta_2,\theta_0^2,\lambda_2)}{\partial\theta_1} &= \frac{-A_1 e^{k_1\theta_1}}{\omega_0^2 + \ell(-\theta^2) - \theta_0^2} + \frac{1}{2}\frac{x_{11}}{\theta_1 x_{11} + \theta_0^2} = 0 \\
\frac{\partial G_2(\theta_1,\theta_2,\theta_0^2,\lambda_2)}{\partial\theta_2} &= \frac{A_2 e^{-k_2\theta_2}}{\omega_0^2 + \ell(-\theta^2) - \theta_0^2} + \frac{1}{2}\frac{-x_{22}}{\omega_{12}^2 - \theta_2 x_{22} + \theta_0^2} + \lambda_2\delta x_{22} = 0 \\
\frac{\partial G_2(\theta_1,\theta_2,\theta_0^2,\lambda_2)}{\partial\theta_0^2} &= \frac{-1}{\omega_0^2 + \ell(-\theta^2) - \theta_0^2} + \frac{1}{2}\frac{1}{\theta_1 x_{11} + \theta_0^2} + \frac{1}{2}\frac{1}{\omega_{12}^2 - \theta_2 x_{22} + \theta_0^2} - \lambda_2 = 0 \\
\frac{\partial G_2(\theta_1,\theta_2,\theta_0^2,\lambda_2)}{\partial\lambda^2} &= \theta_0^2 - \delta\theta_2 x_{22} \geq 0.
\end{aligned}$$

Due to complementary slackness and dual feasibility,

$$\begin{aligned}\lambda_1[\theta_0^1 - \delta\theta_1 x_{11}] &= 0\\ \lambda_2[\theta_0^2 - \delta\theta_2 x_{22}] &= 0\\ \lambda_1 &\geq 0\\ \lambda_2 &\geq 0.\end{aligned}$$

Substituting the liquidation values of the portfolios, the conditions are

$$\frac{A_1 e^{-k_1\theta_1}}{\omega_0^1 - \frac{A_1}{k_1}\left(e^{-k_1\theta_1} - 1\right) + \frac{A_2}{k_2}\left(1 - e^{k_2\theta_2}\right) - \theta_0^1} - \frac{\frac{1}{2}x_{11}}{\omega_{11}^1 - \theta_1 x_{11} + \theta_0^1} + \lambda_1\delta x_{11} = 0$$

$$\frac{-A_2 e^{k_2\theta_2}}{\omega_0^1 - \frac{A_1}{k_1}\left(e^{-k_1\theta_1} - 1\right) + \frac{A_2}{k_2}\left(1 - e^{k_2\theta_2}\right) - \theta_0^1} + \frac{\frac{1}{2}x_{22}}{\theta_2 x_{22} + \theta_0^1} = 0$$

$$\frac{-1}{\omega_0^1 - \frac{A_1}{k_1}\left(e^{-k_1\theta_1} - 1\right) + \frac{A_2}{k_2}\left(1 - e^{k_2\theta_2}\right) - \theta_0^1} + \frac{\frac{1}{2}}{\omega_{11}^1 - \theta_1 x_{11} + \theta_0^1} + \frac{\frac{1}{2}}{\theta_2 x_{22} + \theta_0^1} - \lambda_1 = 0$$

$$\theta_0^1 - \delta\theta_1 x_{11} \geq 0$$

$$\frac{-A_1 e^{k_1\theta_1}}{\omega_0^2 + \frac{A_1}{k_1}\left(1 - e^{k_1\theta_1}\right) - \frac{A_2}{k_2}\left(e^{-k_2\theta_2} - 1\right) - \theta_0^2} + \frac{\frac{1}{2}x_{11}}{\theta_1 x_{11} + \theta_0^2} = 0$$

$$\frac{A_2 e^{-k_2\theta_2}}{\omega_0^2 + \frac{A_1}{k_1}\left(1 - e^{k_1\theta_1}\right) - \frac{A_2}{k_2}\left(e^{-k_2\theta_2} - 1\right) - \theta_0^2} - \frac{\frac{1}{2}x_{22}}{\omega_{12}^2 - \theta_2 x_{22} + \theta_0^2} + \lambda_2\delta x_{22} = 0$$

$$\frac{-1}{\omega_0^2 + \frac{A_1}{k_1}\left(1 - e^{k_1\theta_1}\right) - \frac{A_2}{k_2}\left(e^{-k_2\theta_2} - 1\right) - \theta_0^2} + \frac{\frac{1}{2}}{\theta_1 x_{11} + \theta_0^2} + \frac{\frac{1}{2}}{\omega_{12}^2 - \theta_2 x_{22} + \theta_0^2} - \lambda_2 = 0$$

$$\theta_0^2 - \delta\theta_2 x_{22} \geq 0.$$

Analysis of change in equilibrium upon introduction of a regulatory requirement

When comparing the two equilibria, I use the notation of Proposition 3.4. If the regulatory requirement is redundant, i.e. the optimum determined without the regulatory requirement complies with the new constraint, then $\lambda_1 = \lambda_2 = 0$. We get back the first order conditions of the model without regulatory requirements. The optimization problem of the market maker is unchanged, and the MSDCs describing market liquidity do not change upon the introduction of the regulatory requirement *(Proposition 3.3)*. If $\lambda_1 \neq 0$ and $\lambda_2 \neq 0$, then

$$\begin{aligned}\theta_0^{**1} - \delta\theta_1^{**} x_{11} &= 0\\ \theta_0^{**2} - \delta\theta_2^{**} x_{22} &= 0.\end{aligned}$$

due to complementarity. When optimizing, market agents hit the new constraint, which will be binding. The optimal decisions of the agents change, thus the constraints limiting the decision of the market maker are also modified *(Proposition 3.4)*.

Compare the new equilibrium under regulatory constraints to the one without regulation. For portfolios determined without regulatory constraints, inequations

$$\begin{aligned} \theta_0^{*1} - \delta\theta_1^* x_{11} &< 0 \\ \theta_0^{*2} - \delta\theta_2^* x_{22} &< 0 \end{aligned}$$

hold true. In the new equilibrium, the constraint on asset 1 can be binding if

- $\theta_0^{**1} > \theta_0^{*1}$
- $\theta_0^{**1} = \theta_0^{*1}$ and $\theta_1^{**} < \theta_1^*$
- $\theta_0^{**1} < \theta_0^{*1}$ and $\theta_1^{**} < \theta_1^*$.

If $\theta_1^{**} < \theta_1^*$, the market maker sets a less liquid MSDC when maximizing transaction cost. With the new MSDC, θ_1^{**} would be optimal even without regulatory constraints. The question is whether $\theta_1^{**} = \theta_1^*$ or $\theta_1^{**} > \theta_1^*$ can hold true for equilibral positions of asset 1 while $\theta_0^{**1} > \theta_0^{*1}$. Analogously, for asset 2, can $\theta_2^{**} = \theta_2^*$ or $\theta_2^{**} > \theta_2^*$ hold true while $\theta_0^{**2} > \theta_0^{*2}$?

3.3.2 Solution to the specific two-agent model

The specific model in which the endowments of agents are inverse in period 1 ($\omega_{12}^2 = \omega_{11}^1 := \omega_1$) and identical in period 0 ($\omega_0^2 = \omega_0^1 := \omega_0$) simplifies calculations and is suitable for the analysis of the relationship between regulatory constraints and market liquidity. In this case, the problem is symmetric, and for consumption, $c_0^1 = c_0^2 := c_0$, $c_{11}^1 = c_{12}^2$ and $c_{11}^2 = c_{12}^1$ hold true, while $\theta_0^1 = \theta_0^2 := \theta_0$ and $\theta_1 = \theta_2 := \theta$ hold for the optimal portfolios. Suppose that $x_{12} = x_{21} = 0$ and $x_{11} = x_{22} := x$ for the payoffs of the two assets. The market maker prices two assets with inverse payoffs, and the target portfolios of the agents are inverted. The market maker buys and sells quantities θ of both assets; therefore, setting the same exponential MSDC ($A_1 = A_2 := A$ and $k_1 = k_2 := k$) for both assets is a precondition to the existence of an equilibrium.[8]

[8] The market maker prices assets with inverse payoffs; therefore, it could seem intuitive to substitute the two assets for a single one with payoff x in state of nature 1 and payoff $-x$ in state of nature 2. The market maker would price an asset with payoff $[x, -x]$ by setting parameters A és k of the exponential MSDC. However, the liquidation values of the portfolios of the two agents with symmetric positions trading through the same market maker would be different, thus this would not be an equilibrium. In the case of bid-ask spread, the assumption of a single asset will be possible.

In case of capital market equilibrium, the portfolio of risky assets of Agent 1 is $\theta^1 = (-\theta, \theta)$, and that of Agent 2 is $\theta^2 = (\theta, -\theta)$, thus the liquidation values of the portfolios are identical in equilibrium.

$$\ell(-\theta^1) = \ell(-\theta^2) = \int_0^{\theta} Ae^{-kx}\mathrm{d}x + \int_0^{-\theta} Ae^{-kx}\mathrm{d}x =$$
$$-A\left[\frac{1}{k}e^{-kx}\right]_0^{\theta} + A\left[\frac{1}{k}e^{-kx}\right]_{-\theta}^{0} = -\frac{A}{k}\left(e^{-k\theta} - 1\right) + \frac{A}{k}\left(1 - e^{k\theta}\right)$$
$$= \frac{A}{k}\left(2 - e^{-k\theta} - e^{k\theta}\right).$$

We can suppose that, when optimizing, market agents know that they can only choose portfolios $\theta^1 = (-\theta, \theta)$ and $\theta^2 = (\theta, -\theta)$. In this case, the conditional optimization problems of the agents are identical and can be defined as

$$\max_{\theta,\theta_0} \ln\left(\omega_0 + \frac{A}{k}\left(2 - e^{-k\theta} - e^{k\theta}\right) - \theta_0\right) + \frac{1}{2}\ln(\omega_1 - \theta x + \theta_0) + \frac{1}{2}\ln(\theta x + \theta_0)$$

subject to

$$\theta_0 \;\geq\; \delta\theta x.$$

The first order conditions are as follows:

$$\frac{Ae^{-k\theta} - Ae^{k\theta}}{\omega_0 + \frac{A}{k}\left(2 - e^{-k\theta} - e^{k\theta}\right) - \theta_0} - \frac{\frac{1}{2}x}{\omega_1 - \theta x + \theta_0} + \frac{\frac{1}{2}x}{\theta x + \theta_0} + \lambda\delta x \;=\; 0 \quad (3.27)$$
$$\frac{-1}{\omega_0 + \frac{A}{k}\left(2 - e^{-k\theta} - e^{k\theta}\right) - \theta_0} + \frac{\frac{1}{2}}{\omega_1 - \theta x + \theta_0} + \frac{\frac{1}{2}}{\theta x + \theta_0} - \lambda \;=\; 0 \quad (3.28)$$
$$\theta_0 - \delta\theta x \;\geq\; 0 \quad (3.29)$$
$$\lambda[\theta_0 - \delta\theta x] \;=\; 0 \quad (3.30)$$
$$\lambda \;\geq\; 0. \quad (3.31)$$

Example 3.1. Similarly to the case of the speficic model, assume two agents, two assets and $S = 2$ states of nature in period 1. Agents have endowments of $\omega_0 = 10$ in period 0 and $\omega_1^1 = (20, 0)$ and $\omega_1^2 = (0, 20)$ in period 1. The payoffs of the risky assets are $x_1 = (2, 0)$ and $x_2 = (0, 2)$, respectively. The regulatory parameter is $\delta = 0.3$. For a start, suppose that the market maker sets an endogenous exponential MSDC in the functional form of $m(\theta) = Ae^{-k\theta}$.

The equilibrium portfolios of market agents using exponential MSDCs with various values of parameters k can be calculated. For a given parameter k, increasing the value of parameter A results in an increase in transaction cost (Table 3.1). With transaction cost increasing, the smoothing of the stochastic endowment of period 1

A	θ	θ_0	c_0	$c^1_{12}=c^2_{11}$	$c^1_{11}=c^2_{12}$	$u^1=u^2$	$\theta_0-\delta\theta x$
2000	0.12	3.49	6.4	3.7	23.2	4.09	3.41
1000	0.24	3.32	6.5	3.8	22.8	4.10	3.18
500	0.47	3.00	6.7	3.9	22.1	4.13	2.72
100	1.86	1.23	7.7	5.0	17.5	4.28	0.12
50	2.37	1.42	7.7	6.2	16.7	4.36	0.00

Table 3.1:
Equilibrium for $k=0.003$ and exponential MSDCs at various values of parameter A

is less and less feasible. The difference between the consumptions of the two future states of nature increases and the attainable level of utility decreases. No transaction is carried out when the level of utility drops to the level attainable without trading. Trading and, consequently, risk sharing are constrained by the introduction of regulatory requirement. In a favourable transaction environment ($k=0.003$ and $A\leq 50$), traded position θ in risky assets is constrained by regulatory requirements in reaching the optimum from risk sharing point of view. The impact is similar in case of a fixed parameter A and an increasing parameter k (Table 3.2).

k	θ	θ_0	c_0	$c^1_{12}=c^2_{11}$	$c^1_{11}=c^2_{12}$	$u^1=u^2$	$\theta_0-\delta\theta x$
0.1	0.01	3.64	6.35	23.6	3.7	4.08	3.63
0.01	0.14	3.45	6.4	23.2	3.7	4.09	3.37
0.005	0.28	3.25	6.5	22.7	3.8	4.11	3.08
0.003	0.47	3.00	6.7	22.1	3.9	4.13	2.72
0.001	1.25	1.96	7.3	19.4	4.5	4.21	1.20
0.0001	3.07	1.84	7.7	15.7	8.0	4.45	0.00

Table 3.2: Equilibrium
in case of exponential MSDCs with $A=500$ and various values of parameter k

When setting the optimal MSDC, the market maker considers the optimal decisions of agents with the given MSDC. Therefore, for the maximization of transaction revenue, we can assume that the first order conditions to the consumption-portfolio choice problem of market agents are met. Without regulatory constraints, the optimization problem of the market maker is as follows:

$$\max_{A,k} -\ell(-\theta^1)-\ell(-\theta^2)=\frac{2A}{k}\left(e^{-k\theta}+e^{k\theta}-2\right),$$

subject to

$$\begin{aligned}
\frac{Ae^{-k\theta}-Ae^{k\theta}}{\omega_0+\frac{A}{k}\left(2-e^{-k\theta}-e^{k\theta}\right)-\theta_0}-\frac{\frac{1}{2}x}{\omega_1-\theta x+\theta_0}+\frac{\frac{1}{2}x}{\theta x+\theta_0} &= 0\\
\frac{-1}{\omega_0+\frac{A}{k}\left(2-e^{-k\theta}-e^{k\theta}\right)-\theta_0}+\frac{\frac{1}{2}}{\omega_1-\theta x+\theta_0}+\frac{\frac{1}{2}}{\theta x+\theta_0} &= 0.
\end{aligned}$$

, With regulatory constraints, the market maker optimizes the transaction cost function

$$-\ell(-\theta^1) - \ell(-\theta^2) = \frac{2A}{k}\left(e^{-k\theta} + e^{k\theta} - 2\right)$$

under first order conditions 3.27 and 3.31. Let us see whether the optimal MSDC of the market maker changes upon introduction of regulatory constraints. If, in equilibrium, $\lambda = 0$, then $\theta_0 - \delta\theta x \geq 0$ is met, and the optimization problem of the market maker does not change. However, $\theta_0 - \delta\theta x = 0$ and $\lambda > 0$ can also apply in equilibrium. In this case, the inequation $\theta_0 - \delta\theta x < 0$ applies in the equilibrium determined without regulatory constraints. The constraint can be binding in the new equilibrium provided that

- θ_0 increases or
- θ_0 is unchanged and θ decreases or
- θ_0 increases and θ decreases.

If θ decreases, the market maker sets a less liquid MSDC to maximize transaction revenue. We have to investigate whether θ can remain unchanged or increase in the new equilibrium. By rearranging first order condition 3.28, we get

$$c_0 = \frac{2(\lambda + 1)}{\frac{1}{c_{11}^1} + \frac{1}{c_{12}^1}} \tag{3.32}$$

for the consumptions of the periods. As $\lambda > 0$, the smoothing of consumption across periods is less feasible; the relative consumption of period 0 will be higher. For increasing θ_0 and unchanged θ, condition 3.32 is breached. As liquidation value $(2 - e^{-k\theta} - e^{k\theta})$ decreases upon an increase in θ, the condition cannot be met if θ_0 and θ increase, either. θ necessarily decreases in the new equilibrium, i.e. the introduction of a regulatory constraint results in a decrease in market liquidity.

Example 3.2. Continue example 3.1. Supposing a fixed parameter $k = 0.003$, determine the optimal parameter value A of the market maker for various parameters δ of introduction of regulatory constraints (table 3.3).

If parameter A is fixed rather than k, the market maker will determine the optimal value of k for each δ. For a given parameter δ, it will attain the same level of transaction cost by setting optimal MSDCs of various shapes. The optimal portfolio of agents does not change (Table 3.4).

In the example, the optimal parameters k (and A) of the market maker are lower without regulatory constraints, the MSDC set is flatter and the risky asset is more liquid. In the optimum on a more liquid market, agents trade a higher quantity θ of

δ	A	θ	θ_0	c_0	$c_{12}^1 = c_{11}^2$	$c_{11}^1 = c_{12}^2$	$u^1 = u^2$	$T(\theta, -\theta)$	$\theta_0 - \delta\theta x$
0.6	140.7	1.4	1.7	10.0	18.8	4.6	4.54	1.75	0
0.5	126.0	1.6	1.6	10.3	18.4	4.7	4.56	1.86	0
0.3	95.0	1.9	1.2	11.1	17.3	5.0	4.63	2.12	0
0.2	78.2	2.2	0.9	11.4	16.5	5.3	4.66	2.26	0
0.1	59.3	2.6	0.5	11.9	15.3	5.7	4.71	2.37	0
0	53.1	2.7	0.4	12.0	14.9	5.9	4.72	2.38	

Table 3.3: Optimal decision of the market maker for exponential MSDCs with parameter $k = 0.003$ and various values of parameter δ

δ	k	θ	θ_0	c_0	$c_{12}^1 = c_{11}^2$	$c_{11}^1 = c_{12}^2$	$u^1 = u^2$	$T(\theta, -\theta)$	$\theta_0 - \delta\theta x$
0.6	0.0042	1.4	1.7	10.0	18.8	4.6	4.54	1.75	0
0.5	0.0038	1.6	1.6	10.3	18.4	4.7	4.56	1.86	0
0.3	0.0029	1.9	1.2	11.1	17.3	5.0	4.63	2.12	0
0.2	0.0023	2.2	0.9	11.4	16.5	5.3	4.66	2.26	0
0.1	0.0018	2.6	0.5	11.9	15.3	5.7	4.71	2.37	0
0	0.0016	2.7	0.4	12.0	14.9	5.9	4.72	2.38	

Table 3.4: Optimal decisions of the market maker for an exponential MSDC with parameter $A = 100$ and various/changing values of parameter δ

risky assets, and their utility increases due to the partial realization of risk sharing. The market maker is compensated through higher traded volume on the more liquid market: the transaction fee collected is higher than in a less liquid market.

Using the MSDC optimal for the market maker, the regulatory constraint introduced will be binding for all values of regulatory parameters δ in tables 3.3 and 3.4 ($\theta_0 - \delta\theta = 0$). The equilibrium changes due to the regulatory constraint. When determining their optimal portfolios, market agents hit the constraint, and the prior optimal portfolio is not attainable any more with the optimal MSDC without regulatory constraints. Due to regulation, market makers trade a lower quantity θ of risky assets, thus the optimal MSDC of the market maker is less liquid.

If the value of the regulatory parameter is lower than the 0.1 used in the table, for example $\delta = 0.05$, the regulatory constraint is also met in the optimum determined without constraint ($\theta_0 - \delta\theta = 0, 12$). Equilibrium and market liquidity do not change.

3.3.3 Model with bid-ask spread

As a specific case of the model framework outlined in Chapter 3, I measure market liquidity as the price difference between bid and ask offers in Chapter 5 of the dissertation. The quantity of assets available at the best prices is assumed to be unlimited, i.e. the MSDC is horizontal on both the supply and demand side. I show that the optimal bid-ask spread of the market maker increases and market liquidity decreases in response to the regulatory constraint. I calculate solutions and present results for both the N- and the two-agent cases. This chapter relies on the results published in the paper Hevér (2020).

I use the notation introduced in section 3.2.1 for the model with bid-ask spread. In order to facilitate the solution, the following additional concepts are defined: vectors of bids, asks, bid-ask spreads and short and long positions. In the analytical derivation, I first introduce the bid-ask spread, and subsequently the regulatory constraint.

In case of direct trading ($t_1 = t_2 = 0$), risk sharing is complete. Due to the increase in bid-ask spread, the future payoff of the traded portfolios of market agents is gradually decreasing. Beyond a certain level, agents stop trading with each other and diversification does not occur. In the example used as illustration, the introduction of bid-ask spread leads to borrowing becoming the optimal strategy. As the spread increases, saving becomes optimal due to the dampening of trading. This is due to the fact that positions held in risk-free assets play a role in risk sharing among both periods and states of nature. If the level of bid-ask spread is set by a profit-maximizing market maker, there will be trading among market agents in equilibrium. The difference between the payoffs in future states of nature is reduced, but no complete risk sharing occurs.

I distinguish two possible outcomes of the introduction of regulatory requirement. In the first one, where the equilibrium portfolio determined without the constraint meets the new regulatory constraint, the optimal decisions of agents remain unchanged. Adding to the results with MSDCs, I examine what level of regulatory parameter δ keeps the equilibrium unchanged. As the bid-ask spread increases, even a stricter regulatory requirement (represented by higher δ) remains redundant. In the second case, where the optimal decisions of market agents change due to the introduction of the regulatory constraint, the optimization problem of the market maker also changes. Equilibrium spread increases, i.e. market liquidity decreases.

In the specific model framework of Chapter 4 of the dissertation, I confirm the conclusions reached in Chapter 3 through analytical derivation. Due to limitations on length, the present abstract solely covers the general model of Chapter 3 of the dissertation in detail.

3.4 Institutions and economic growth: the effect of transaction costs on risk sharing

Chapter 5 of the dissertation uses the general equilibrium model in the context of the literature on institutions and economic growth, examining the relationship between the introduction of transaction costs and risk sharing. Fernández and

Tamayo (2017) measure the level of development of the system of financial intermediation by market frictions (information asymmetry and transaction costs). In their model, market frictions are determined by the institutions resulting from colonization strategy, historical junctures and legal origins. Market frictions are represented by financing constraints, incomplete risk-sharing, liquidity shortages and poor market discipline, influencing the accumulation and allocation of capital. Yang and Borland (1991) construct a dynamic general equilibrium model which captures economic growth through the development of division of labor. The paper underlines that the efficiency of transactions determines the evolution of the division of labor and thereby the performance of the economy of the country.

The general equilibrium model is presented in the form of a decision tree. The two representative market agents have a choice between division of labor and self-sufficiency. Division of labor offers a higher but uncertain future payoff occurring only in one state of nature. Without transaction costs, the division of labor is complete, aggregate endowment (which can be regarded as a measure of productivity) and the consumption and utility of agents increase. Upon introduction of transaction costs into the model, agents trade fewer risky assets in equilibrium, and the elimination of the risk of stochastic endowment as natural exposure is not carried out, thus consumption and utility decrease. In extreme cases, the increase of transaction cost results in choosing self-sufficiency; therefore, the economic growth enabled by the division of labor does not take place. If the objective of the state is profit maximization, its self-interest will determine the level of transaction costs in a way that results in division of labor as the optimal choice of agents in equilibrium. Whether all or only part of the surplus from the division of labor is captured by the state depends on the level of endowments.

3.5 Conclusions

My aim has been to introduce a framework that enables the examination of the relationship between regulation and market liquidity. In the first part of the chapter, I outlined modelling options and model variants, leaving several possible avenues of research open. Subsequently, I solved a specific two-agent model and illustrated the results through examples. Although the core of the general equilibrium model builds on the book of Le Roy and Werner (2001), I have made significant additions in order to enable the examination of the research question.

I embed market liquidity into portfolio allocation decisions of agents because the purchase and sale of portfolios takes place at liquidation values determined using the MSDC. The impact of introduction of regulatory constraints on market liquidity can be examined using an endogenous MSDC. Accordingly, offers are matched by a market maker and transaction costs are maximized through setting the MSDC. The market maker decides on the parameters of the MSDC of a shape defined exogenously (linear, exponential, ladder). As market agents optimize simultaneously, an additional assumption is needed to ensure that each asset has a single seller and a single buyer in equilibrium. One solution applied is a two-agent model, another one is the use of bid-ask spread.

As a prerequisite to the introduction of a regulatory requirement setting the minimum level of risk-free asset, I distinguish the risk-free asset from risky assets. The conditions to capital market equilibrium do not apply to it: market agents can hold cash or take (0% interest) loans independently of each other. Regulatory requirements can be defined as functions of expected shortfall in assets or portfolios. Different regulatory constraints result in different equilibrium portfolios.

The second part of the chapter presents and examines the results of the two-agent model. The aims of trading of market agents are to reduce the risk arising from the uncertaintly of future payoffs, smoothing of consumption and maximization of their utility. I introduce regulatory constraint as a function of expected shortfall defined at portfolio level into the model. Parameter δ defines what portion of ES shall be covered by the regulatory requirement. It is trivial that as an additional constraint is added to the conditional maximization problem, the situation of the optimizing agent cannot improve. As long as the regulatory constraint is redundant, the equilibrium does not change. If the regulatory constraint results in a new equilibrium, trading and risk sharing are reduced, and the utility of agents drop.

Regulatory requirement also changes the problem of the optimizing market maker. If the regulatory constraint is binding at a given value of the regulatory parameter, the market maker will keep increasing transaction cost up to the point where the regulatory constraint becomes redundant. In case of an exponential MSDC, the market maker increases the variable parameter of the MSDC, and market liquidity decreases. As regulatory parameter δ increases, the market maker determines less and less liquid MSDCs. The stricter the regulatory requirement is, the larger the negative impact on market liquidity is. The model underlines the importance of considering the impact of regulation on liquidity when choosing regulatory measures.

Chapter 4

Conclusions

Liquidity is a key consideration in financial markets, especially in times of liquidity crises. For this reason, regulatory attention to this field has been on the rise for the past years.

Acerbi and Scandolo (2008) formalize liquidity risk at portfolio level to incorporate liquidity into the theory of coherent risk measures. In order to determine the value of an illiquid portfolio, their theory uses the marginal supply-demand curves of the assets within the portfolio and a liquidity policy, defining the set of acceptable portfolios. When deriving optimal execution strategies, institutional investors should take into consideration the impact of their trading and its permanent price impact (Almgren and Chriss, 2001). To handle this problem, we modified portfolio valuation under liquidity constraints by permanent price impact in Chapter 2. Our method can be applied in all cases where there is a need to calculate risk measures or capital requirements. (Csóka and Hevér, 2018)

Based on practical experience and theoretical models, it seems that regulation aiming at reducing liquidity risk and risk-taking has the side-effect of reducing market liquidity. We used a general equilibrium model with transaction costs (using MSDCs and bid-ask spreads) to formalize the opposite dynamics between regulation and liquidity. In the model, the introduction of regulatory requirements results in higher endogenous bid-ask spread (a measure of lower market liquidity), which lowers traded volumes and risk sharing among agents in equilibrium. As a result, consumption profiles remain riskier and utility levels are reduced.

Although each Chapter of the dissertation is a standalone study, they are closely interrelated through the applied methodology, uniform notation and research questions. Each Chapter starts with a comprehensive review of literature which introduces and lays the foundation for research. After the presentation of methodology developed by others, I contribute new model variants and results to literature. The results presented in Chapter 2 and in Section 3.3.3 have already been published (Hevér, 2017; Csóka and Hevér, 2018; Hevér, 2020), while Section 3.4 is under review and Chapter 3 is work in progress.

There are many further possible directions for future research. Using permanent price impact in portfolio valuation under liquidity policy, one could model nonlinear price impact or a price impact which does not shift all the points of the MSDC in a parallel way. In fact, one of the reasons why we assume that short positions must be closed is that in case of short-selling of an extreme volume of an asset, the parallel shift of its MSDC results in negative prices at which short positions could be closed, and then the optimization problem has no solution. For industry implementation, the underlying optimization problem could be further investigated, even for more general liquidity policies. Finally, our method could also be adjusted to generate liquidity in high-frequency trading, since there is also permanent price impact on a smaller timescale.

In the general equilibrium model of Chapter 3, I do not discuss the conditions to the existence and uniqueness of an equilibrium. Most model variants are only outlined; detailed analysis can be subject of further research. At the level of the general model, it is key to compare the two equilibria defined in Proposition 3.4 from the market liquidity point of view. A relevant question is how to choose between regulations at the level of assets and portfolios. It can be subject of further analysis how liquidity and the demand for assets can be influenced by asset-level regulation and different values of the regulatory parameter. Taking natural exposure into account is relevant due to its importance in theory. Regulatory options based on expected shortfall can be contrasted with ones introducing collateral requirements.

Bibliography

Csóka P. and Hevér J. (2018), Portfolio valuation under liquidity constraints with permanent price impact, *Finance Research Letters*, 26, 235–241. https://doi.org/10.1016/j.frl.2018.02.019

Hevér J. (2020), A piaci likviditás és a szabályozás kapcsolatának vizsgálata általános egyensúlyelméleti modellkeretben, *Közgazdasági Szemle*, LXVII (7-8), 708–733. https://doi.org/10.18414/ksz.2020.7-8.708

Hevér J. (2015), Yakov Amihud-Haim Mendelson-Lasse Heje Pedersen: Market Liquidity. Asset Pricing, Risk and Crises, Cambridge University Press, Cambridge, 2013, 289 o., Könyvismertetés, *Közgazdasági Szemle*, LXII (7-8), 871–875. DOI:10.18414/KSZ.2015.9.871

Hevér J. (2017), A likviditás és a permanens árhatás szerepe a portfólióértékelésben, *Közgazdasági Szemle* LXIV (6), 594–611. https://doi.org/10.18414/ksz.2017.6.594

Acerbi, C., and G. Scandolo (2008), Liquidity risk theory and coherent measures of risk, *Quantitative Finance*, 8 (7), 681–692. https://dx.doi.org/10.1080/14697680802373975

Acharya, V. V. and L. H. Pedersen (2005), Asset pricing with liquidity risk, *Journal of Financial Economics*, 77 (2), 375–410. https://doi.org/10.1016/j.jfineco.2004.06.007

Almgren, R. (2003), Optimal execution with nonlinear impact functions and trading-enhanced risk, *Applied Mathematical Finance*, 10 (1), 1–18. https://doi.org/10.1080/135048602100056

Almgren, R., C. Thum, E. Hauptmann and H. Li (2005), Equity market impact, *Quantitative Trading*, 10, 21–28.

Almgren, R. and N. Chriss (2001), Optimal execution of portfolio transactions, *Journal of Risk*, 3 (2), 5–39. https://doi.org/10.21314/jor.2001.041

Alzahrani A. A., A. Gregoriou and R. Hudson (2012), Can market frictions really explain the price impact asymmetry of block trades? Evidence from the Saudi Stock Market, *Emerging Markets Review*, 13, 202–209.
https://doi.org/10.1016/j.ememar.2012.02.003

Amihud, Y. (2002), Illiquidity and stock returns cross-section and time-series effects, *Journal of Financial Markets*, 5 (1), 31–56.
https://doi.org/10.1016/s1386-4181(01)00024-6

Amihud, Y., H. Mendelson and L. H. Pedersen (2013), Market Liquidity: Asset Pricing, Risk and Crises, *Cambridge University Press*.

Amihud, Y., H. Mendelson and R. Wood (1990), Liquidity and the 1987 Stock Market Crash, *Journal of Portfolio Management*, 16 (3), 65–69.
https://doi.org/10.3905/jpm.1990.409268

Basel Committee on Banking Supervision (2013), Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools, *Bank for International Settlements*, 2013 január.

Begenau, J. (2019), Capital Requirements, Risk Choice, and Liquidity Provision in a Business-Cycle Model, *Journal of Financial Economics*, 136 (2), 355–378.
https://doi.org/10.1016/j.jfineco.2019.10.004

Bigio, S. (2015), Endogenous Liquidity and the Business Cycle, *American Economic Review*, 105 (6), 1883–1927.
https://doi.org/10.1257/aer.20110035

Bank for International Settlements (1999), Market liquidity: Research findings and selected policy implications, *CGFS Papers*, 11, 1999. május 1.
https://www.bis.org/publ/cgfs11.htm

Boyd, S. and L. Vandenberghe (2004), Convex Optimization, *Cambridge University Press*.

Brunnermeier, M. K. and L. H. Pedersen (2008), Market Liquidity and Funding Liquidity, *Review of Financial Studies*, 22 (6), 2201–2238.
https://doi.org/10.1093/rfs/hhn098

Castellano, R. and R. Cerqueti (2011), The optimal bid/ask spread in a specialist System, *Economic Modelling*, 28 (5), 2247–2253.
https://doi.org/10.1016/j.econmod.2011.06.019

Çetin, U., R.A. Jarrow and P. Protter (2004), Liquidity risk and arbitrage pricing theory, *Finance and Stochastics*, 8, 311–341.

Csóka P. (2003), Koherens kockázatmérés és tőkeallokáció, *Közgazdasági Szemle*, L (10), 855–880.

Csóka P. (2017), Fair Risk Allocation in Illiquid Markets, *Finance Research Letters*, 21, 228–234.
https://doi.org/10.1016/j.frl.2016.11.007

Csóka P. and P.J.J. Herings (2014), Risk Allocation under Liquidity Constraints, *Journal of Banking and Finance*, 49, 1–9.
https://doi.org/10.1016/j.jbankfin.2014.08.017

Csóka P., P. J. J. Herings and L. Á. Kóczy (2007), Coherent Measures of Risk from a General Equilibrium Perspective, *Journal of Banking and Finance*, 31 (8), 2517–2534.
https://doi.org/10.1016/j.jbankfin.2006.10.026

Csóka P., P. J. J. Herings and L. Á. Kóczy (2009), Stable allocations of risk, *Games and Economic Behavior*, 67 (1), 266–276.
https://doi.org/10.1016/j.geb.2008.11.001

De Nicolò, G., A. Gamba and M. Lucchetta (2014), Microprudential Regulation in a Dynamic Model of Banking, *The Review of Financial Studies*, 27 (7), 2097–2138.
https://doi.org/10.1093/rfs/hhu022

Erb T. and Havran D. (2015), Mit veszítünk a piaci súrlódásokkal? A pénzügyi piacok mikrostruktúrája *Közgazdasági Szemle*, LXII (3), 229–262.

Faias, M. and J. Luque (2018), Developers, Wall Street and the Taxmen: A Theory of Real Estate Development

Fernández, A. and C. E. Tamayo (2017), From Institutions to Financial Development and Growth: What Are the Links?, *Journal of Economic Surveys*, 31 (1), 17-57.
https://doi.org/10.1111/joes.12132

Gromb D. and D. Vayanos (2010), A model of financial market liquidity based on intermediary capital, *Journal of the European Economic Association*, 8 (2/3), 456–466.
https://doi.org/10.1111/j.1542-4774.2010.tb00516.x

Han, J., Doojin Ryu, Doowon Ryu and R. I. Webb (2016), The price impact of futures trades and their intraday seasonality, *Emerging Markets Review*, 26, 80–98.
https://doi.org/10.1016/j.ememar.2016.01.002

Havran D. and Szűcs B. Á. (2016), Árjegyzői viselkedés belső kockázatmegosztás mellett, *Szigma*, XLVII (1–2).

Havran D. and Váradi K. (2015), Price impact and the recovery of the limit order book: Why should we care about informed liquidity providers? *Magyar Tudományos Akadémia Közgazdasági and Regionális Kutatóközpont Műhelytanulmányok*, (MT-DP 1540), ISBN 978-615-5594-04-5.

Huberman, G. and W. Stanzl (2004), Price manipulation and quasi-arbitrage, *Econometrica*, 72 (4), 1247–1275.
https://doi.org/10.1111/j.1468-0262.2004.00531.x

International Organization of Securities Commissions (2018), Recommendations for Liquidity Risk Management for Collective Investment Schemes, FR01/2018 február.

International Organization of Securities Commissions (2019), Liquidity in Corporate Bond Markets Under Stressed Condition, FR10/2019 június.

International Organization of Securities Commissions (2019), Sustainable finance in emerging markets and the role of securities regulators, CR01/2019 február.

Jarrow, R. A. and P. Protter (2005), Liquidity risk and risk measure computation, *Review of Futures Markets*, 11, 27–39.

Kitamura, Y. (2016), The probability of informed trading measured with priceimpact, price reversal, and volatility, *Journal of International Financial Markets, Institutions & Money*, 42, 77–90.
https://doi.org/10.1016/j.intfin.2016.02.001

Kiyotaki, N. and J. Moore (2019), Liquidity, Business Cycles, and Monetary Policy, *Journal of Political Economy*, 127 (6), 2926-2966.
https://doi.org/10.1086/701891

Le Roy, S. F. and J. Werner (2001), Principles of Financial Economics, *Cambridge University Press*.
https://doi.org/10.1017/cbo9781139162272

Mitchell, M., L. H. Pedersen and T. Pulvino (2007), Slow Moving Capital, *American Economic Review*, 97 (2), 215–220.
https://doi.org/10.1257/aer.97.2.215

Sommer, K. and P. Sullivan (2018), Implications of US Tax Policy for House Prices, Rents, and Homeownership *American Economic Review*, 108 (2), 241–274.
https://doi.org/10.1257/aer.20141751

Tian, Y., R. Rood and C. W. Oosterlee (2013), Efficient Portfolio Valuation Incorporating Liquidity Risk, *Quantitative Finance*, 13 (10), 1575–1586.
https://doi.org/10.1080/14697688.2013.779013

Yang, X. and J. Borland (1991), A Microeconomic Mechanism for Economic Growth, *Journal of Political Economy*, 99 (3), 460–482.
https://doi.org/10.1086/261762